



# Design of a Controller Using Hedge Algebra to Control Position of Hydraulic Cylinders

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The manuscript was received on 5 December 2023 and was accepted after revision for publication as an original research article on 9 December 2024.

## Abstract:

Currently, hydraulic actuators are widely used in both civil and military industries that require high response speed and capacity. The problem of accurately controlling the position of hydraulic actuators, especially hydraulic cylinders, is a typical problem and that is why it is of research interest. Fuzzy controllers are successful with complex objects that have incomplete object information and are easier to change than other controllers, but the problem of choosing fuzzy rules, membership functions, as well as the amount of calculation, is still limited. Hedge algebra is a new approach to fuzzy logic computing and has achieved some success in the field of control. In this study, the authors propose a control model using algebraic control algorithms for hydraulic servo systems to overcome the disadvantages of classical PID controllers and the nonlinearity of hydraulic systems. The quality of the hedge algebraic controller has been simulated on Matlab/Simulink software and tested on a real model.

## Keywords:

*hedge algebra, hedge algebra control, position control, hydraulic cylinders*

## 1 Introduction

Electro hydraulic servo systems are widely used in precision machining machines, industrial automation systems and surgical robots where the requirements for capacity, cost, cleanliness and conformity are relatively high. However, electro hydraulic servo systems are known to be highly nonlinear system because the applied force and torque are very large. The nonlinear characteristics of the system cause many difficulties in control [1]. To solve this problem, there have been many research works on it for instance, in 2010, Vladimir Milić and colleagues focused on using techniques based on linear matrix inequalities to synthesize Position Controller  $H_\infty$  Synthesis of an electro hydraulic servo

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system [2]. In their research, an adaptive fuzzy sliding mode controller was implemented to solve the trajectory tracking. The bounded and convergent properties of the closed-loop system have been proven by Lyapunov stability theory and Barbalat's lemma [3]. Active disturbance rejection control was designed to improve the position tracking performance of electro hydraulic drive systems in the presence of parameter uncertainty, parametric uncertainty, and external disturbances [4]. In 2020, C. Cheng also used second-order sliding control to solve this problem [5]. In the paper [6], adaptive control is used to deal with system parameter uncertainties, and the radial basis function neural network algorithm is introduced to approximate the unknown nonlinearity combined. In the work [7], the authors used the adaptive sliding mode control method based on the RBF neural network to solve this problem.

These studies can be sufficient to solve nonlinear problems for electro hydraulic servo systems. In this article, we introduce a new approach using hedge algebra to address this problem. Hedge Algebra (HA) effectively solves problems in uncertain environments, so researchers aim to apply it in the field of control and automation. HA has been studied in a number of identification and diagnosis problems [8-11] and has had significant successes in the field of control (applied to some simple model control and approximation problems [12-14]). However, using HA in control problems is still quite a new problem. Successful research and application of a controller using HA will further confirm the effectiveness of HA theory, opening up the possibility of practical application.

Starting from the above mentioned goal, the research team has designed a controller using HA for a number of motion control systems. The content of the article presents results verified on a physical model of a specific system in practice.

## 2 Modeling Hydraulic Systems

In the hydraulic system model in Fig. 1, the hydraulic cylinder is a cylinder with a piston shaft on one side with the following specific system parameters in Tab. 1.

