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Determining High-Payoff Targets in Combat Models Via Competing Poisson Processes with Interruptions

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Abstract:

We present a method for generating a high-payoff target list for tactical-level combat engagements involving agents represented primarily by artillery units. The method formulates target selection as an optimal firepower allocation problem across different types of agents. To this end, we propose a framework that combines generalized interacting Poisson processes with non-Markovian interruptions to model battlefield agents, together with a set of external discrete rules that govern their interactions. Focusing on assault operations in which the attacker has a numerical advantage, we use numerical simulations to demonstrate the existence of a non-trivial optimal targeting strategy for the defender. This strategy simultaneously maximizes the area in which enemy advance is denied while minimizing the defender's losses.

Keywords:

high-payoff target, targeting, Poisson process, Monte Carlo

1 Introduction

The armed conflict aims to achieve strategic (operational) goals (objectives) by destroying enemy targets, capturing them, and creating decisive conditions under which the enemy cannot achieve its intentions (objectives). The main content of armed conflict is the process of destroying enemy targets with weapons (rockets, shells, missiles, electronic warfare, etc.) from various components (air, sea, land).

To conduct armed conflict, the enemy forms a Joint Task Force (JTF) to achieve its military objectives. Such a JTF is a temporary, multi-service, or multi-national military formation established to achieve a specific, limited objective which may in-

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clude a large number of both individual and group entities that interact with each other. These entities are usually grouped according to their purpose into categories of warfighting functions [1].

Each warfighting function is represented by a set of interconnected and interacting objects which perform that function to the enemy and friendly JTF (Fig. 1) [1].

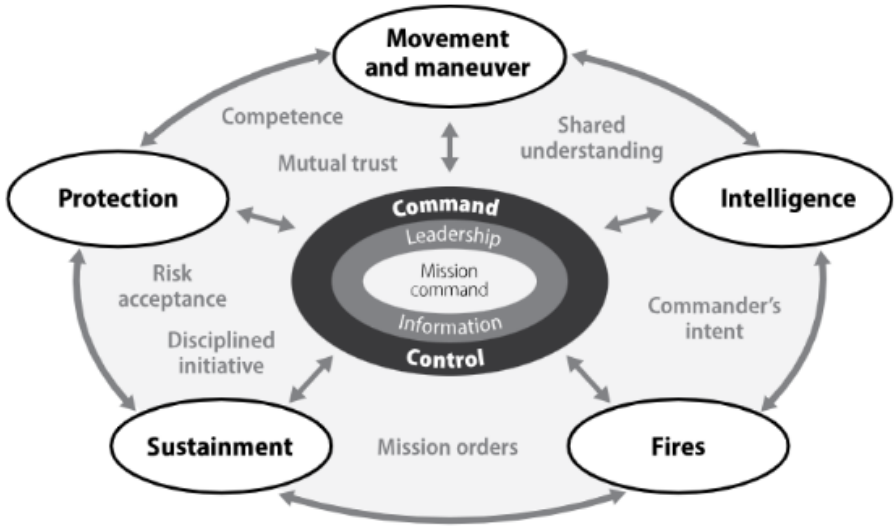


Fig. 1 Warfighting functions [1]

Each warfighting function (subsystem) can include hundreds of such objects. In turn, warfighting functions, taken together and interacting with each other, form a complex military system that enables enemy and friendly JTF to perform their tasks and achieve specific military objectives. Viewing the enemy's JTF as a complex military system makes it possible to predict how this system might function. It also makes it possible to determine which specific objects within this combat function (subsystem) need to be targeted to neutralize the subsystem's ability to perform its main function for the enemy. After all, destroying all enemy objects requires both a large amount of time and a large number of destructive resources. This necessitates the formation of a task to determine a high-payoff targets list and their prioritization, which will determine the distribution of the share of destructive resources among the corresponding categories of enemy objects, the sequence of their destruction, etc.

The best way to prioritize enemy targets is to simulate the most intense (decisive) phase of the operation (battle).

To simulate a military operation, one has to calculate statistically feasible parameters of a specific agent (e.g. hitting probabilities, areas of effect) [2] and to describe interactions in the entire system of all agents (objects/targets) present on the battlefield. The framework of the theory of operations is typically used for models targeting the second task [2, 3]. In the case of an averaged description of a large number of combat units, systems of Lanchester differential equations [3, 4] and the method of averages in the theory of continuous-time Markov chains [3–5] are classical references. An alternative direction typically considers each combat unit separately and,

besides the above-mentioned continuous-time Markov chains, includes statistical models [6]. Such models primarily rely on numerical simulations.

In [7] in the spirit of Lanchester theory, the authors derive, for the first time, force-parity equations for various combinations of the effects and obtain general implications and trade-offs. They showed that more units and better weapons (higher attrition rate) are preferred over improved targeting capability and relaxed deployment restrictions unless these are poor. However, when facing aimed fire and unable to deploy more than half of one's force, it is better to be able to deploy more existing units than to have either additional reserve units or the same increase in attrition effectiveness. Likewise, more relaxed deployment constraints are preferred over enhanced loss-tolerance when initial reserves are greater than the force level at which withdrawal occurs.

In [8] a detailed description is provided of the solution to an example of a stochastic model of a "disorganized battle" for use in teaching of specific sections of applied mathematics and for supplementing traditional teaching methodology with the aim of increasing the motivation of cadets and students at military universities to study mathematical disciplines.

The proposed example examines the system states, constructs the corresponding state-transition graph, derives the transition intensities between states, and formulates the Kolmogorov system of differential equations. It also outlines the solution method and presents the resulting solutions. Finally, formulas are provided for calculating the expected final number of combat units on each side and the probability of victory for each side.

