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Closed-Form Expressions for Predicting Certain Projectile Trajectories

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Abstract:

Accurate prediction of the trajectory of a projectile passing through air almost always requires the aid of computer-assisted numerical procedures. Herein are derived closed-form expressions for a certain class of such trajectories, which can provide insights not readily obtainable with computer codes alone. For example, the expressions reveal errors in widely-promulgated formulas for predicting the effects of Coriolis acceleration and of uphill or downhill launches. An example is presented showing negligible differences between results from expressions derived herein and those from a well-developed and widely-used code that uses numerical integration. Such agreement can be expected for flat trajectories of projectiles with shapes resembling that of a standard, so-called G-7 projectile, in the supersonic speed range.

Keywords:

ballistics, trajectories, small arms

1 Introduction

A projectile's trajectory is the path it creates after its launch. It is determined by the properties of its launch and the physical laws that govern changes in its motion during flight. The basis for predicting quantities of interest (e.g. velocity, deviation from line of sight, etc.) consists of accurate mathematical representation of the physical laws as they act along the path. To reach useful predictions from these equations of motion requires determination of the accumulative effects from local changes along the path.

When certain conditions lead to sufficiently simple equations of motion, these can lead to closed-form expressions for quantities of interest. An element that almost always prevents such simplification is aerodynamic drag. The mathematical expression of its effect depends upon a factor called the projectile's drag coefficient, values of which are determined by experiments and can vary in complicated ways.

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Absent our ability to derive closed-form expressions, modern computer-aided numerical methods can supply quantitively accurate values for items of interest. Codes such as that of [1] have been developed with extensive experimental verification, so provide meaningful comparisons for predictions from the model developed herein.

The class of trajectories analysed herein is among those effected by aerodynamic drag but is restricted to flat trajectories of projectiles with certain shapes and range of speeds.

2 Reference Frames, Forces, and Accelerations

The earth is assumed to form a reference frame E rotating in a Newtonian reference frame N about the north-south polar axis at one revolution per sidereal day. As shown in [2], the accelerations of a moving point P in N and in E are related by

$$^{N}\boldsymbol{a}^{P} = {}^{N}\boldsymbol{a}^{PE} + {}^{E}\boldsymbol{a}^{P} + 2\,{}^{N}\boldsymbol{\omega}^{E} \times {}^{E}\boldsymbol{v}^{P}$$

The first term on the right expresses the N-observed acceleration of a point fixed in E and located coincident with P.

In flight, the forces acting on the projectile are aerodynamic drag $\boldsymbol{f}_{\mathrm{d}}$ and the gravity force $\boldsymbol{f}_{\mathrm{g}}$. In terms of these, Newton's second law, $\boldsymbol{f}_{\mathrm{d}} + \boldsymbol{f}_{\mathrm{g}} = m\,{}^{N}\boldsymbol{a}^{P}$, can be combined with the above kinematic relationship to yield

$$^{E}\boldsymbol{a}^{P} = \frac{\boldsymbol{f}_{\mathrm{d}}}{m} + \left(\frac{\boldsymbol{f}_{\mathrm{g}}}{m} - {}^{N}\boldsymbol{a}^{PE}\right) - 2\,{}^{N}\boldsymbol{\omega}^{E} \times {}^{E}\boldsymbol{v}^{P} \tag{1}$$

The free-fall acceleration ($\boldsymbol{f}_{\mathrm{g}}/m - {}^{N}\boldsymbol{a}^{PE}$) establishes the local vertical direction; its magnitude g varies negligibly with geographic location, around 9.80665 m/s^2. The last term gives rise to what is known as the Coriolis displacement.

As in most "point-mass" models, the force $\boldsymbol{f}_{\mathrm{d}}$ is assumed to act opposite the difference $\boldsymbol{v} - \boldsymbol{w}$ between the velocities of the projectile and of the wind. Fig. 1 shows vectors of $\boldsymbol{v}$, $\boldsymbol{w}$, and $\boldsymbol{f}_{\mathrm{d}}$. $\boldsymbol{e}_{\mathrm{t}}$ designates a unit vector tangent to the path of the projectile. It will be further assumed that $|\boldsymbol{w}|$ is much smaller than $\boldsymbol{v}$.