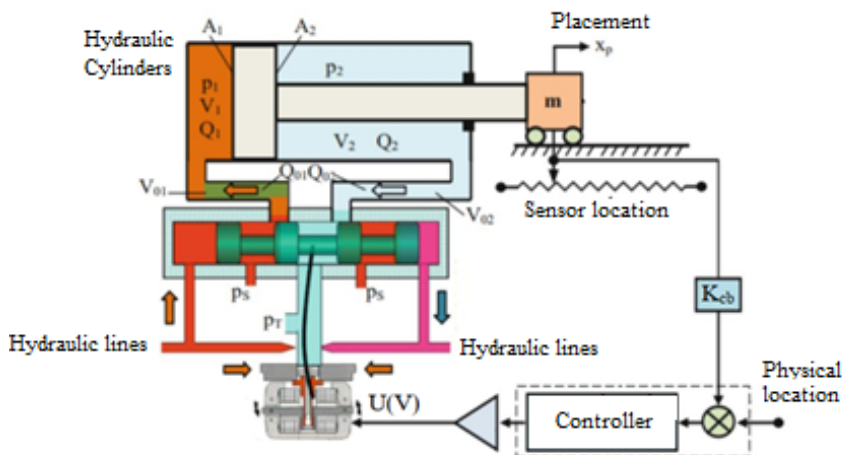


Fig. 1 Model of hydraulic cylinder position control system

Tab. 1 Hydraulic system model parameters

Symbol	Description	Value	Unit
$L_{Max}$	Piston shaft displacement stroke	800	mm
$H_{Max}$	Working distance of position sensor	800	mm
$m$	Mass	3	kg
$f$	Viscosity reduction coefficient	$250 \times 10^5$	Ns/mm
$\lambda_1$	Internal leakage coefficient	$4.6 \times 10^{-10}$	$\text{mm}^3 / (\text{s}/\text{Pa})$
$\lambda_2$	External leakage coefficient	$1.6 \times 10^{-10}$	$\text{mm}^3 / (\text{s}/\text{Pa})$
$\beta$	Fluid volume elastic modulus	$8 \times 10^8$	Pa
$V_{01} + V_1$	Volume of oil from the valve to the working chamber	$13 \times 10^5$	$\text{mm}^3$
$A_1$	Piston cross section on the left side of the cylinder	1 265	$\text{mm}^2$
$A_2$	Piston cross section on the right side of the cylinder	625	$\text{mm}^2$
$K_{sv}$	Servo valve coefficient	$4 \times 10^5$	$\text{mm}^3 / (\text{s}/\text{A})$
$\omega_{sv}$	Angular velocity of servo valve	100	$\text{s}^{-1}$
$Q_{01}$	Flow through servo valve	Eq. (1)	l/min
$Q_v$	Flow of hydraulic oil	Eq. (2)	l/min

The flow through the servo valve to the left working chamber of the hydraulic cylinder is determined by Eq. (1), published in [1]:

$$Q_{01} = A_1 \frac{dx_p}{dt} + \lambda_1 p_L + \lambda_2 p_1 + \frac{V_{01} + V_1}{\beta} \frac{dp_1}{dt} \quad (1)$$

where  $V_1 = A_1 x_p$ ;  $p_L = p_1 - p_2$ . The flow of hydraulic oil through the valve is determined based on the voltage supplied to the valve coil and the pressure difference between the pressure of the two working chambers of the cylinder [12]:

$$Q_v = K_v u - K_c p_L \quad (2)$$

Equation to balance the force acting on the piston:

$$A_1 p_1 - A_2 p_2 = m \frac{d^2 x_p}{dt^2} + f \frac{dx_p}{dt} \quad (3)$$

If we mark  $R = A_1/A_2$ , we receive  $Q_1 = K\sqrt{p_s - p_1}$ ;  $Q_2 = K\sqrt{p_s - p_T}$ . From there, the pressure in the right working chamber of the cylinder:

$$p_2 = \frac{p_s - p_1}{R^2} \quad (4)$$

From Eq. (1), Eq. (2) and Eq. (4) we get:

$$p_L = p_1 \left( 1 + \frac{1}{R^2} \right) - \frac{1}{R^2} p_s \quad (5)$$

with  $E = -(A_2/R^2)p_s$ .