Article [9] describes a game-theoretic model that can be used during the planning and decision-making stages regarding the deployment of military units of the State Border Guard Service of Ukraine. The focus of the model is the study of the "victory function" in combat (special) operations based on the parameters of the combat (moral and technological) advantage of the first side over the second, the balance of forces between the sides, and the scale of combat (special) operations. In addition, with the "victory function", the model allows for solving the inverse problem given a specified probability of victory (degree of task completion), to determine the required balance of forces (advantage over the enemy). The presented theoretical game model is fundamental. With its help, it is possible to justify individual elements of the battle plan (special operations), as well as to predict their course and outcome.

In article [10], a mathematical model of the problem has been constructed and implemented, based on a combination of the dynamic programming method with the solution of Lanchester's systems of differential equations of battle dynamics with certain initial conditions at each of the three stages of the battle. With the help of a numerical experiment, the admissibility of the parameters of the optimization problem (the number of combat units of side A, which are appropriately distributed, transferred from area to area or from the reserve at each stage of the battle) is analyzed. The developed Python program allows, for any initial data, to give an answer to the optimal allocation of resources of party A, including from the reserve, at three stages of the battle, and to calculate the corresponding largest enemy losses at a given time or to give an answer that there are no valid values of the problem parameters, i.e. the problem has no solution for certain initial data.

Article [11] presents an analytical model of combat between opposing forces, the dynamics of which are described by a system of linear differential equations. Explicit solutions to the system of differential equations are derived for the given initial condi-

tions. Calculation formulas have been derived to determine the average numbers of combat units of the opposing sides remaining at any given moment during the battle, and a functional relationship between their average numbers has been established. An advantage coefficient has been introduced, which makes it possible to assess the superiority of one side over the other and predict the outcome of the battle. A practical example of the model's operation is considered, and the dynamics of losses in the average numbers of combat assets of each side at an arbitrary time during the battle are presented, both for simultaneous fire and taking into account preemptive fire.

The aim of the article [12] was to determine the long-range assets effectivity dependence on the method of their employment. To achieve this goal, the mathematical model of combat employment of long-range assets based on the theory of Markov processes with continuous time and discrete states was improved. In contrast to the existing ones, the improved mathematical model includes a state in which the asset launches a second missile strike from the firing position from which the previous missile strike was launched. This enables the modeling of a combat employment method for the assets that increases the number of missile strikes these assets can execute within a given period. This is the essence of long-range assets effectivity employment.

The article [13] presents an experiment performed in the MasaSword version 6.17.1 environment in evaluating the effectiveness of an attack on ground targets within a close air support mission to an opponent on the march or in a defensive position, or in an attack. The ground unit commander has performed an analysis of available firepower based on aerial reconnaissance and he decided to use the allocated aircraft with different types of ammunition. The key element is the effect of selected weapons and ammunition on the enemy's targets according to the required level of target elimination. The article also evaluates the possibilities offered using MasaSword in the decision-making process of the commander when making decisions.

Additional relevant studies include [14] (a stochastic combat model of an infantry engagement using real-life geodata), [15] (accelerated simulation runs for a specific type of wargames), [16] (a stochastic model of artillery fire), among others.

The list above includes only a small number of scientific works devoted to the study of combat simulation. This underscores the high relevance of this area of research.

These studies conduct independent research; however, they do not focus on prioritizing enemy targets whose destruction would lead to a battle outcome in which the enemy could capture only a minimal portion of the territory defended by our troops.

The framework developed in this paper combines tools from reliability theory and practically important characteristic properties of simulation models, using an agent-based structure of a discrete battlefield. Agents run separate distorted inhomogeneous and interacting Poisson-like processes that mimic the flow of commands (for instance, fire missions in the case of artillery pieces). Recommendations on better distribution of firepower can be obtained by analyzing the numerical results.

The rest of the introduction is devoted to a survey of the mathematical aspects of combat models relevant to our study and a discussion of the differentiating features of this framework besides the decision-making procedure.

Poisson processes are a standard tool in reliability theory [17]. However, practice often requires distorting them by adding delays or interruptions. This results in the loss of the Markov property, which makes it impossible to deploy standard results and necessitates additional calculations [18].

Such models (including those obtained from Poisson processes by introducing events of fixed length instead of instantaneous ones) were also used in military science. In particular, [19] uses statistical data to introduce events with random length into a system of Poisson process to model the historical processes of changes in the composition of naval forces, while [20] studies the problem of distributing objects between flows of tasks to simulate military operations. Non-stationary Poisson processes are used in [21] to produce more flexible Lanchester models by averaging.

However, while Lanchester-type models concentrate on capturing the averaged behavior, the model considered in the paper belongs to the wide class of numerically oriented models and thus allows one to obtain a set of random battlefield snippets for further analysis, possibly including the investigation of rare or atypical events or scenarios.

The model proposed in the paper is derived from agent-based ones and differs from previous results. It uses Poisson processes to describe the flow of Markov events as a basis and extends this model in two directions.

Firstly, we add competition, intervals of inactivity and heterogeneity of activity intensities (in our case, attempts to execute fire missions etc.), which ruin the Markovian nature of pure Poisson processes and produce an inhomogeneous and heterogeneous flow of events.

Secondly, by supplying explicit contextual rules to describe interactions between objects, we can simulate the presence of various classes of objects, including command and control objects.

One specific drawback of such extensions is that obtaining explicit expressions for the distributions present in the model, except for significantly simplified cases, is typically unfeasible.

Compared to semi-Markov models with embedded proper Markov chains (e.g. [22]), the proposed framework discards the set of fixed distributions used to fully define such models, sacrificing the use of analytical expressions, and replaces them with a selection of rules of an arbitrary nature, thus focusing on pure simulation techniques. The authors believe this allows for the inclusion of more varied interaction mechanisms and more distinct types of agents than more predefined and thus more rigid models admit. In particular, transitions between states no longer follow a Markov chain and can depend on a number of external factors and actions of other agents in a nonlinear way, which is highly reminiscent of the approach of the celebrated JCATS model [23]. Numerical characteristics of the agents and the corresponding flows of events can be obtained statistically or described by a user explicitly as a set of instructions. One can add agents of significantly different nature, in particular, by introducing pools of available resources and critical hubs, including virtual ones.

