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ROCKET MOTOR WITH TUBULAR SOLID PROPELLANT CHARGE AND PROFILED OUTER SURFACE

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A b s t r a c t :

The paper introduces the procedure of main solid propellant charge dimensions having the tubular geometric shape with grooves at the outer surface. Main attention is paid to the general description of solid propellant rocket motor modification with the mentioned solid propellant charge.

1. Introduction

Solid **p**ropellant charges (SP) without a profiled surface in **s**olid **p**ropellant **r**ocket **m**otors (SPRM) are applied very frequently, because they have a big burning surface and can be produced simply. In cases when they are protected on SP grain fronts they burn neutrally. SP charges without the protection of fronts (in case of long slender SP charges) the slight degressive character of burning can be omitted. Other situation can be seen when using these SP charges with small slenderness. In these cases the degressive character of burning cannot be omitted and it is necessary to compensate this effect by convenient manner. One of the possibilities is the creation of longitudinal profiling in the form of grooves on outer SP grain surface.

The shape of the SP charge is evident from Fig. 1. The system of n- longitudinal grooves is created at the outer charge surface and their width influences the time and quality of degressive character of the SP charge.

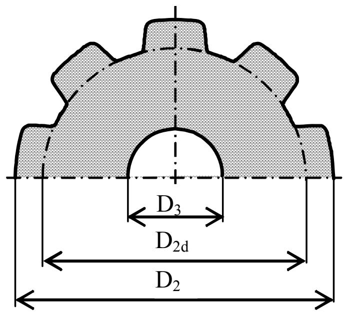

Fig. 1. Solid propellant grain cross-section

2. Determination of Main Dimensions of Solid Propellant Charge

For the determination of SP charge main dimensions are used the following relationships.

SP charge outer diameter is deduced from the rocket calibre as follows:

$$D_2 = D A_D \tag{1}$$

and the inner diameter of grooves is then:

$$D_{2d} = D c A_D , \tag{2}$$

where A_D is dimensionless SP charge diameter, c is the coefficient of the groove depth. Diameter of inner channel of the SP charge is as follows:

$$D_3 = D_{2d} - 4e_0 = D c A_D - 4e_0 , \tag{3}$$

where e_0 is the initial thickness of SP charge burning.

The shape of longitudinal groove is introduced in Fig. 2. The groove wall is inclined by the angle δ; outer groove edge and inner groove corner are rounded. Their radii dimensions are generally different. Dimensionless rounding radius of edges and inner corners will generally be:

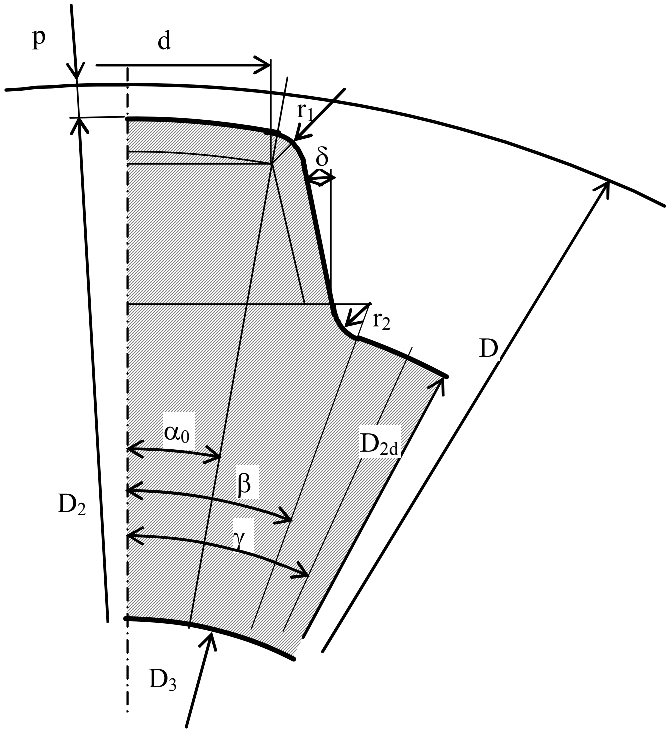

Fig. 2. Detail of the SP grain groove

$$\bar{r} = \frac{r}{D}. \tag{4}$$

Modified dimensionless rounding radius of edges and corners will then be as follows:

$$r_N = \frac{r}{D A_D} = \frac{\bar{r}}{A_D}. \tag{5}$$

For the groove shape design is important the correct choice of the groove teeth width d. During deduction can be applied with the advantage of so called half dimensionless width b [1]:

$$b = \frac{d}{2 D A_D}.$$

For chosen value of d, respectively b, the determination of angles α_0, β and γ is important for the groove dimensions. The angle of groove wall inclination δ is from technologic reasons as a rule greater than zero and its value is chosen till 10°. The angle of groove pitch is as follows:

$$\gamma = \frac{\pi}{n} .$$

The angle of groove teeth width is for selected value d given by expression:

$$\alpha_0 = \arcsin \frac{d}{D A_D - 2r_1} = \arcsin \frac{2b}{1 - 2r_{1N}} .$$

The other possibility of the groove teeth width is the choice of teeth width coefficient k_t, which is less than one. Therefore there is correct:

$$\alpha_0 = k_z \gamma = k_z \frac{\pi}{n} .$$

The half groove teeth width in such a case will be:

$$b = \frac{1 - 2r_{1N}}{2} \sin \alpha_0 .$$

The angle of the groove teeth base is given by equation [3]:

$$\beta = \arccos \left\langle \frac{y_{0N}}{c + 2r_{2N}} \left\{ \frac{\sin(2\delta)}{2} + \sqrt{\left(\frac{\sin(2\delta)}{2} \right)^2 - \cos^2(\delta) \left[1 - \frac{(c + 2r_{2N})^2}{y_{0N}^2} \right]} \right\} \right\rangle , \qquad (6)$$

where y_{0N} is as follows:

$$y_{0N} = 2b + (1 - 2r_{1N}) \cos \alpha_0 \operatorname{tg}\delta + \frac{2(r_{1N} + r_{2N})}{\cos\delta} .$$

The resulting value of the groove teeth base has to fulfil the condition $\beta \le \gamma$.

For known groove angles the initial SP charge perimeter can be determined by the equation [3]:

$$\Pi = nD(O_1 A_D + O_2) + \pi D_3 , \qquad (7)$$

where used substitutions are as follows:

$$O_1 = \alpha_0 + c(\gamma - \beta) + \frac{\cos\alpha_0 - c \cdot \cos\beta}{\cos\delta} ,$$

$$O_2 = 2\bar{r}_1 \left(\frac{\pi}{2} - \alpha_0 - \delta + \operatorname{tg}\delta - \frac{\cos\alpha_0}{\cos\delta} \right) + 2\bar{r}_2 \left(\frac{\pi}{2} - \beta - \delta + \operatorname{tg}\delta - \frac{\cos\beta}{\cos\delta} \right) .$$

Initial SP charge cross-section is given by equation:

$$A_P = \frac{n}{4} D^2 \left(P_1 A_D^2 + P_2 A_D + P_3 \right) - \frac{\pi}{4} D_3^2 , \tag{8}$$

where substitutions used are as follows:

$$P_1 = \alpha_0 + c \sin(\beta - \alpha_0) + c^2 (\gamma - \beta);$$

$$P_2 = 2(\bar{r}_1 - \bar{r}_2) \frac{\cos\alpha_0 - c\cos\beta}{\cos\delta} - 2(\bar{r}_1 c - \bar{r}_2)\sin(\beta - \alpha_0);$$

$$P_3 = 4\left[\bar{r}_1 \bar{r}_2 \left(\frac{\cos\alpha_0 - \cos\beta}{\cos\delta} - \sin(\beta - \alpha_0) \right) + \bar{r}_1^2 \left(\frac{\pi}{2} - \alpha_0 - \delta + \mathrm{tg}\delta - \frac{\cos\alpha_0}{\cos\delta} \right) - \right.$$
$$\left. - \bar{r}_2^2 \left(\frac{\pi}{2} - \beta - \delta + \mathrm{tg}\delta - \frac{\cos\beta}{\cos\delta} \right) \right].$$