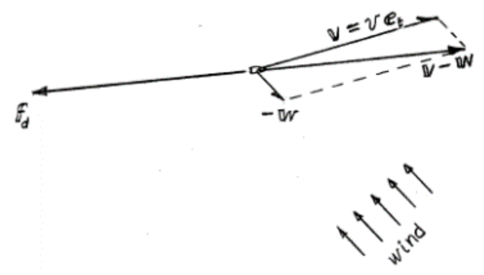

Fig. 1 Vectors representing projectile and wind velocities and drag force

The figure indicates that the drag force can be expressed as

$$\boldsymbol{f}_{\mathrm{d}} = -|\boldsymbol{f}_{\mathrm{d}}|\frac{\boldsymbol{v} - \boldsymbol{w}}{V} = |\boldsymbol{f}_{\mathrm{d}}|\left(-\boldsymbol{e}_{\mathrm{t}} + \frac{\boldsymbol{w}}{V}\right)$$

The relatively small component of the drag force perpendicular to the flight path,

$$|\boldsymbol{f}_{\mathrm{d}}|\left[\frac{\boldsymbol{w}}{V} - \boldsymbol{e}_{\mathrm{t}} \cdot \frac{\boldsymbol{w}}{V}\boldsymbol{e}_{\mathrm{t}}\right] \tag{2}$$

contributes importantly to the centripetal component of acceleration. Its contribution to the Coriolis displacements is missing from the formulas in [3].

3 Definitions for Location and Directions

The following relationships will be used to specify positions and directions:

λ = the angle above the equator locating the latitude of the shooting site. Then, the angular velocity of the earth in a Newtonian reference frame is given by

$$^{N}\boldsymbol{\omega}^{E} = \Omega\left(\cos\lambda\boldsymbol{e}_{\mathrm{N}} + \sin\lambda\boldsymbol{e}_{\mathrm{v}}\right)$$

in which Ω = 7.292 × 10^{-5} rad/s [4], and $\boldsymbol{e}_{\mathrm{N}}$ and $\boldsymbol{e}_{\mathrm{V}}$ are unit vectors in the local northward and vertical directions. α = the azimuth of the direction of firing, measured clockwise from the local North. Then, horizontal unit vectors in the azimuth and azimuth + 90° directions are given by

$$\boldsymbol{e}_{\mathrm{A}} = \cos\alpha\boldsymbol{e}_{\mathrm{N}} + \sin\alpha\boldsymbol{e}_{\mathrm{E}}$$
$$\boldsymbol{e}_{\mathrm{z}} = -\sin\alpha\boldsymbol{e}_{\mathrm{N}} + \cos\alpha\boldsymbol{e}_{\mathrm{E}}$$

in which $\boldsymbol{e}_{\mathrm{E}}$ is a unit vector directed to the local eastward direction.

β = the "look angle", or elevation of the line of sight above the horizontal. Then, unit vectors in the directions of the line of sight and 90° above it are given by

$$\boldsymbol{e}_{\mathrm{x}} = \cos\beta\boldsymbol{e}_{\mathrm{A}} + \sin\beta\boldsymbol{e}_{\mathrm{v}}$$
$$\boldsymbol{e}_{\mathrm{y}} = \boldsymbol{e}_{\mathrm{z}} \times \boldsymbol{e}_{\mathrm{x}}$$

4 Equations of Motion

Rectangular Cartesian coordinates *x, y, z*, corresponding to the unit vectors $\boldsymbol{e}_{\mathrm{x}}$, $\boldsymbol{e}_{\mathrm{y}}$, and $\boldsymbol{e}_{\mathrm{z}}$ as defined above, will be used to locate the projectile. Distance along the flight path will be denoted by *s*. The *x*-axis will be taken as the line of sight from the firearm with its origin directly above the muzzle. The following will be restricted to flat trajectories, in which d*x*/d*s* differs negligibly from unity and both $|\mathrm{d}y/\mathrm{d}s|$ and $|\mathrm{d}z/\mathrm{d}s|$ are much smaller than unity. This leads to

$$\boldsymbol{e}_{\mathrm{t}} = \boldsymbol{e}_{\mathrm{x}} + \frac{\mathrm{d}y}{\mathrm{d}x}\boldsymbol{e}_{\mathrm{y}} + \frac{\mathrm{d}z}{\mathrm{d}x}\boldsymbol{e}_{\mathrm{z}} \tag{3}$$