However, the primary objective of this research is to develop a general recommendation procedure for situations in which the nature of command flows and instructions cannot be fully and accurately characterized, either within individual entities or across the battlefield as a whole. Consequently, randomized flows described by inhomogeneous intensities are used as the primary modeling framework, at least for engagements involving heavy artillery or unmanned aerial vehicles (UAVs). This is especially important in the case of UAVs since it might be impossible to properly describe their behavior at the level of a singular entity given the volume of such objects on the modern battlefield.

The results of the recommendation procedure can be used to derive weights for the classes of agents for further use in target prioritization algorithms. At the same

time, such algorithms can be embedded directly into the model after some modifications (such as introducing rounds), possibly to test available assumptions regarding the weight distribution. Still, one should consider the framework presented here as a source of such weights.

2 The Framework

2.1 Competing Poisson Processes as a Model of Combat Units

For the purposes of this study, a hypothetical scenario of combat engagement between two opposing units was developed. According to the scenario, Side A launches an attack on the opposing formation of Side B. Side A's objective is to capture the positions defended by Side B. Side B's objective is, accordingly, to repel the attack and prevent Side A from capturing its positions. The formations of Sides A and B have the same structure, specifically consisting of mechanized units from Side A that carry out the attack and capture of Side B's positions (while Side B defends its positions). To carry out their missions, the mechanized units receive artillery fire support. The mechanized and artillery units are commanded from command posts – headquarters (HQs in short). To demonstrate how the model is constructed in a specific situation, let us first assume that only individual artillery pieces are present on the battlefield, which will carry out fire missions to provide fire support for their mechanized units. Each artillery piece in the model is treated as a separate autonomous agent. This assumption stems from the experience of the Russia-Ukraine war, which shows that, for reasons of survivability, artillery units do not carry out fire missions as part of artillery batteries (battalions), as had been stipulated in operational guidelines prior to the start of the Russian invasion [24].

We consider the case of a tactical level engagement, which motivates a rather small number of units in a synthetic example in Section 3. One can consult [25] for the description of changes in artillery deployment rules on a tactical level for the Russian forces in particular and [26] for a simulation of a tactical level with a comparable number of agents.

Each agent enters the engagement by starting a separate independent Poisson process (PP) whose intensity λ equals the intensity of the calls for fire (fire missions) administered by an external source to the agent in question when the agent is left undisturbed by enemy actions. In particular, the mean number of fire missions issued for this specific unit in time t, provided the unit is safe from any retaliation actions, is equal to λt.

Events of the corresponding PPs are the moments when fire missions are being executed. The duration of the execution of a fire mission is neglected.

To introduce interaction between agents, we need to modify PPs as follows. We start by forming the queue of all ordered events for all PPs on the battlefield and finding the time τ of the earliest event. An additional Bernoulli random variable is rolled to determine the success of this specific fire mission. Here success means hitting the target successfully and destroying it permanently or only suppressing (neutralizing) it for a fixed known period of time, but, depending on the actual setup of the model, it might also include damaging the target without suppressing it etc.

In this study, we assume only suppressing and destruction are possible and the affected unit fully restores its functionality over time unless destroyed. The target is

chosen randomly among all available targets (unsuppressed and non-destroyed enemy agents). We address proper and more advanced target acquisition strategies (instead of random selection as in this section) in Section 2.3.

If the fire mission attempted at time τ was unsuccessful, we proceed to the next event and repeat the entire procedure.

If the fire mission at time τ was successful, then the targeted unit is either fully destroyed and leaves the battlefield permanently or suppressed and then has to spend some time s to recover. E.g. the interval of inactivity s might be calculated as the sum of the time the personnel stay in cover during incoming artillery fire (e.g. 15 min) and the displacement time required for a move to an uncompromised firing position (e.g. a movement rate of 4 min per 1 km at night time etc.).

The corresponding PP of the target is halted at time τ and then restarted after time s, with the same intensity as before, after which the query of all events is updated and is then examined for the next earliest event again.

The modified PPs produced by this recursive procedure can be called competing Poisson processes with interruptions (CPPwI). Such processes consist of intervals of inactivity and intervals when a proper PP is run.

Such processes are no longer Markov. At the same time, one could suggest introducing exponentially distributed intervals of inactivity to preserve the Markovian property of the original system. Unfortunately, realistic waiting times in practice rarely follow an exponential distribution. The reasoning provided above shows that the time a unit spends recovering cannot be arbitrarily small. It must account for the time required for relocation and is also bounded from above.

Additional shifts and interruptions, possibly based on actions of other agents or external forces, can be added into such CPPwI. Such examples will be given later.

For simplicity, we do not consider cases when agents are damaged and always assume their functionality is restored fully unless they were destroyed. Such a decision is driven by an attempt to avoid calculating potential damage dealt by different types of means of destruction, which is a non-trivial question in its own. However, the proposed framework easily accommodates such more advanced rules. The same applies to the time of execution of a fire mission and similar parameters.

We will apply the CPPwIs framework to a scenario where attackers advance assault ground infantry units under the cover of suppressive fire directed at the defender's artillery.

After specifying the full model, the target acquisition process can utilize the simulation results (see Section 2.5 for the description of the statistical procedure and Section 3 for a synthetic example) to ultimately determine high-payoff targets.

2.2 Introducing Other Classes of Agents

To introduce other types of agents (units) into the model, one must specify the way new agents operate (by running new competing PPs or by having alternative internal clocks to produce events etc.). All new events are added into the global queue and are then processed in chronological order according to a collection of rules describing possible interactions between agents.

We start with adding agents that represent the existence of HQs whose purpose is to generate calls for fire by analyzing the situational data and available intelligence. External unobserved reserve command and control channels are present so disabling or suppressing a HQ does not fully prevent artillery units attached to it from acting while

still reducing the effectiveness of such units by decreasing the intensity of calls for fire as reserve channels are present.

