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AMMUNITION SHOT ACCURACY TEST WITH FOULED GUN

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A b s t r a c t :

This article stipulates the purpose, procedure and methodology for evaluation of shooting with ammunition using a fouled gun. The procedure for evaluation is based on statistical analysis of data obtained during shooting process with a clean and a fouled gun (position and dispersion parameters).

1. Introduction

Requirements of our customers regarding the quality of ammunition keep rising. One of the new requirements is to maintain the accuracy of ammunition when shooting with a fouled gun. The need for such requirements and its classification into the group of prescribed customer requirements for ammunition and its acceptance shall be developed in two aspects.

The first one is related to the development trend in the field of maintainability of arms and ammunition. Manufacturers of ammunition, who have succeeded in implementing the maintenance free system for ammunition, still work on this system and try to keep extending the prescribed interval for maintenance works (cleaning, lubrication).

The second issue is that the demand from customers is becoming more and more strict (e.g. commercial shooting ranges), they demand that one of the decisive properties of ammunition determining its quality–accuracy – be maintained for the longest period of shooting possible in order to prevent users from interrupting the shooting process owing to maintenance (cleaning) of their guns.

The requirement for maintenance of shooting accuracy even during shooting with a fouled gun is expressed by customers clearly and in an understandable manner. However, the question still is how to measure the conformance to this requirement. We need to determine methods for quantitative evaluation and expression of this requirement for particular type of ammunition. Methods for testing of ammunition (ČSN 395105) have not included nor described this test.

The aim of this article is to stipulate the purpose, procedure of testing and to determine methods for evaluating test results.

2. Test Purpose

The test shall be performed in order to discover, whether the accuracy of ammunition has not changed during the shooting process led with a particular type of ammunition using a fouled gun, e.g. whether it has not deteriorated compared to shooting with a clean gun (meaning a clean barrel as well). The purpose of this test can be formulated as follows:

"This test is conducted to examine, whether the fouling in bore left during shooting with a functioning gun does not cause any adverse change to the accuracy of the specific ammunition type."

This test is to prove or disprove the fact that once the barrel is fouled during shooting, such situation would not cause a statistically significant change to the accuracy of shooting with the particular type of ammunition.

3. Test Procedure

The test procedure comprises three partial tests:

1) The shooting accuracy test with ammunition shot from a clean gun (barrel). During this test, we will make n_1 shots from a clean, functioning gun fixed properly at a target sheet or an automatic target at the distance conforming to technical conditions for evaluation of shooting accuracy. We record shooting results per hit, i.e. coordinate values (x_{1i}, y_{1i}) for $i = 1,2,\dots n_1$ (side and height of the hit). It is also required to make the test with $n_1 > 30$ shots.

2) Functional testing of ammunition shot from the same gun as the one used for test No. 1. Shooting will ensure fouling of the barrel and gun. The number of rounds shot complies with customers requirements. If the number has not been specified, shooting shall be conducted with the same number of rounds as prescribed for the functional testing of ammunition.

3) Shooting accuracy test for the particular ammunition type with a fouled gun (barrel). During this test, we will shot n_2 rounds from a fouled gun at a target sheet or an automatic target, under same conditions as for test No. 1. Fixing of the gun, distance, ambient conditions and the target point shall be identical as for the test No. 1. We will record shooting results per hit, i.e. coordinate values (x_{2i}, y_{2i}) for $i = 1,2,\ldots n_2$. The number of rounds shot from a clean and fouled gun may equal to ($n_1 = n_2$), yet this is not a prerequisite. That means tests No. 1 and 3 can be taken with different number of shots ($n_1 \neq n_2$). It is important that the number of shots at both tests n_1 and n_2 be greater than 30.

This procedure of performance of three partial tests will provide us with two independent results related to the process of shooting with a clean or fouled gun under identical conditions. These two dispersion patterns represent two independent sets with the scope of n_1 and n_2, which are expected to have normal distribution. We further assume that dispersion values for both sets are unknown and different.