Burning surface of SP charge burning on the whole surface then will be

$$S_0 = \left[nD^2 (A_D O_1 + O_2) + \pi D_3 \right] K_L + 2A_P . \tag{9}$$

Combustion chamber free cross-section is given by equation

$$A_{v0} = \frac{\pi}{4} D^2 \bar{\vartheta}^2 (1 - K_{CC}). \tag{10}$$

Combustion chamber filling coefficient is then [1], [2] :

$$K_{CC} = \frac{A_{P0}}{A_{CCef}} = \frac{\frac{n}{4} D^2 \left(P_1 A_D^2 + P_2 A_D + P_3 \right) - \frac{\pi}{4} D_3^2}{\frac{\pi}{4} D^2 \bar{\vartheta}^2} = \frac{\frac{n}{\pi} \left(P_1 A_D^2 + P_2 A_D + P_3 \right) - \bar{D}_3^2}{\bar{\vartheta}^2} \tag{11}$$

where quadratic equation for unknown value A_D will be obtained after arrangement from:

$$A_D^2 + \frac{P_2}{P_1} A_D + \frac{P_3}{P_1} - \frac{\pi}{nP_1} \left(\bar{\vartheta}^2 K_{CC} + \bar{D}_3^2 \right) = 0 . \tag{12}$$

where $\bar{D}_3 = \frac{D_3}{D}$ is the relative diameter of SP charge inner channel.

Carrying out solution of equation (12) the value A_D is as follows:

$$A_D = -\frac{P_2}{2P_1} + \sqrt{\left(\frac{P_2}{2P_1}\right)^2 - \frac{P_3}{P_1} + \frac{\pi}{nP_1}\left(\overline{\vartheta}^2 K_{CC} + \overline{D}_3^2\right)} . \qquad (13)$$

Relative inner channel diameter can be expressed from the condition of equality of clamping factors for inner and outer solid propelllant charge parts:

$$\overline{D}_3 = \frac{\frac{\pi}{n}\overline{\vartheta}^2 - P_1 A_D^2 - P_2 A_D - P_3}{O_1 A_D + O_2} \qquad (14)$$

SP charge clamping factor for the whole SP charge then will be:

$$\left(\frac{Z}{C}\right) = \frac{D^2 K_L \left[n\left(O_1 A_D + O_2\right) + \frac{\frac{\pi}{n}\overline{\vartheta}^2 - P_1 A_D^2 - P_2 A_D - P_3}{O_1 A_D + O_2} \right]}{\frac{\pi}{4} D^2 \overline{\vartheta}^2 \left(1 - K_{CC}\right)} .$$

Let us introduce the substitution

$$B_1 = \left[n\left(O_1 A_D + O_2\right) + \frac{\frac{\pi}{n}\overline{\vartheta}^2 - P_1 A_D^2 - P_2 A_D - P_3}{O_1 A_D + O_2} \right] .$$

After arrangement we shall get:

$$\left(\frac{Z}{C}\right) = \frac{4 B_1 K_L}{\overline{\vartheta}^2 \left(1 - K_{CC}\right)} .$$

Where the SP charge slenderness ratio is from as follows:

$$K_L = \frac{1}{4}\left(\frac{Z}{C}\right)\overline{\vartheta}^2 \frac{\left(1 - K_{CC}\right)}{B_1} . \qquad (15)$$