Let us first consider only one side, the attackers. We observe artillery pieces of different types so Types 1 and 2 can represent self-propelled and towed howitzers, Type 3 can represent mortars and so on. In what follows we associate superscript 1 with the attackers and superscript 0 with the defenders.

The set of all such agents is denoted by $A^{(1)}$. The set of tactical-level HQs is denoted by H. Let $A(h_i) \subset A^{(1)}$ be the set of artillery pieces attached to an HQ h_i. Each artillery piece is attached to exactly one HQ. The time of recovery of agent a after getting suppressed is denoted by $\tau_{\text{inact}}(a)$.

For each HQ h_i, instead of running a proper CPPwI, we introduce a different process (we will call it a degenerate generalized CPPwI to avoid bloated terminology) as follows. No events are generated by h_i until it is successfully targeted by an enemy unit at time t. Then two rules are used:

- If the HQ h_i is destroyed, then the CPPwI for each agent $a \in A(h_i)$ is restarted with intensity $\nu(h_i) \cdot \lambda(a)$, where $\lambda(a)$ is the original value and $\nu(h_i)$ is a fixed known coefficient. If the CPPwI of such an agent is inactive at time t, its current reactivation time does not change.
- If the HQ h_i is only suppressed, then a single event is created at time $t + \tau_{\text{inact}}(h_i)$, and then the CPPwI for each agent $a \in A(h_i)$ is restarted with intensity $\nu(h_i) \cdot \lambda(a)$ exactly as in the previous case. At time $t + \tau_{\text{inact}}(h_i)$ all such CPPwIs are modified again by restoring their intensities to the initial values.

To fully specify the model, we assume that, in addition to agents described previously, the following agents are also present on the battlefield:

- attackers move infantry units (the set of which is denoted by B),
- defenders deploy artillery units (the set of which is denoted by $A^{(0)}$) of different types exactly as in the case of the attackers (see Section 2.2).

We assume the attackers are not aware of the precise positions of enemy units initially and are thus incapable of attacking them immediately.

The duration of the fight is at most T. For each agent a, including HQs, we introduce the following parameters:

- $P(a;t), t \in [0,T]$, is the process that generates events for the agent a. It is a proper CPPwI for artillery pieces and a one-point at most process for HQs.
- $Q(a)$ is the set of all values produced by process $P(a;\cdot)$.
- $\lambda(a)$ is the current intensity of $P(a;\cdot)$ (if applicable, e.g. $\lambda(a)$ is not defined for HQs).
- $\tau_{\text{inact}}(a)$ is a (possibly random) time of recovery if a gets suppressed.
- $\tau_{\text{rec}}(a)$ is a random time at which the enemy obtains a possibility to target the agent a (used only for $a \in A^{(0)}$). The simplest way to introduce such times is to take them uniformly distributed on $[0, \tau_{\text{rec,max}}]$, where $\tau_{\text{rec,max}}$ is shared between all agents of the same type. Alternatively, we can set $\tau_{\text{rec}}(a) = \min\{\tau_{\max}, \min(Q(a))\}$, where $\tau_{\max}$ is the maximal time needed for the attackers to identify agents in $A^{(0)}$, and $\min(Q(a))$ is the time an agent discloses itself by activity.
- $\nu(h)$ is the coefficient governing the decrease of intensities of all agents in $A(h)$ as described in Section 2.2 (used only for $h \in H$).

- $\tau_{\text{supp}}(b)$ is the time an infantry unit b stays in place without advancing further after being hit (used only for $b \in B$).

Parameters and data required to determine $\tau_{\text{rec}}(a)$ and hitting probabilities should be based on statistical field evidence and are beyond the scope of the present research. In general, various outcomes, including partial destruction, are possible but we consider only a simplified scenario when only a hit and a miss are possible. On hit, a target is either suppressed or destroyed.

CPPwIs for agents in B (infantry units) are trivial and simply encode that such agents are suppressed for fixed periods of time if successfully attacked. We do not discuss precise values of damage inflicted by a successful hit, but one can consider 40 min intervals for armored units, which corresponds to light losses up to 30 %, for example. It is also possible to introduce cumulative gains in suppression times under consecutive successful attacks and so on. Such a model of this specific combat situation should be considered as an illustration of the general framework proposed in the paper.

To produce more advanced combat models, one would introduce other types of agents, the corresponding generalized CPPwIs and sets of rules that govern how different agents interact with each other in a similar manner, so such generalized CPPwIs generate separate events that are added to the joint queue. When processed, each event uses explicit rules to affect the CPPwIs of other agents, possibly stopping them, restarting with altered intensities and so on.

The model offers a certain degree of flexibility in terms of the decentralized picture of the combat. While separate objects of the same class run initially independent Poisson processes of events to simulate the possibility of independent decision-making processes and actions of separate units, the command-and-control flows are generated by separate command units (HQs) whose states affect all the units attached to specific HQs. The reserve channels guarantee that some form of independent target acquisition is still available to units to mimic modern surveillance options, and the degree of centralization is controlled in particular via $\nu(h)$. On the other hand, to consider a fully centralized picture of the combat, one can reduce the independence between CPPwIs either by including correlations or even by running the same underlying process for all units within a class, for instance. Removing HQs would lead, conversely, to a fully decentralized situation. The example in Section 3 uses the format described in this section.

2.3 Target Acquisition Strategies and Priorities

Proper distribution of resources is one of the largest and most complex tasks on the battlefield. Due to the statistical nature of our framework, we employ a randomized targeting procedure to emulate this process. We will show how it can be used to determine an optimal strategy with respect to a cost function.

The strategy will be different for those who are in defense and those who are in offence.

Let p_{inf}, p_{hq}, $p_{\text{art}} \in [0,1]$ be such that $p_{\text{art}} + p_{\text{inf}} + p_{\text{hq}} = 1$. For each fire mission for the defenders, an active agent a is targeting a random active agent in H with probability p_{hq}, a random active agent in $A^{(1)}$ with probability p_{art} and a random infantry unit with probability p_{inf}. If no active agent in H is present, a chooses between a random

agent in $A^{(1)}$ and a random infantry unit with probability $\frac{p_{\text{art}}}{1-p_{\text{hq}}}$ and $\frac{p_{\text{inf}}}{1-p_{\text{hq}}}$ respectively and so on.