Performing the Laplace transformation, from Eq. (1) and Eq. (5) we get:

$$Q_{01} = A_1 x_p(s) s + \left[ \lambda_1 \left( 1 + \frac{1}{R^2} \right) + \lambda_2 + \frac{V_{01} + V_1}{\beta} s \right] p_1(s) \tag{6}$$

$$x_p(s) (ms^2 + fs) = \left( A_1 + \frac{A_2}{R^2} \right) p_1(s) + E(s) \tag{7}$$

We marked  $A = A_1 s; B = \lambda_1 \left( 1 + \frac{1}{R^2} \right) + \lambda_2 + \frac{V_{01} + V_1}{\beta} s; C = ms^2 + fs; D = A_1 + \frac{A_2}{R^2}$ , Eqs (6) and (7) is rewritten as:

$$\begin{cases} Q_{01}(s) = Ax_p(s) + Bp_1(s) \\ Cx_p(s) = Dp_1(s) + E(s) \end{cases} \tag{8}$$

The hydraulic cylinder block diagram is built as shown in Fig. 2.

The hydraulic cylinder transfer function  $W_{XL}$  is determined according to the following expression in Eq. (9) [12]:

$$W_{XL} = \frac{x_p(s)}{Q_{01}(s)} = \frac{K_{XL}}{s(T_1^2 s^2 + T_2^2 s + 1)} = \frac{K_{XL}}{s \left[ \left( \frac{s}{\omega_0} \right)^2 + 2\xi \left( \frac{s}{\omega_0} \right) + 1 \right]} \tag{9}$$

where  $K_{XL} = \frac{A_1 + \frac{A_2}{R^2}}{\lambda_1 f \left( 1 + \frac{1}{R^2} + \lambda_2 \right) + A_1^2 + \frac{A_1 A_2}{R^2}}$ ;

$$T_1 = \sqrt{\frac{V_{01} + V_1}{\beta \left[ \lambda_1 f \left( 1 + \frac{1}{R^2} + \lambda_2 \right) + A_1^2 + \frac{A_1 A_2}{R^2} \right]}}; T_2 = \frac{m \left[ \lambda_1 \left( 1 + \frac{1}{R^2} \right) + \lambda_2 f \frac{V_{01} + V_1}{\beta} \right]}{\lambda_1 f \left( 1 + \frac{1}{R^2} + \lambda_2 \right) + A_1^2 + \frac{A_1 A_2}{R^2}}$$

$$\omega_0 = \frac{1}{T_1} = \sqrt{\frac{\beta \left[ \lambda_1 f \left( 1 + \frac{1}{R^2} + \lambda_2 \right) + A_1^2 + \frac{A_1 A_2}{R^2} \right]}{V_{01} + V_1}} = \sqrt{C_H};$$

$$\xi_{XL} = \frac{T_2}{2T_1} = \frac{m \left[ \lambda_1 \left( 1 + \frac{1}{R^2} \right) + \lambda_2 f \frac{V_{01} + V_1}{\beta} \right]}{\sqrt{\left( V_{01} + V_1 \right) \left[ \lambda_1 f \left( 1 + \frac{1}{R^2} + \lambda_2 \right) + A_1^2 + \frac{A_1 A_2}{R^2} \right]}}$$

From Eq. (9) and input data we get:

$$W_{XL} = \frac{x_p(s)}{Q_{01}(s)} = \frac{K_{XL}}{s(T_1^2 s^2 + T_2^2 s + 1)} = \frac{3.54 \times 10^{-4}}{1.65 \times 10^{-10} s^3 + 4.6 \times 10^{-6} s^2 + s} \tag{10}$$

Transmission function of servo valve [12]:

$$W_{VTL} = \frac{Q_{01}(s)}{U(s)} = \frac{K_{VTL}}{\left( \frac{s}{\omega_{VTL}} \right)^2 + 2\xi_{VTL} \left( \frac{s}{\omega_{VTL}} \right) + 1} = \frac{40 \times 10^4}{10^{-4} s^2 + 0.014 s + 1} \tag{11}$$