3. Ballistic and Mass Design of the Rocket with Solid Propellant Rocket Motor

Ballistic design of the rocket [3] results from the determination of the maximum firing range or from final rocket velocity as function of total SPRM impulse I_T and initial rocket mass (weight) m_0. Firing range or final rocket velocity can be determined by numerical solution of differential equations system describing the unguided rocket trajectory:

$$F = a_0 m_0 \,; \quad t_k = \frac{I_T}{F} \,;$$

$$\frac{dv}{dt} = \frac{F}{m} - c_b\, H_\tau(y)\, v\, G(v_\tau) - g \sin\theta \,;$$

$$\frac{d\theta}{dt} = -g \frac{\cos\theta}{v} \,; \tag{16}$$

$$\frac{dx}{dt} = v\cos\theta \,; \quad \frac{dy}{dt} = v\sin\theta \,;$$

$$\frac{dm}{dt} = -\frac{F}{i_s} \,.$$

For the solution it is necessary to choose the initial rocket acceleration a_0 and SPRM specific impulse i_s. The initial angle of elevation θ_0 has to correspond either to maximum firing range or to principle requirements laid on rocket trajectory for required final rocket velocity. Further on the proposal of booster **r**ocket **m**otor (RM) for anti-tank or air-defence guided rocket will be assumed where the reaching of required final velocity for chosen angle of inclination is necessary. The mass of SP charge can be determined from input data, as well as the final rocket mass, i.e.

$$m_P = \frac{I_T}{i_S} \,; \quad m_F = m_0 - m_P = m_0 - \frac{I_T}{i_S} \,.$$

The ratio of SP charge mass and final rocket mass then will be:

$$\frac{m_P}{m_F} = \frac{\dfrac{I_T}{m_0}}{i_s - \dfrac{I_T}{m_0}} \,,$$

Introduced masses can also be deduced from the mass analyses of the construction, i.e.:

$$m_P = a\, K_{CC} K_L D^3 \,; \quad m_F = m_N + b\, K_L D^3 = D^3 (C_N + b\, K_L) \,;$$

where m_N is the rocket invariable mass and C_N is relative rocket invariable mass, i.e.

$$C_N = \frac{m_N}{D^3} \,, \tag{17}$$

The coefficients are as follows

$$a = \frac{\pi}{4}\overline{\vartheta}^2\rho_P\,,\quad b = \frac{\pi}{4}\xi_C\left[\left(1-\vartheta^2\right)\rho_m + \left(\vartheta^2 - \overline{\vartheta}^2\right)\rho_{is}\right].$$

The ratio of both the masses then will be:

$$\frac{m_P}{m_F} = \frac{a\,K_{CC}K_L}{C_N + b\,K_L}\,. \tag{18}$$

The used solution is based on maximum attainable velocity, which can be found from the following function extreme $dv_F/dK_{CC} = 0$. This corresponds to the function extreme:

$$\frac{d}{dK_{CC}}\left(\frac{m_P}{m_F}\right) = \frac{d}{dK_{CC}}\left(\frac{aK_{CC}K_L}{C_N + bK_L}\right) = 0 \tag{19}$$

Marking in eq. (15)

$$K_T = \frac{1}{4}\left(\frac{Z}{C}\right)\overline{\vartheta}^2$$

then holds:

$$K_L = K_T\,\frac{1-K_{CC}}{B_1}\,, \tag{20}$$

and its derivative with respect to K_{CC}:

$$\frac{dK_L}{dK_{CC}} = -\frac{K_T}{B_1}\,,$$

Substituting the introduced equations into equation (19) and after arrangement will hold:

$$C_N = \frac{b\,K_T\left(1-K_{CC}\right)^2}{B_1\left(2K_{CC}-1\right)}\,, \tag{21}$$

and substituting eq. (21) into eq. (18) will hold:

$$\left(\frac{m_P}{m_F}\right) = \frac{aK_{CC}K_T\,\dfrac{1-K_{CC}}{B_1}}{\dfrac{bK_T\left(1-K_{CC}\right)}{B_1\left(2K_{SK}-1\right)} + bK_T\,\dfrac{1-K_{CC}}{B_1}} = \frac{a}{b}\left(2K_{CC}-1\right). \tag{22}$$

The filling coefficient is then:

$$K_{CC} = \frac{1}{2}\left[\frac{b}{a}\left(\frac{m_P}{m_F}\right)+1\right], \tag{23}$$