Consider an ordering of $A^{(0)}$ according to some external rule. E.g., self-propelled artillery can be given lower priorities than towed artillery, guns can be given lower numbers than mortars and so on. The attackers always concentrate fire on an active enemy agent with the lowest number. If the times $\tau_{\text{rec}}(a), a \in A^{(0)}$, are given, the attackers can target at time t only active agents satisfying $\tau_{\text{rec}}(a) < t$.

In general, the defenders might also be allowed to redistribute ammunition, which directly affects probabilities of hitting enemy objects. Introducing such new aspects into the model (including logistics hubs and important terrain features as separate agents) and using more complex strategies (e.g. targeting critical logistics hubs) would allow one to develop a more realistic representation of the battlefield.

Due to the continuous command flows of fire missions, discrete combinatorial weapon-target assignment algorithms cannot be integrated into the model directly. One solution is to consider a turn-based modification of the presented framework. Such a model will be discussed in a separate paper.

2.4 Cost Functions and Evaluation of Results

The main goal of such combat models is to develop recommendations and to propose optimal strategies and decisions. In our case, this means finding the best weights $\left(p_{\text{hq}}^{*}, p_{\text{inf}}^{*}\right) = \text{argmin}_{p_{\text{inf}}, p_{\text{hq}}} L$ for a specific cost function L under the assumption that the targeting strategy of Section 2.3 is used. In relation to the real battlefield, this is equivalent to determining the targets with a better payoff.

We introduce a selection of new cost functions to illustrate possibilities. To adhere to standard naming conventions, all suggested functions are non-positive.

Recalling our assumptions regarding the superior firepower of the attackers, we are primarily interested in minimizing the area of the territory taken by the enemy infantry units.

Note that it is not a priori clear what values of $p_{\text{inf}}, p_{\text{hq}}$ are optimal: concentrating fire on enemy HQs and artillery agents reduces their firepower and thus expected losses of the defenders while simultaneously draining resources used to attack enemy infantry directly.

In this case, the simplest cost function L_1 to be minimized can be built as

$$L_1 = -\sum_{b \in B} v(b) \max\left\{0, T - \tau_{suppr}(b) S(b)\right\}$$

where $v(b)$ is the average speed of an infantry unit b, $\tau_{suppr}(b)$ is the time the unit b stays in cover under artillery fire and $S(b)$ is the average number of successful attacks on b. Under the natural assumption of infantry units operating in separate stripes of comparable width, the cost function L_1 calculates how the area the enemy infantry units were expected to capture was reduced.

Including logistical aspects as costs of ammunition leads to a modified cost function

$$L_2 = \frac{L_1}{\sum_{a \in A^{(0)}} x(a) n(a)}$$

where $x(a)$ is the cost of a fire mission of the agent a (essentially, average amount of ammunition used) and $n(a)$ is the number of fire missions executed by an agent a, so the denominator is the total expected value of resources used to reduce the enemy advancement (measured by L_1).

The next loss function evaluates the effectiveness of the suppression of enemy infantry versus expected losses:

$$L_3 = \frac{L_1}{\sum_{a \in A^{(1)}} n(a)}$$

where the denominator is proportional to the absolute losses of the defenders.

Due to the statistical nature of the framework, one is not limited to cost functions expressible in terms of averages only. Let us consider an example of a more involved compound cost function. We define global success as a situation when enemy infantry is unable to move for more than 50 % of the time with probability 95 %. Then one can try to minimize a cost function provided the situation of global success is achieved.

2.5 Monte Carlo Simulations

An important feature of the model of competing and interacting agents is the loss of the Markov property, which heavily diminishes the possibility of describing analytical properties of such a model. Even though one can still try to calculate distributions of important statistics in some basic and simplified cases, we suggest using a universal approach of Monte Carlo simulations to study the proposed model instead. What is even more important, using Monte Carlo simulations allows one to easily modify the model and include elaborate rules while preserving a possibility to calculate complex statistics and parameters.

This section outlines a statistical analysis of the simulation results in general and suggests methods for obtaining statistically valid conclusions.

Let us briefly recall the setting (details are well known, see e.g. [27], while the authors used the method successfully in [28]). Software implementation of the suggested algorithm run N computer simulations as follows.

For the initialization stage:

- start CPPwIs as independent Poisson processes for all agents in $A^{(0)}$, $A^{(1)}$ and as empty processes for agents in H,
- calculate random times $\tau_{\text{rec}}(a)$, $a \in A^{(0)}$ according to the rules in Section 2.2.

Then until $t > T$ repeat the following:

- find $a^* = \operatorname{argmin}_{a \in A^{(0)} \cup A^{(1)} \cup H} Q(a)$ (a^* is the next agent to act),
- set $t = \min Q(a^*)$,
- use rules from Sections 2.1-2.2 and the targeting strategy described in Section 2.3 to process the event at time t,
- remove the event at time t from $Q(a^*)$.

