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THE NEW SOLUTION ASPECTS OF ELECTRIC SMALL LOOP

Reviewer: Ivan Kneppo

A b s t r a c t :

The paper deals with the vector potential calculation of an electric small loop. This type of integral occurs in antenna theory. Presented method is more simple than the standard method and makes the connection between the radiation theories of the point source at the far field and the loop at the near field.

1. Problem formulation

In antenna theory it is often needed to express electric respectively magnetic field in scattering area via wave equations. For the possibilities of the attainment of this preparation so-called vector potential was introduced. This vector potential significantly simplified the radiation solution. For a scatters located as in Fig. 1, the retarded vector potential of the electric current has three components. Its value is:

$$A \approx \iiint_V J(x', y', z') \frac{e^{-jkR}}{R} dV \tag{1}$$

where R is the dimension of vector displacement from the radiation place to observation place as shown Fig 1. In the figure it is possible to define R which is dependent on the displacement of the observation point in the centre of the coordinate system and the current element.

At the most complicated geometry of the emitter is solving of the equation (1) relatively lengthy and exacting. For this reason there are only approximate solutions in the sources [1].

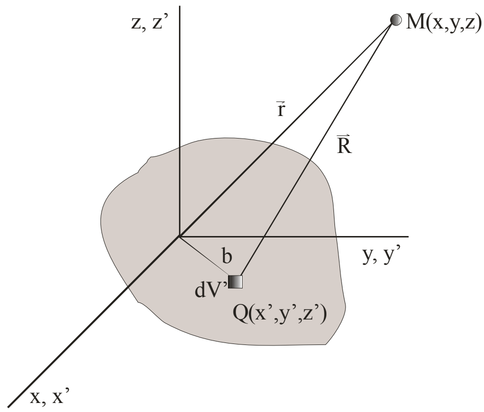

Fig. 1. The determination of the displacement from the observation point

The aim of this paper is to find an accurate solution to vector potential for a real emitter type. The name for this emitter is an electric small loop.

2. Electric small loop antenna

The dimensions of this antenna type are much smaller than the wavelength. According to [2] is the current distribution along the loop constant:

$$I_{\Phi} = \text{const.} \tag{2}$$

It means that the solution has to focus on the integral solving:

$$\iiint_V \frac{e^{-jkR(b)}}{R(b)} dV \, . \tag{3}$$

This is directly tied with loop antenna geometry. For example, the classic solution can be explained now. On the basis the loop antenna geometry (see Fig. 2) the dimensions of the position vector R(b) can be explained:

$$R(b) = \sqrt{r^2 + b^2 + 2br \sin\Theta \cos\Phi} \, . \tag{4}$$

Substituting (4) to the integrand in (3) we obtain:

$$g(b) = \frac{e^{-jk\sqrt{r^2+b^2+2br\sin\Theta\cos\Phi}}}{\sqrt{r^2+b^2+2br\sin\Theta\cos\Phi}}\,. \tag{5}$$

The integration of this function is in most sources solved by the

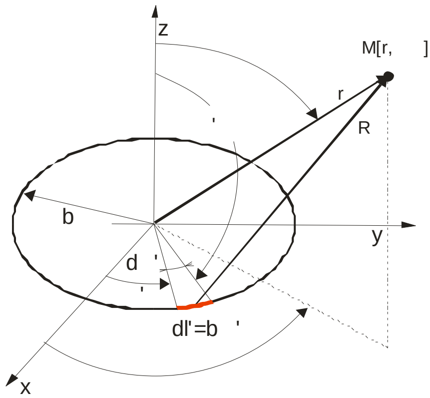

Fig. 2. Geometry of electric small loop

approximation of suitable series. The equation (3) is simplified. The sources [1] describe the evolution of function to Mac-Lauren series in point b=0. In real case b is not equal to zero. This approximation is valid only for a big distance of r. In this case the loop appears to be infinitely small:

$$\text{if} \quad r \to \infty \quad \text{then} \quad b \to 0\,. \tag{6}$$

In this case the loss of solution objectivity can be seen and the expression of function g(b) is unnecessarily demanding.