Ballistic coefficient can be solved from equation as follows:

$$c_b = \frac{iD^2 10^3}{m_F} = \frac{iD^2 10^3}{m_P}\left(\frac{m_P}{m_F}\right).$$

After arrangement it will hold:

$$\frac{c_b}{i \cdot 10^3} = \frac{D^2}{m_F} = \frac{D^2}{m_P}\left(\frac{m_P}{m_F}\right) = \frac{D^2}{D^3 a K_{CC} K_L}\left(\frac{m_P}{m_F}\right) = \frac{B_1(2K_{CC}-1)}{D\, b\, K_{CC} K_T (1-K_{CC})}.$$

And further on

$$D^3 = \frac{m_F B_1 (2K_{CC}-1)}{b\, K_{CC} K_T (1-K_{CC})}. \tag{24}$$

From the equation which expresses the invariable mass we shall get

$$D^3 = \frac{m_N B_1 (2K_{CC}-1)}{b\, K_T (1-K_{CC})^2}. \tag{25}$$

Substituting into previous equation we get:

$$\frac{m_F B_1 (2K_{CC}-1)}{b\, K_{CC} K_T (1-K_{CC})} = \frac{m_N B_1 (2K_{CC}-1)}{b\, K_T (1-K_{CC})^2}.$$

After some arrangement we shall get the relation for invariable mass as follows:

$$m_N = \frac{m_F (1-K_{SK})}{K_{SK}}. \tag{26}$$

By interpolation from counted values for required invariable mass can be determined the most convenient solution of the rocket for given input data.

First of all, the controlled magnitude of coefficient A_D of SP charge should be chosen when substituting the value of filling coefficient into the equation (13). When newly computed value doesn't agree with the originally chosen one, then the part of computation being formed by equations (13) and (14) has to be repeated, till the difference of two successive values A_D isn't less than the chosen accuracy of iteration.

Further on, the SP charge dimensions and main SP charge characteristics are determined.

From equations (25) and (20) the SP charge coefficient K_T should be expressed, i.e.

$$K_T = \frac{m_N B_1 (2K_{CC}-1)}{D^3 b (1-K_{CC})^2} = K_L \frac{B_1}{(1-K_{CC})},$$

where the SP charge slenderness ratio can be determined from, i.e.:

$$K_L = \frac{m_N(2K_{CC}-1)}{D^3 b(1-K_{CC})} \qquad (27)$$

4. Procedure of solution

The rocket with SPRM proposal flows out from the allocation, which contains the required maximum rocket flight velocity when the rocket SPRM is stopped and required mass of payload (rocket warhead). For the solution are chosen: the rocket calibre, required invariable rocket mass, SPRM specific impulse and required initial rocket acceleration. Further on, the set of SPRM total impulse values and independent set of initial rocket mass values is chosen. The number of chosen values will not be exactly the same:

$$I_T \in (I_{T1}, \quad I_{T2}, \quad \ldots \quad I_{Tm});$$

$$m_0 \in (m_{01}, \quad m_{02}, \quad \ldots \quad m_{0n}).$$

We can proceed during numerical solution in the following sequence. For each pair of values (I_C, m_0) by numerical solution of the set of equations (16) the rocket velocity in the moment of SPRM stopping is determined. The solution result is the matrix of reached values of rocket flight velocities in the moment of SPRM end of operation, i.e.:

	m_{01}	m_{02}	…	m_{0n}
I_{T1}	v_{F11}	v_{F12}	…	v_{F1n}
I_{T2}	v_{F21}	v_{F22}	…	v_{F2n}
…	…	…	…	…
I_{Tm}	v_{Fm1}	v_{Fm2}	…	v_{Fmn}

From this matrix by interpolation in matrix rows for required final rocket velocity we assign to each value of total impulse the corresponding initial rocket mass. For performance of interpolation in every matrix row has to be fulfilled the condition $(v_{Ki,j})_{MIN} < v_{Kreq} < (v_{Ki,j})_{MAX}$. If such a demand isn't fulfilled, it is necessary to go back to the solution beginning and to select the other extents of values I_T and m_0.