One observes N values $\eta_1, \ldots, \eta_N$ of some statistics (say, the number of successful attacks on infantry units), and by the strong law of large numbers the expected

value of η is approximately equal to $\tilde{\eta} = \frac{1}{N}\sum_{i\le i}^{N}\eta_i$. At the same time, by the central limit theorem

$$\mathrm{P}\left(\left|\tilde{\eta}-\eta\right| \ge \varepsilon\right) \approx \mathrm{P}\left(\left|N\left(0,1\right)\right| \ge \frac{\sqrt{N}}{\sqrt{\mathrm{Var}\left(\eta\right)}}\varepsilon\right)$$

To guarantee the accuracy and reliability of the conclusions based on the Monte Carlo method one needs to find confidence intervals for η in terms of *N*, that is, to determine a sufficient number of simulations. The corresponding methods can be found in many sources. In particular, such discussion of the Berry-Esseen inequality can be found in [27]. We only recall that, in general, it might be sufficient to have an upper bound for mean value $\mathrm{E}\left|\eta-\mathrm{E}\left(\eta\right)\right|^3$ and a lower bound on variance $\mathrm{Var}\left(\eta\right)$ since, by the Berry-Esseen inequality, the uniform error of replacing the distribution of $\frac{\tilde{\eta}-\eta}{\sqrt{\mathrm{Var}\left(\eta\right)}}$ with the standard Gaussian distribution is bounded by $\frac{\mathrm{E}\left|\eta-\mathrm{E}\left(\eta\right)\right|^3}{2\mathrm{Var}(\eta)^{\frac{3}{2}}\sqrt{N}}$. In general, it is hard to propose a universal approach here given the complex nature of interactions between agents.

Alternatively, one can estimate the aforementioned moments by using the results of the Monte Carlo simulations. An iterative scheme usable here is given e.g. in [27]. Still, explicit estimates for such moments can be given in some situations. E.g. let us return to the original η. It is sufficient to estimate $\mathrm{E}\left(\eta^3\right)$, which can be done via

$$M^3\mathrm{E}\left(\eta^3\right) \le \mathrm{E}\left(\sum_{a\in A^{(0)}}\sum_{1\le i}^{\pi(a)}\gamma_i\left(a\right)\right)^3 \tag{1}$$

where *M* is the number of infantry units, $\pi\left(a\right)$ is a Poisson random variable with intensity $\lambda\left(a\right)T$ (the maximal number of fire missions such an agent could execute in ideal conditions), $\gamma_i\left(a\right)$ is a Bernoulli random variable that encodes a successful hit on the *i*-th mission.

To obtain the lower estimate, the following simplification can be used. Assume that the defenders are only targeting infantry units, leaving the artillery of the attackers unanswered. Using the previous calculations in the terms of Poisson and Bernoulli random variables, we can obtain the expression for the random maximal possible number of successful attacks of the attackers. By dividing this number among all the defending agents, we obtain a virtual flow of interruptions for these agents (say, with intensity μ).

Each defending artillery unit *a* will then suffer periods of inactivity, which would, on average, reduce the effective intensity of $P\left(a;\cdot\right)$ to $\lambda\left(a\right)\left(1-\mu\tau_{\mathrm{inact}}\left(a\right)\right)$. These corrected intensities can be used to estimate necessary moments similarly to Eq. (1).

However, in the case study presented in Section 3, we consider only observed statistics to avoid a lengthy discussion. For rigorous justification of the conclusions

presented below, the combinations of the methods discussed in this section and the nonparametric Mann-Whitney U test [29] can be used to test whether two target acquisition strategies lead to the same distribution of cost functions as the null hypothesis. The test is statistically consistent against the alternative hypothesis that one target acquisition strategy stochastically dominates the other. The corresponding statistical procedure is illustrated via a synthetic example in Section 3.

As an effect size measure, the standard probability of superiority is recommended due to its clear interpretation and explicit relation to the test statistic.

The discussion presented in this section should be seen as a general tunable methodology since every installment of the framework developed in the paper might need its own nonparametric analysis.

In particular, we mostly omit the details in the example presented below due to its purely synthetic nature. However, a brief discussion of multiple-comparison corrections and guidance on interpreting the sensitivity analysis are provided below. These should be regarded as separate components of the general statistical procedure described in this section. Different setups might lead to a significantly divergent analysis, particularly in regard to the clusters of target acquisition strategies introduced in Section 3.

3 Case Study: Results and Discussion

We consider a purely hypothetical example of such a combat engagement to illustrate the flow of the statistical analysis and to demonstrate the corresponding decision-making process.

To include practical intuition, we assume that the attackers (side A) hold firepower superiority and the defenders are consequently expected to be suppressed or eventually destroyed unless they withdraw their forces at some point. Still, the defenders (side B) are expected to execute enough fire missions to noticeably reduce the advance of the enemy infantry units.

In what follows, the values of parameters (including hitting probabilities, intensities of command flows and precise composition of the opposing forces) are synthetic and should be replaced with valid statistical estimates obtained from the real data for practical usage. Possible further research directions aimed at obtaining proper values of parameters (in particular, hitting probabilities) are listed in Section “Conclusions”.

The following assumptions are used:

- 2 types (Type 1 – Arty1 and Type 2 – Arty2) of artillery units are present.
- Probability of hitting a target, the time of inactivity after getting suppressed and the starting rate of the corresponding CPPwI are the same for all artillery units of the same type regardless of the side.
- 2 HQs are present on the side of the attackers.
- The probability of successfully attacking an HQ is lower than probability of successfully attacking an artillery piece, and we introduce a new special coefficient c_{cor} to measure this reduction: $p_{\text{corrected}} = p_{\text{orig}} c_{\text{cor}}$; the coefficient c_{cor} is assumed to be the same for all HQs and artillery agents.
- The time of the simulations is 3 hours.
- Agents of the side in defense cannot be targeted until they disclose themselves by executing a fire mission or τ_{rec} has passed.

Precise numbers are as follows:

- The defenders (side B) have 6 agents of Arty1 and 4 agents of Arty2. The attackers (side A) have 10 agents of Arty1 and 4 agents of Arty2. Among them, 6 agents of Arty1 are attached to the first HQ, and the rest are attached to the second one. All units of the defenders are attached to 1 HQ. At the same time reserve channels are present as described in Section 2.2.
- Agents of Arty1 can execute 2 fire missions per hour on average, $\tau_{\text{inact}} = 15$ min and $\tau_{\text{rec,max}} = 20$ min.
- Agents of Arty2 can execute 2.5 fire missions per hour on average, $\tau_{\text{inact}} = 20$ min and $\tau_{\text{rec,max}} = 30$ min.
- Probabilities to hit for Arty2 are 1.5 times higher than those for Arty1.
- For all HQs $\nu(a) = 0.3$, $\tau_{\text{inact}} = 30$ min.