Useful relationships are obtained by projecting each member of Eq. (1) onto three selected directions. First, dot-multiplication with $\boldsymbol{e}_{\mathrm{t}}$ leads to

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{|\boldsymbol{f}_{\mathrm{d}}|}{m}\left(1 - \frac{\boldsymbol{e}_{\mathrm{t}} \cdot \boldsymbol{w}}{v}\right) + g\left(\sin\beta + \frac{\mathrm{d}y}{\mathrm{d}x}\cos\beta\right)$$

Small values of $\boldsymbol{e}_{\mathrm{t}}\,\boldsymbol{w}/v$ and d*y*/d*x* justify reducing this to

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{|\boldsymbol{f}_{\mathrm{d}}|}{m} - g\sin\beta$$

Further, it is known that the drag force is much greater than that of gravity, suggesting further neglecting the term $g \sin \beta$. This will have no effect except on predictions for uphill or downhill firing, where its neglect will slightly overestimate the speed with uphill firing and slightly underestimate the speed with downhill firing. In the following, the approximation

$$\frac{|f_\mathrm{d}|}{m} = -v\frac{\mathrm{d}v}{\mathrm{d}x} \tag{4}$$

will be used.

Relationships for predicting displacements in the y- and z (horizontal)-directions are obtained by dot-multiplying Eq. (1) throughout by $\boldsymbol{e}_\mathrm{y}$ and $\boldsymbol{e}_\mathrm{z}$. For the y-direction, this leads to

$$\theta' = -\frac{g\cos\beta}{v^2} - \frac{v'}{v^2}w_\mathrm{y} + \frac{2C_y\Omega}{v} \tag{5}$$

wherein primes denote d/dx, $\theta = y'$, $w_\mathrm{y} = \boldsymbol{e}_\mathrm{y}\,\boldsymbol{w}$, and $C_\mathrm{y} = \cos\lambda\sin\alpha$. Similarly, dot-multiplication with $\boldsymbol{e}_\mathrm{z}$ into Eq. (1) leads to

$$\Psi' = -\frac{v'}{v^2}w_\mathrm{z} + \frac{2C_\mathrm{z}\Omega}{v} \tag{6}$$

wherein $\Psi = z'$, $w_\mathrm{z} = \boldsymbol{e}_\mathrm{z}\,\boldsymbol{w}$, and $C_\mathrm{z} = \sin\lambda\cos\beta - \cos\lambda\cos\alpha\sin\beta$.

5 Rate of Diminishing Speed

If there are no forces in the direction of $\boldsymbol{e}_\mathrm{t}$ other than aerodynamic drag (see above comment regarding uphill and downhill firing.), Newton's second law and the definition of the drag coefficient led to the rate of change of speed given by

$$\frac{\mathrm{d}v}{\mathrm{d}x} = -C_\mathrm{D}\frac{\pi\rho v^2}{8m/d^2} \tag{7}$$

in which C_D is the projectile's drag coefficient, ρ is the air density, and m and d are the mass and diameter, respectively, of the projectile. Modern computer codes use tabulated values of C_D for numerical predictions of bullet speed and related elements of trajectories. An alternative to tabulated values has been suggested by R. L. McCoy [5] and others, as

$$C_\mathrm{D} = \frac{K}{\sqrt{M}} \tag{8}$$

wherein M is the Mach number and K is a constant selected to best-fit the tabulated values for the projectile in the speed range of interest. Many (possibly most) modern, spitzer-type projectiles exhibit a variation of drag coefficient with Mach number that is well-approximated by that of a selected standard shape called the G-7 projectile, which forms a basis of many computer codes for external ballistics predictions. The drag coefficient of the projectile under examination here is assumed to follow Eq. (8), which, within the supersonic range, closely follows that of the G-7 projectile, C_D7.