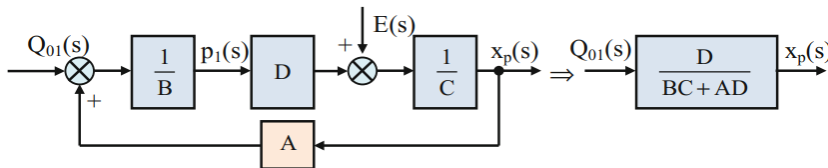


Fig. 2 Block diagram of hydraulic cylinder

## 2.1 Building a Controller using Hedge Algebra to Control the Position of Hydraulic Cylinders

HA is an algebraic structure that quantifies the semantics of the linguistic value domain of a linguistic variable [9, 11, 12, 15]. With the input-output relationship according to fuzzy logic, the membership functions must be determined discretely. With HA, there is an algebraic structure in the form of a functional relationship, allowing the formation of an infinitely large set of linguistic values in such a way that the resulting structure simulates well the semantics of the language which helps human reasoning processes. HAC controller – Hedge Algebra based Controller consists of 3 blocks as shown in Fig. 3:  $x$  - input set value;  $x_s$  - input semantic value;  $u$  - the control value and  $u_s$  the control semantic value.

The HAC suite includes the following blocks:

- Block I – Normalization (Semanticization): linearly transforms  $x$  to  $x_s$ .
- Block II – SQMs and HA-IRMd (Semantic inference): performs semantic interpolation from  $x_s$  to  $u_s$  on the basis of quantitative semantic mapping and rule system conditions.
- Block III – Denormalization: linearly transforms  $u_s$  to  $u$ .

The HA controller is built with 2 inputs: position error and change in position over time. The output of the controller using electronic algebra is the voltage supplied to the servo valve coil.

- Position error  $e = [-800 \ 800]$  [mm].
- Position error variation  $de = [-8000 \ 8000]$  [mm/min].
- Voltage value  $u = [-10 \ 10]$  [V].

Built membership functions for input and output variables  $e = \text{NB, NM, NS, Z, PS, PM, PB}$ . In there: Negative Big (NB), Negative Medium (NM), Negative Small (NS), Zero (Z), Positive Small (PS), Positive Medium (PM) and Positive Big (PB).

The linguistic labels in hedge algebra for the variables E, IE, U are as follows: Very Very Negative (VVN), Very Negative (VN), Negative (N), Little Negative (LN), W, Little Positive (LP), Positive (P) and Very Positive (VP), Very Very Positive (VVP). Select parameters for variables E, IE, U in Tab. 2.

The control rule table for HAC language labels is in Tab. 3.

Calculating the semantic quantification values for the system variables E, IE and U, we get table system which is based on hedge algebra as shown in Tab. 4. The quantitative semantic curve represents the input-output relationship and it is shown in Fig. 4.