The situation thus can be seen as maneuver-based defense as the attackers (side A) noticeably outnumber the defending forces (side B) and the agents on the side of the defenders are expected to be driven away (suppressed, destroyed) with time. As a result, the enemy's advance (side A) is expected to be only impeded.

We denote the probability of hitting an artillery unit for Arty1 as p_{l}. Simulations are run for parameter p_{l} taking values in [0.05, 0.5] with a step 0.05, parameter c_{cor} taking values in [0.2, 0.8] with a step 0.1 and for all meaningful combinations of values $p_{\text{hq}}, p_{\text{inf}}$ in [0.2, 0.8] with a step 0.2. This reflects the fact that real probabilities depend on a number of external factors (including logistics, technical issues and weather) and might not be taken as fixed. In this example, we make an artificial assumption that real-life values of p_{l} and c_{cor} are uniformly distributed within the stated ranges. Proper analysis of distributions of such parameters requires a separate study.

For each combination of values of these parameters the Monte Carlo simulations were run 100 times using the future.apply package with a fixed pseudo-random number generator seed (123) and the results were averaged.

Essentially, one can view $(p_{\text{hq}}, p_{\text{inf}})$ as shares of resources (fire capabilities, ammunition stocks and reconnaissance information) allocated to neutralize specific types of enemy targets. The rest of the resources are used to counter separate artillery pieces. Disabling HQs can also be interpreted as constraining reconnaissance capabilities of the attackers (since more time is needed for proper target acquisition) etc.

For illustrative purposes, we consider only two simplified cost functions:

- the number of successful attacks executed by the defenders on infantry units in the assumption that each infantry unit advances in a sector of the same width (referred to as denied enemy advancement in visual materials),
- the losses suffered by the defenders (measured in units).

Two extreme modes are considered:

- suppression: after an attack an agent fully recovers after inactivity,
- destruction: a successful attack removes an agent from the battlefield.

In fact, such grid-based Monte Carlo simulations can be used as a practical version of sensitivity analysis to determine crucial parameters or at least instability in the behavior of the model. However, due to dependencies present in the model and the complex nature of real combat, one should not be quick to draw simplistic conclusions about most influential parameters (say, by considering exclusively certain probabilities or coefficients). We consider randomness of these values as a part of the presented statistical model instead and compare the resulting cost function by splitting all target acquisition strategies into clusters depending on the value of p_{hq}. We did not detect

instability in the averaged behavior of the model and obtained a rather continuous transition both for reasonable changes of the compositions of the opposing forces and within the aforementioned clusters for a fixed initial battlefield. The latter can be deduced from Figs 2–4 below. However, we emphasize the synthetic nature of these simulations again.

This leads to the following remark on multiple comparison corrections. Despite a significant number of target acquisition strategies tested, one can compare only the clusters introduced above instead of separate pairs of $(p_{\mathrm{hq}}, p_{\mathrm{inf}})$, since the cost functions are essentially monotonic within the clusters, so only best representatives should be compared directly, reducing the number of comparisons. Assuming preservation of the forces is another expected and essential goal of the defenders, this might lead to 2-step analysis using both cost functions. Otherwise, one can use the primary cost function to obtain regions of values of $(p_{\mathrm{hq}}, p_{\mathrm{inf}})$ where the standard Benjamini-Hochberg or Holm-Bonferroni methods can be used to adjust p-values accordingly. In particular, if losses are to be minimized, the more conservative Benjamini-Hochberg method could be recommended in general and is used in this example.

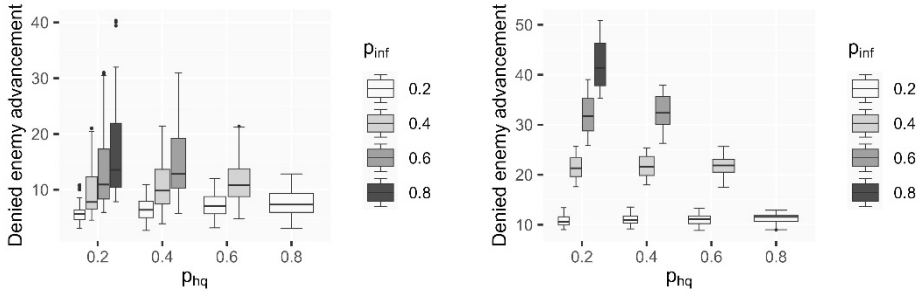
We use standard boxplots to illustrate the results in Figs 2–4 (so, for instance, the horizontal line denotes the median etc.). Note that each boxplot aggregates the data for a fixed pair $(p_{\mathrm{hq}}, p_{\mathrm{inf}})$, assuming uniform priors for p_{l} and c_{cor} over the discretized ranges given above.

The presence of outliers is another feature of such statistical model which is absent in the Lanchester-type averaged dynamics. It allows one to analyze critical and atypical cases, too. In particular, a separate study might address the problem of determining what key parameters should be controlled to reduce the appearance of rare runs that are disastrous for the defenders etc., which is related to compound cost functions in Section 2.4.

Overall, the losses decrease when p_{hq} increases. This effect is more pronounced in the destruction mode.

In the suppression mode, the best strategy is $p_{\mathrm{hq}} = 0.2$, $p_{\mathrm{inf}} = 0.8$. Therefore, the priority of high-payoff targets will be as follows: 1st priority – fire missions to destroy enemy infantry; 2nd priority – HQs; 3rd priority – artillery. That is, simply concentrating all the resources on blocking the advance of infantry is sufficient in a simplified situation of minor effects of artillery fire.