Along with the chain rule, dv/dt = v dv/ds, Eq. (7) and Eq. (8) can be combined and integrated to yield a useful expression for speed in terms of distance s:

$$v = v_0\left(1 - Cs\right)^2$$

The significance of the constant C can be revealed upon examination of Fig. 2, where the distance b can be recognised to equal

$$\left(\frac{v}{-\mathrm{d}v/\mathrm{d}s}\right)_{s=0}=\frac{1}{2C}$$

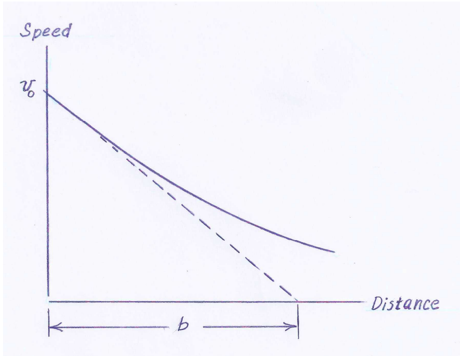

Fig. 2 Variation of speed with distance.

This leads us to define a dimensionless measure of range as

$$r=\frac{s}{2b} \tag{9}$$

in terms of which the speed varies as

$$v=v_0\left(1-r\right)^2 \tag{10}$$

From Eq. (10) we find that a projectile, initially moving at Mach $M_0 > 1$, slows to Mach 1 at $r=1-1/\sqrt{M_0}$, after which the approximation Eq. (8) cannot be relied upon.

As Section 8 below reveals, the effects of the drag force on elements of the trajectory are determined by the characteristic distance b. Toward an evaluation of b, observe that from Eq. (7)

$$-v/v'=\frac{8\,m/d^2}{\pi\rho C_\mathrm{D}} \tag{11}$$

Because the drag coefficient is usually unknown, we turn to a related *ballistic coefficient* relative to the G-7 projectile. This is defined as the ratio of the deceleration of a one-pound, one-inch diameter G-7 projectile to that of the projectile under consideration, both at the same speed and atmosphere:

$$C_\mathrm{B7}=\frac{C_\mathrm{D7}\left(m/1\,\mathrm{lb}\right)}{C_\mathrm{D}\left(d/1\,\mathrm{in}\right)^2} \tag{12}$$

Elimination of m/d^2C_D from Eq. (11) and Eq. (12) leads to

$$-v/v'=\frac{8C_\mathrm{B7}\left(0.45359237\ \mathrm{kg}\right)}{E\rho C_\mathrm{D7}\left(0.0254\ \mathrm{m}\right)^2}$$

If the constant K is selected so that Eq. (8) agrees exactly with C_D7-data at Mach 2.2, where C_D7 = 0.286, the corresponding approximation $C_\mathrm{D7}\approx 0.286\sqrt{2.2a/v}$ can be expected to be acceptably accurate throughout the supersonic range. This, along with the values of density and sonic speed of the standard ICAO atmosphere at sea-level, ρ_s = 1.2250 kg/m^3, a_s = 340.294 m/s, lead to

$$2b = \sqrt{373.532}\,[\mathrm{ms}]^{1/2}\,\frac{p_s}{p}\left(\frac{T}{T_s}\right)^{3/4} C_{B7}\sqrt{v_0} \tag{13}$$

wherein the pressure and absolute temperature of the standard ICAO atmosphere are 760 mm Hg and 288.15 K, respectively. C_{B7} is the G-7 ballistic coefficient of the projectile and v_0 is its initial speed.

Other means of estimating b can be obtained from measured or independently-predicted flight data, from the following: the time of flight can be determined from Eq. (4) by integrating $\mathrm{d}t = \mathrm{d}s/v$:

$$t = \int_0^s \frac{\mathrm{d}\sigma}{v_0\left(1-\frac{\sigma}{2b}\right)^2} = \frac{s}{v_0\left(1-\frac{s}{2b}\right)} \tag{14}$$

from which an estimate of b follows in terms of distance and flight time or speed

$$2b = \frac{s}{1-\frac{s}{v_0 t}} = \frac{s}{1-\sqrt{\frac{v}{v_0}}}$$

Tab. 1 provides comparisons of the predictions of Eq. (10) and Eq. (14) with those from a widely-used code [1]. The example there is for a bullet of G-7 ballistic coefficient C_{B7} = 0.288, launched at 1 021 m/s into an ICAO atmosphere at sea level, for which Eq. (13) predicts a distance $2b$ = 3 437.4 m.