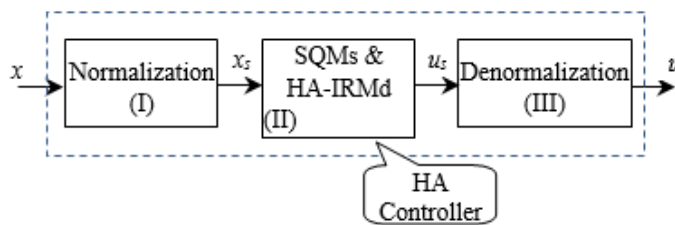


Fig. 3 Diagram of controller for hedge algebra application

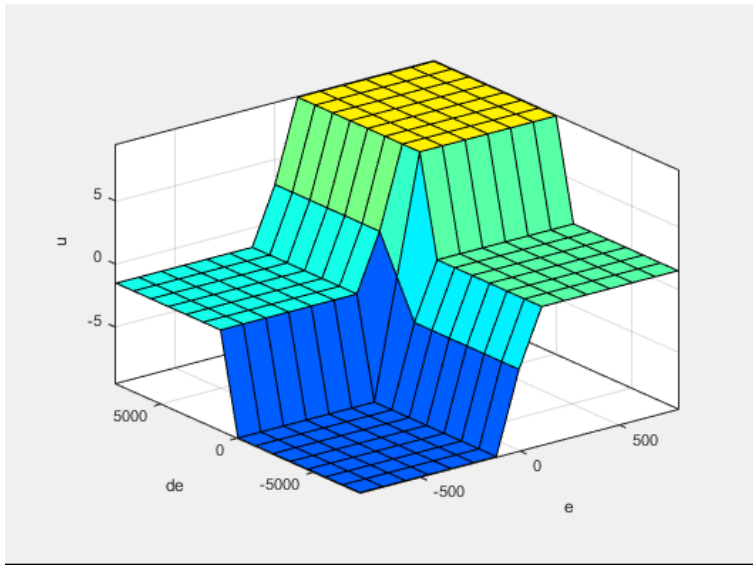


Fig. 4 Quantitative semantic surface

**2.2 Simulation on Simulink**

From the above observations, we proceed to simulate the system for different cases shown in Fig. 5. The results are shown in Fig. 6. From the results we see:

- With PID, PI-fuzzy (FLC) and HAC controllers with two inputs and one output. Simulation results show that the system response with all three controllers is stable, following the set value and after a specified period of time, the system error gradually approaches 0.
- HAC has created an algebraic structure in the form of functional relationships, allowing the formation of an arbitrarily large set of language variables to describe input/output relationships, so the quality of the achieved control system can be better than FLC.
- System response with HAC controller has small over-adjustment and fast setup time. This shows the feasibility of applying the HAC suite to control systems with continuous reversing requirements and verifying the research results experimentally.

Tab. 2 Table Type Styles

	Input 1 (E), Input 2 (IE)		Output (U)	
H	$\mu[h]$		$\mu[h]$	
H- Little(L)	$\alpha$	0.6	$\alpha$	0.55
H+ Very(V)	$\beta$	0.4	$\beta$	0.45

Tab. 3 Control law

U	IE							
E	NB	NM	NS	Z	PS	PM	PB	PB
E	NB	NB	NB	NB	NB	NB	NM	NS
E	NB	NB	NB	NB	NB	NB	NM	NS
E	NS	NM	NS	NS	NS	NM	NS	Z
E	Z	NM	NS	Z	PS	NS	PS	PM
E	PS	Z	PS	PS	PS	PS	PS	PM
E	PM	PS	PM	PB	PB	PS	PB	PB
E	PB	PS	PM	PB	PB	PB	PB	PB

Tab. 4 Table system is based on hedge algebra

U	IE							
E	0.108	0.180	0.300	0.500	0.500	0.700	0.820	0.890
E	0.108	0.045	0.120	0.162	0.460	0.225	0.720	0.625
E	0.180	0.045	0.250	0.250	0.120	0.456	0.750	0.500
E	0.300	0.045	0.250	0.102	0.250	0.384	0.750	0.954
E	0.380	0.120	0.102	0.375	0.500	0.775	0.450	0.954
E	0.500	0.225	0.500	0.682	0.450	0.957	0.645	0.775
E	0.620	0.376	0.375	0.375	0.896	0.954	0.322	0.954
E	0.700	0.500	0.628	0.500	0.654	0.125	0.954	0.954