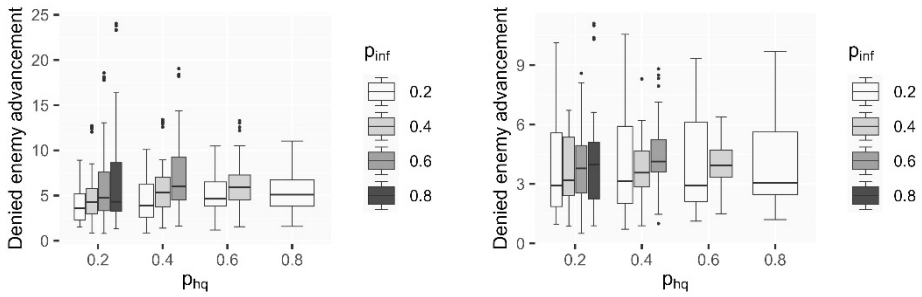
The rest of the paper discusses the destruction mode.



(a) Mode: destruction *(b) Mode: suppression*

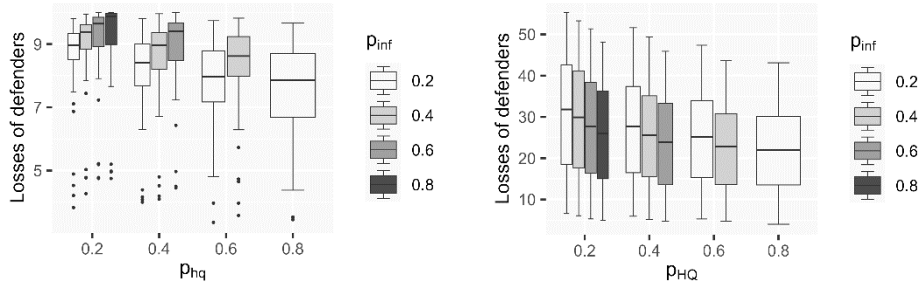
Fig. 2 Denied enemy advancement

DOI 10.3849/aimt.02085



(a) Over last 2 hours *(b) Over last hour*

Fig. 3 Denied enemy advancement, long-term development



(a) Mode: destruction *(b) Mode: suppression*

Fig. 4 Losses of the defenders

Though it might seem that $p_{hq} = 0.2, p_{inf} = 0.8$ is still the optimal strategy, a more detailed analysis of later stages of the combat suggests using $p_{hq} = 0.4, p_{inf} = 0.6$ as the optimal strategy: the defenders maintain a better presence on the battlefield later and obtain better results in suppressing enemy infantry compared to the case of $p_{hq} = 0.2, p_{inf} = 0.8$. In the latter case, the attackers start dominating the battlefield faster. This is shown in subplots in Fig. 3 by restricting the data to the last 2 hours and the last hour, respectively.

Therefore, allocating resources to targeting HQs (essentially, concentrating on counter-battery fire) leads to better survivability of defending forces and even increases the number of successful attacks on infantry in later stages of the combat engagement. This occurs because damaging the enemy's command post primarily weakens the controllability of enemy attacks. Damage to enemy artillery reduces the losses of artillery and infantry on the defending side, which ultimately enables the defenders to execute more fire missions in the long-term perspective. Since real life combat operations can contain multiple extensive stages, preserving the forces of the defenders for later stages while still achieving good results in countering enemy infantry should be considered as the optimal choice.

In the terms of statistical analysis, the losses for $p_{hq} = 0.2, p_{inf} = 0.8$ are higher than the losses for $p_{hq} = 0.4, p_{h_i z} = 0.6$ ($p < 0.01$) (Fig. 4), while the null hypothesis for the values of denied enemy advancement cannot be rejected ($p = 0.42$, effect size 45 %). This solidifies the recommendation of suggesting $p_{hq} = 0.4, p_{inf} = 0.6$ as an

optimal strategy and provides an example of how more than one metric can be used to better reflect real-life situations.

The priority of high-payoff targets is essentially the same in both modes (suppression and destruction). The only difference is in the percentage distribution of firepower between high-payoff targets.

The mesh for $(p_{\text{hq}}, p_{\text{inf}})$ is quite coarse. To obtain more precise optimal values one can use the search on the mesh with smaller steps, possibly even using dynamic steps by introducing gradient descent. However, such models are not expected to be extremely precise and are instead expected to provide intuition about possible strategies the sides of the combat encounter can employ.

4 Conclusions

The paper presents a description of a new framework for constructing stochastic combat models. Each acting agent on the battlefield is associated with a generalized Poisson process with non-Markovian interruptions and a set of rules is used to govern the interactions between such agents. In particular, suppression of an agent is equivalent to a pause of the corresponding generalized Poisson process and events of such processes correspond to fire missions or acts of relocating and recovery after an agent was suppressed by enemy fire. This approach makes it possible to change the paradigm of fire support planning from predicting enemy losses to influencing the intensity of its advance and the execution of combat (fire) missions. A model can be tuned into each specific situation by introducing different sets of rules and providing parameters of the agents appearing in the model.

The Monte Carlo method is then used to simulate results of a combat encounter and statistical analysis is used to determine high-payoff targets and to find optimal distribution of resources between them assuming that a cost function to evaluate the quality of an employed strategy is given.

The method and the decision-making process are illustrated by a synthetic example of a combat encounter in which one side launches attacks by using artillery to cover the advancement of infantry units while the other side attempts to minimize both the area captured by the enemy infantry and their own losses. The target acquisition logic for the defenders is modeled via a statistical decision procedure for redistributing fire between different types of enemy agents, so each type is chosen randomly according to the corresponding probability.

Further research in this direction may involve:

- Introducing more realistic distributions of hitting probabilities (e.g. via Bayesian inference for calculating weighted cost functions), including dynamic changes under the actions of other agents.
- Introducing other types of agents to better represent modern warfighting functions. In particular, a better representation of reconnaissance operations might be obtained by introducing proper separate independent units while logistical systems can be represented both as critical objects (such as bridges) and pools of resources available to agents and depleted on use. The latter can constrain the probability of successful engagement of enemy agents due to insufficient ammunition stocks etc.

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Data Availability Statement

The data used in the current study are available upon reasonable request.

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