3. Modification of the solution

In this modification the solution does not use the approximation of relation (5) but only approximation of function *R(b)* and $e^{-jkR(b)}$. Each of these function calculate near the point 0. While multiplication of estimations complete approximation can be obtained:

$$f(b) = e^{-jkR(b)} \cdot \frac{1}{R(b)}\,. \tag{7}$$

According to Taylor series:

$$g(b) = g(0) + g'(0)b + g''(\xi)\frac{b^2}{2} \tag{8}$$

where ξ is point from interval (0,b). Function g(b) can be defined from the following expression:

$$g(b) = g(0) + g'(0)b \ . \tag{9}$$

and the approximation fault is:

$$\varepsilon(b) = g''(\xi)\frac{b^2}{2} \quad 0 \le \xi \le b \ . \tag{10}$$

Choose

$$g(b) = \frac{1}{R(b)} \tag{11}$$

and compute the derivation of displacement vector. The result of calculation was used in the expression of Taylor series:

$$g'(b) = -\frac{R'(b)}{R^2(b)} \tag{12}$$

from this

$$\frac{1}{R(b)} = \frac{1}{r} + \frac{1}{r^2} b\left(\sin\Theta\cos\Phi\right) + \varepsilon_1(b) \tag{13}$$

where $\varepsilon_1(b)$ is the approximation error and its valuation is:

$$g''(b) = \frac{-R''(b)R^2(b) + 2R(b)R'^2(b)}{R^4(b)} = -\frac{R''(b)}{R^2(b)} + \frac{2R'^2(b)}{R^3(b)} \tag{14}$$

next:

$$R'(b) = \frac{b - r\sin\Theta\cos\Phi}{R(b)} \tag{15}$$

after modification:

$$R''(b) = \frac{1}{R(b)} - \frac{\left(b - r\sin\Theta\cos\Phi\right)^2}{R^3(b)} \tag{16}$$

from this:

$$g''(b) = \left(\frac{1}{R(b)}\right)'' = -\frac{1}{R^3(b)} + \frac{(b - r\sin\Theta\cos\Phi)^2}{R^5(b)} + \frac{2(b - r\sin\Theta\cos\Phi)^2}{R^5(b)} . \quad (17)$$

If $b \to 0$, then $R(b) \to r$ and $g''(b) \to g''(0)$ obtained result is:

$$g''(0) = \frac{-1 + 3\sin^2\Theta\cos^2\Phi}{r^3} . \quad (18)$$

The approximation error can be obtained:

$$|\varepsilon_1(b)| \to \left|\frac{-1 + 3\sin^2\Theta\cos^2\Phi}{r^3}\frac{b^2}{2}\right| \le \frac{2b^2}{r^3} . \quad (19)$$

Choose:

$$g(b) = e^{-jkR(b)} \quad (20)$$

and use Taylor series, obtain this:

$$g(b) = e^{-jkR(b)} = e^{-jkR(0)} + \left(e^{-jkR(b)}\right)'_{b=0} b + \varepsilon_2(b) \quad (21)$$

where

$$\varepsilon_2(b) = \left(e^{-jkR(b)}\right)''_{\xi} \frac{b^2}{2} . \quad (22)$$

Similarly as above case relevant derivations can be calculated:

$$\left(e^{-jkR(b)}\right)' = e^{-jkR(b)}\left(-jkR'(b)\right) = -jke^{-jkR(b)} \frac{b - r\sin\Theta\cos\Phi}{R(b)} \quad (23)$$

and

$$\left(e^{-jkR(0)}\right)' = jke^{-jkr}\sin\Theta\cos\Phi . \quad (24)$$

The approximation of the exponential function has acquired the form:

$$e^{-jkR(b)} = e^{-jkr} + jkbe^{-jkr}\sin\Theta\cos\Phi + \varepsilon_2(b) . \quad (25)$$

Approximation error $\varepsilon_2(b)$ require the same manner as in the previous case. The calculation type has the same way. For this reason is presented only result.

$$|\varepsilon_2(b)| = \left|\left(e^{-jkR(b)}\right)''\right| \frac{b^2}{2} \le \left(k_1^2 + \frac{2k_1}{r}\right)\frac{b^2}{2} \quad (26)$$

Now we can create the Taylor approximation of the vector potential integrand (condition: constant current distribution):

$$e^{-jkR(b)}\frac{1}{R(b)} = \left(e^{-jkr} + jkbe^{-jkr}\sin\Theta\cos\Phi + \varepsilon_2(b)\right)\left(\frac{1}{r} + \frac{b}{r^2}\left(\sin\Theta\cos\Phi\right) + \varepsilon_1(b)\right)$$

$$= \frac{e^{-jkr}}{r}\left(1 + \frac{jkb^2}{r}\sin^2\Theta\cos^2\Phi + \left(jk + \frac{1}{r}\right)b\sin\Theta\cos\Phi\right) + \varepsilon(b)$$