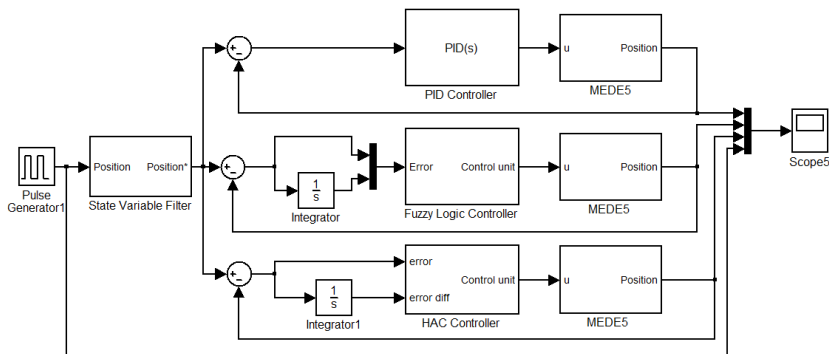


Fig. 5 Simulation diagram on Simulink

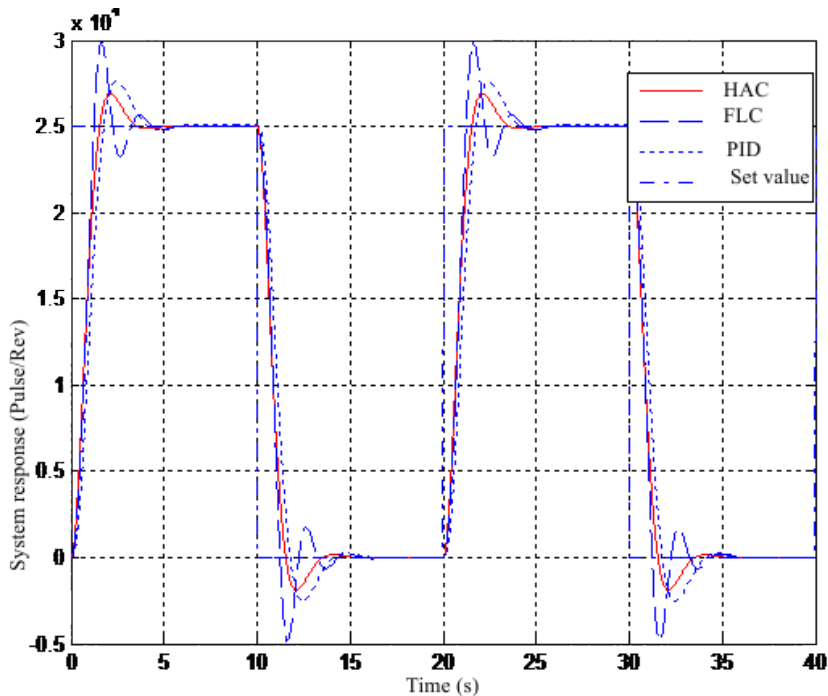


Fig. 6 Simulation results

### 3 Experimental Results on Real Models

The experimental model is described in Fig. 7. In this model, the S7-300 PLC and EM235 expansion module act as data acquisition devices (This block will receive data from the rheostat position sensor and will output a control signal to the hydraulic valve system). The computer is connected to the PLC using RS232 PC-PPI cable, with Modbus RTU communication protocol. The control algorithm for the Hedge algebra application is built on a computer using the Microsoft Visual Studio programming environment with the C programming language.

Experimental results in the case of the actuator moving from the starting point to a position of 300 mm and in two cases where the system's set value is linear and the system's set value is stepped are shown in Fig. 8 and Fig. 9.

Simulation and experimental results show that the gain parameters of the HAC controller have an important influence on the convergence speed and accuracy, HAC can increase the convergence speed and the displacement curve achieves good conditions tracking. After many repetitions in the experiment, the position error is very small, proving that the proposed algorithm is suitable. From the results we can see the following:

- System quality with HAC is very good, the output signal can follow the set value, the system response in steady-state has over-adjustment, and static deviation is very small and insignificant (the simulation process does not have a static bias).