For final set of pairs of values, i.e. I_T, m_0 are determined the values m_{Pi}, m_{Fi} and $(m_P/m_F)_i$, at the base of those values are from equation (23) determined the values of total combustion chamber filling coefficients K_{CCi} and further on from equation (26) the values of the rocket invariable mass m_{Ni} are determined. By interpolation for required invariable mass can be determined the most convenient values of the SPRM total impulse, as well as the rocket initial mass. With the help of these values the most convenient value of combustion chamber filling coefficient K_{CC} is determined, being after that applied for SP charge final dimensions, i.e. D_2, D_{2d}, D_3. These values are obtained from equations (1), (2) and (14). SP charge length is determined from slenderness ration definition, i.e.:

$$L_P = K_L D .$$

The initial thickness of burning is then determined from the equation:

$$e_0 = \frac{1}{4}\left(D c A_D - D_3\right).$$

SP charge initial burning surface is determined from equation (9). The time of SPRM operation for chosen chamber pressure and kind of SP is determined then:

$$t_F = \frac{e_0}{u_1 p_{CC}^{\alpha}} .$$

SPRM nozzle critical cross-section is as follows:

$$A_{CR} = \frac{S_0 u_1 c^* \rho_P}{p_{CC}^{1-\alpha}}$$

and average SPRM thrust then will be:

$$F = \frac{i_S}{c^*} p_{CC} A_{CR} .$$

In introduced equations u_1 is unit burning rate, α is burning law exponent and c^* is SP characteristic velocity.

5. Conclusion

The performed analysis of SP charge of tube shape with profiled outer surface main characteristics represents the contribution to the theory and design of SP charges.

Ballistic and mass design of the rocket, being the base for SPRM proposal is in the case assumed modified with regard to geometric shape peculiarity of the SP charge discussed in previous paragraphs. For example of above described SPRM solution, the parameters are determined for following chosen values: required rocket final velocity v_F = 500 ms^{-1}, initial angle of elevation θ_0 = 40°, rocket calibre D = 0.1 m, rocket shape coefficient i = 1.1, initial acceleration a_0 = 300 ms^{-2}, solid propellant specific impulse i_s = 2200 $Nskg^{-1}$. The total impulse and initial rocket mass values are chosen from ranges: $I_T \in$ (7,500 ÷ 10,000) Ns and $m_0 \in$ (15 ÷ 22) kg. The number of grooves is n = 10, the radius r1 = 0,001 m, r2 = 0,002 m, the coefficient of the groove depth c = 0.9. The solution is carried out for required invariable rocket mass m_N = 12 kg.

The results after the 1st interpolation for required final velocity are introduced in tab. 1.

Tab. 1.

I_T	m_0	K_{CC}	m_N
7,500.0	15.718	0.55371	9.921
7,812.5	16.426	0.55349	10.387
8,125	17,146	0.55324	10.863
8,437.5	17.839	0.55311	11.314
8,750	18.512	0.55307	11.745
9,062.5	19.156	0.55302	12.149
9,375	19.869	0.55295	12.619
9,687.5	20.520	0.55288	13.029
10,000	21.245	0.55277	13.510

We obtain from these values by 2nd interpolation for the required rocket invariable mass the following results:

Tab. 2.

I_T	8,949	Ns	m_0	18.920	kg
m_p	4.0478	kg	m_F	14.812	kg
K_{CC}	0.55311	-	K_L	6.4155	-
L_P	0.64155	m	D_2	0.0801	m
D_{2d}	0.07209	m	D_3	0.0279	m
F	7,308	N	t_F	1.225	s

Finally, we can check the value of the final rocket velocity at the end of calculation from these resulting values of solution. The resulting value $v_F = 507.7\ ms^{-1}$ differs from the required one only in 1%. This fact confirms the validity of derived relations introduced above.

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