Tab. 1 Predicted Speeds and Flight Times for C_{B7} = 0.288

	Speed, [m/s]		Flight Time, [s]	
Range, [m]	Ref. [1]	Eq. (10)	Ref. [1]	Eq. (14)
0	1 021	1 021	0.000	0.0000
400	797	797	0.4431	0.4434
800	602	601	1.0206	1.0212
1 200	435	433	1.8019	1.8057
1 600	315	292	2.9079	2.9317

6 Projectile Path

The near-agreements exhibited in Tab. 1 suggest substituting Eq. (10) into Eq. (5) and Eq. (6), so that closed-form expressions for the effects of gravity, cross-wind and Coriolis displacement can be obtained. With s replaced with x in Eq. (9), these substitutions lead to

$$\frac{\mathrm{d}\theta}{\mathrm{d}q}=\frac{2bg\cos\beta}{v_0^2q^4}-\frac{2w_{\mathrm{y}}}{v_0q^3}-\frac{4bC_{\mathrm{y}}\Omega}{v_0q^2} \tag{15}$$

$$\frac{\mathrm{d}\psi}{\mathrm{d}q}=-\frac{2w_{\mathrm{z}}}{v_0q^3}-\frac{4bC_{\mathrm{z}}\Omega}{v_0q^2} \tag{16}$$

wherein the variable $q = 1 - x/2b$ specifies the position along the flight path.

Integration of Eq. (15), with the assumption that $\boldsymbol{w}$ is constant along the flight path, yields an expression for the slope of the trajectory:

$$\theta=\theta_0-2b\left[-\frac{g\cos\beta}{3v_0^2}\left(\frac{-1}{q^3}+1\right)+\frac{w_{\mathrm{y}}}{2bv_0}\left(\frac{-1}{q^2}+1\right)+\frac{2C_{\mathrm{y}}\Omega}{v_0}\left(\frac{-1}{q}+1\right)\right]$$

Here, θ_0 is the value, at $x = 0$, of y', the small angle between the line of sight and the line of departure. This is determined by the elevation component of the sighting system. Another integration leads to

$$y=-h+\theta_0x-\left[\frac{1-2r/3}{(1-r)^2}\right]\frac{g\cos\beta x^2}{2v_0^2}+\frac{r}{1-r}\cdot\frac{w_{\mathrm{y}}x}{v_0}+\frac{2\left(\ln\frac{1}{1-r}-r\right)}{r^2}\cdot\frac{C_{\mathrm{y}}\Omega x^2}{v_0} \tag{17}$$

in which $r = x/2b$ and h is the height of the line of sight above the muzzle. The first two terms specify the line of departure (the path the projectile would follow in the absence of lateral acceleration). With the help of Eq. (14), the expression for contribution from cross wind can be rewritten as $(t - x/v_0)\boldsymbol{w}$, the well-known "delay-time" formula. The next term represents what will be herein called *gravity drop*, i.e., the drop below the line of departure. "Drop" is sometimes used to refer to that below the line of sight, unsurprisingly creating confusion.

The expression for horizontal displacement z can be obtained from Eq. (17) by replacing (h, θ_o, g, w_{y}, C_{y}) with (0, ψ_0, 0, w_{z}, C_{z}), in which ψ_0 is determined by the windage component of the sight setting.

7 The Coriolis Force Contribution

The y- and z-components of the Coriolis displacement are distinguished by the factors C_{y} and C_{z}, which depend on the geographic location and direction of firing. A common multiplier (which, incidentally, is the lateral Coriolis displacement that would result if the same launch were initiated at the North pole) gives the corresponding displacement components. The formula in [3] for the horizontal component gives this multiplier as $x\Omega\, t$, which, with the help of Eq. (14), can also be expressed as

$$\tilde{C}=\frac{1}{1-r}\cdot\frac{\Omega x^2}{v_0} \tag{18}$$

For comparison, the last term in Eq. (17) shows the multiplier to be

$$C=\frac{2\left(\ln\frac{1}{1-r}-r\right)}{r^2}\cdot\frac{\Omega x^2}{v_0} \tag{19}$$

Now, the expression $x\Omega\, t$ is exactly one that would be valid if the projectile were to follow a path that is straight *in the inertial frame*. But the Coriolis effect causes the earth-observed path to take on a curvature, which will induce a lateral component of the drag force in still air. Hence, we can expect the expression $x\Omega\tau$ to lead to an over-estimate of the displacement.