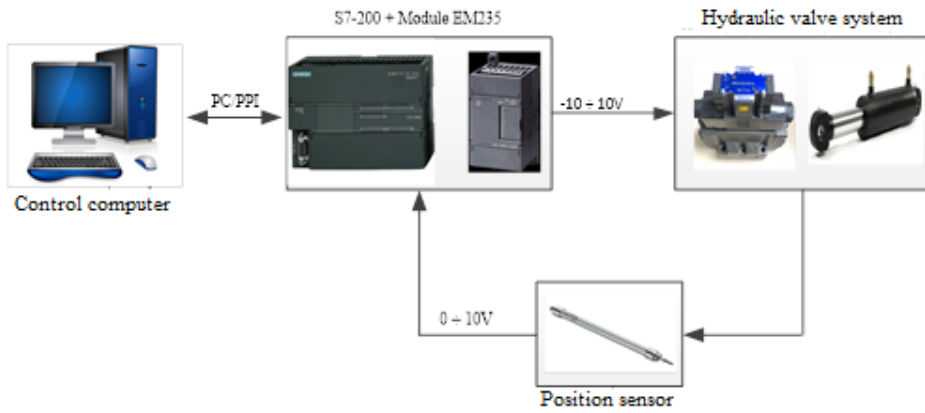


Fig. 7 Diagram of the test setup

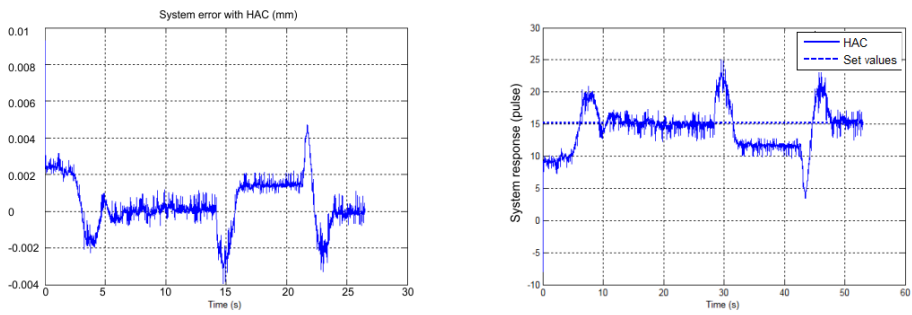


Fig. 8 Experimental results when the input is linear

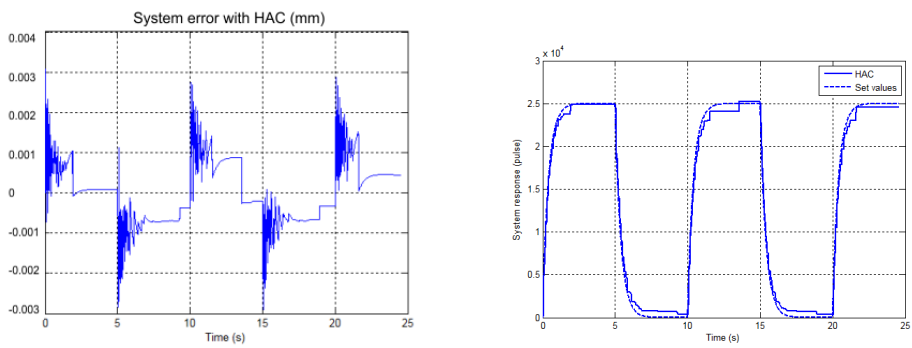


Fig. 9 Experimental results when the input is ladder shaped

- Experimental results show that the HA controller can be applied in motion control systems following a sample trajectory with requirements for quick action and continuous reversal. Therefore, HAC can be applied in the industry ensuring that the system meets the requirements of quality assurance.

## 4 Conclusion

In this study, the authors performed modeling of the hydraulic system including servo valves and hydraulic cylinders. At the same time, the authors built a controller to apply hedge algebra on Matlab/Simulink software. The results showed that the quality of the controller met the requirements. In addition, the control law has been realized on a real model using Siemens S7-300 PLC and EM235 expansion module. The controller is built on the control computer. Specific results show that the system using the HAC controller meets quality requirements and opens up application possibilities.

During the system design process, the advantage of using HAC is shown, that is, using HA in controller design can create an algebraic structure in the form of a functional relationship, allowing the formation of a set of variables arbitrarily large language. Thus, the quality of the achieved control system will be much better than other controllers. However, we also realize a disadvantage of HAC: if the fuzzy controller through each design step can consult experts, HAC cannot do this, so the design will be difficult or there must be an automatic design solution according to a preset quality criterion.

What can be solved by improving the quality of the HAC without: increasing the input and reducing the control law and designing the controller according to the squared error integral criterion using genetic algorithms. These issues will be researched and published in the future.

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