(27)

where ε(b) is the approximation error:

$$\left|\varepsilon(b)\right| \le \frac{2b^2}{r^3}\left(1+b\right) + \left(k_1^2 + \frac{2k_1}{r}\right)\frac{b^2}{2}\left(\frac{1}{r} + \frac{b}{r^2}\right) + \frac{2b}{r^3}\left(1+b\right)\left(k_1{}^2 + \frac{2k_1}{r}\right)\frac{b^2}{2} \quad (28)$$

4. Vector potential of electric small loop

In this paper the mathematical apparatus for vector potential calculation of electric small loop has been prepared. For the concrete loop characteristics computing the current distribution is needed.

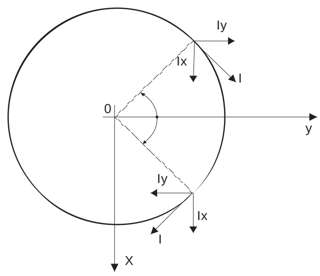

Fig. 3. Determination of electrical current in a small loop

The standing wave on the electric small loop doesn't exist. In this case the loop current distribution is constant $I_\Phi = \text{konst}$ and the current has everywhere the same amplitude and phase.

For computing there must be chosen two elements on loop, the localization of which are symmetric because of the y axis. In these elements will be a constant current but its orientation in space is different. In Fig. 3 the current I_Φ can be seen which is possible divided on two components I_x and I_y. Component I_x is affect in the same orientation and are added. The component in I_y is affect in mutually opposite orientation and their effect is eliminated. The result of this consideration is: the field in radiation zone is created by I_x component. This current can be as follows:

$$I_x = I_\Phi \cos\Phi \tag{29}$$

Put the equation (29) into the integral of vector potential and the obtained result is:

$$A_\Phi = \frac{\mu b I_\Phi}{4\pi} \int_0^{2\pi} \cos\Phi \frac{e^{-jkr}}{r} \left(1 + \frac{jkb^2}{r} \sin^2\Theta \cos^2\Phi + \left(jk + \frac{1}{r} \right) b \sin\Theta \cos\Phi \right) d\Phi$$

$$= \frac{\mu b I_\Phi}{4r} e^{-jkr} \left(jk + \frac{1}{r} \right) \sin\Theta \tag{30}$$

5. Conclusion

In this paper the alternative possibilities of the vector potential solution for an electric small loop antenna are presented. In comparison with classic solution where the integrand of vector potential is extended into Mac-Lauren series and the approximation is sufficiently exact only for the loop whose dimensions are very close to zero. In this case the information about field shape in the far zone is losing at calculation. The loop is no appearing in this far zone as infinitesimally small. Approximation which was used in our solution is significantly exact for the real loop dimension. The reinstate approximation into the integral of vector potential is the obtained relation (30) which has the same expression as is asserted in [1]. From these verities two conclusions can be deduced:

- the right calculation of the vector potential
- the field solution in the far zone for environs of electric small and elemental electric loop is the same

The presented method is original. The substitution of the integrand of till used Mac-Laurin series is exact, but only about zero point of view thus only for the loop size approaching to zero. The applied Taylor series gets correct results also for the neighborhood of the point of view thus for the real loop size. The method can be used in the analytical description of the antennas created on the base of small loop antennas as are disc antennas, cylindrical antennas and its clones.

References:

[1] Balanis, A., C.: *Anthenna Theory Analysis and Design*, Haper & Row, Publishers, New York, 1982

[2] Vavra, Š., Turán, J.: *Antény a šírenie elektromagnetických vĺn*, [Antennas and Propagation] Alfa, Bratislava, 1989

[3] Jarník, V.: *Diferenciální počet I* [Differential Calculus I], Academia, Praha 1974

[4] Petrek, J.: *Interference Simulation for Ultra Wideband Signal*, Comite 2005, Prague, Czech Republic, Sept. 26 – 28, 2005, pp. 195 – 198

[5] Harťanský, R., Maga, D.: *Possibilities of Electrical Equipment Radiation Interference Measurement,* PCIM 2003, Nurmberg, May 20 -22, 2003, pp. 377 - 381

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