Tab. 2 presents predictions of gravity drop and the Coriolis multiplier, for the same trajectory as that of Tab. 1. Comparison between results from [1] and from Eq. (17) for gravity drop suggest that Eq. (8) leads to accurate results. Comparisons of the Coriolis multiplier predicted by Eq. (17) with that from [3] illustrate the possible extent of the overestimate from [3].

Tab. 2 Gravity Drop and Coriolis Multiplier for Trajectory of Tab. 1

Range, [m]	Gravity Drop, [m]		Coriolis Multiplier, [m]	
	Ref. [1]	Eq. (17)	Eq. (18)	Eq. (19)
400	0.887	0.889	0.0129	0.0124
800	4.315	4.320	0.0596	0.0543
1200	12.235	12.266	0.1580	0.1355
1600	28.880	29.066	0.3420	0.2716

As pointed out above, the formula in [3] for the horizontal component neglects the influence from the lateral component of the drag force. The formula for the vertical component neglects variation in both the magnitude and direction of the velocity; hence it predicts the result as if aerodynamic drag were absent. Comparisons with the prediction from Eq. (17) indicate that the formula for the vertical component can over-estimate the effect by more than 58 %.

8 Effects from Surrounding Air

Surrounding air affects several elements of the trajectory, including speed, time of flight, gravity drop, and Coriolis displacement. The effect on each of these is determined by a *drag factor*, that depends only on the single parameter $r = x/2b$, which is composed of range, ballistic coefficient, initial speed and atmospheric pressure and temperature. The factors determine how each is related to that of a "vacuum" trajectory, i.e., one with gravity the only force. These functions are readily identified as factors in Eq. (10), Eq. (14) and Eq. (17), with some values shown in Tab. 3.

9 Summary and Conclusions

Tab. 1 and Tab. 2 indicate that errors resulting from replacing the G-7 drag law with Eq. (8) will be negligible with appropriate K within the speed range $1.0 < M < 3.0$. Tabulated values of $C_{D7}/0.286(2.2\, a/v)^{1/2}$ confirm this.

Tab. 3 Drag Factors

Range Parameter $r = x/2b$						
Element	0.0	0.100	0.200	0.300	0.400	0.500
Speed	1.0	0.810	0.640	0.490	0.360	0.250
Flight Time	1.0	1.111	1.250	1.429	1.667	2.000
Gravity Drop	1.0	1.152	1.354	1.633	2.037	2.667
Coriolis	1.0	1.072	1.157	1.259	1.385	1.545

Elements of trajectories presented here are easily predicted today using one of a number of well-developed computer codes that use numerical procedures to integrate the equations of motion. An excellent one is listed as [1]. More detailed predictions than provided by "point-mass" codes like that of [1] require models of more than three degrees of freedom, instead of the assumptions described in the paragraphs following Eq. (1).

While available codes easily provide quantitatively accurate predictions, there are advantages to closed-form expressions when it comes to displaying trends associated with varying parameters. For example, these expressions show clearly how, in terms of the single parameter $r = x/2b$, aerodynamic drag modifies each element of a vacuum trajectory. Or, the increase in elevation resulting from firing uphill or downhill with a sight setting fixed for horizontal firing, is readily predicted from Eq. (17), as $\Delta y = (1 - \cos \beta)$(gravity drop) $\beta = 0$ (The irrationally-based but widely-promulgated "equivalent-horizontal-distance" method can significantly under- or over-predict the necessary correction).

Finally, Eq. (17) provides an accurate (within the supersonic range) predictor of Coriolis displacements, including heretofore-neglected effects of the drag force. Whether this will be of importance to the long-range shooter will depend on the uncertainty within the knowledge of crosswind, which may be too great to justify accounting for Coriolis displacement.

Acknowledgement

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