The aim of this test is to find out, whether these two results of shooting differ with respect to position and dispersion (variability) parameters or not. The instrument used here will be the statistical analysis of both shooting results.

4. Test Results Evaluation Methodology

The methodology for evaluation of test results is based on testing of statistical hypotheses. We formulate a couple of hypotheses, which will concern the shooting accuracy of a particular type of ammunition shot using a clean and a fouled gun. The first hypothesis is called the zero hypothesis and marked with H_0. The second hypothesis is called the alternate hypothesis and marked with H_1.

We formulate the zero hypothesis H_0 as follows: the shooting accuracy of the particular type of ammunition using a clean and a fouled gun has not changed (see *Figure 1*).

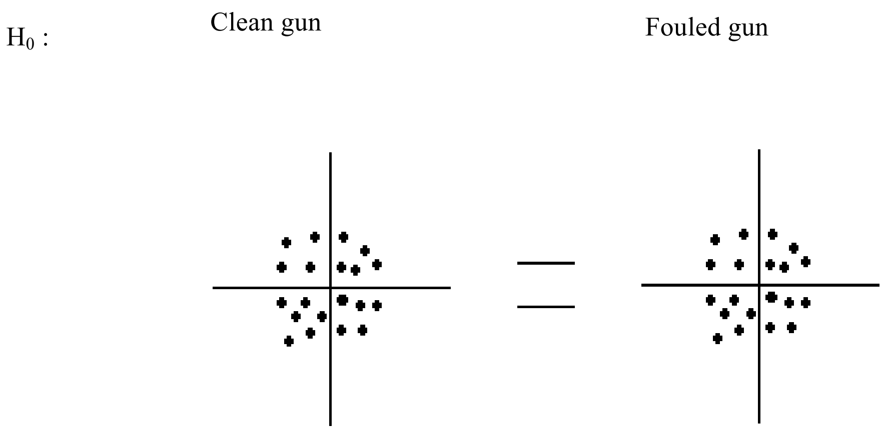

Figure 1 Zero hypothesis H_0, the shooting accuracy has not changed

This zero hypothesis is opposed by the so called alternate hypothesis H_1 saying that the gun fouling caused change to the shooting accuracy with the particular type of ammunition (see *Figure 2*). The alternate hypothesis denies the shooting accuracy stated in the zero hypothesis.

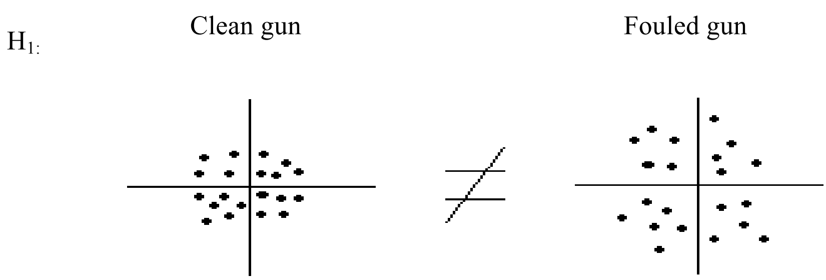

Figure 2 Alternate hypothesis H_1, the shooting accuracy has been changed

To verify the correctness or incorrectness of the formulated hypotheses we need to use a convenient testing criterion, which shall comprise results obtained from both shooting processes. This test criterion will then represent our statistics, i.e. random variable, which can assume only certain values for approval or refusal of hypotheses.

As the testing of hypotheses is associated with conclusions, which are drawn according to data obtained from random sampling – dispersion patterns formed in the shooting process with number of hits being n_1 and n_2, our consideration can also lead to a wrong conclusion that we refuse the zero hypothesis tested, even though it is actually true and valid. The probability of occurrence of such mistake during testing is equal to the level of significance of α, which will help us verify the correctness or incorrectness of formulated hypotheses. Then the probability of drawing a wrong conclusion will be α. For example in case the $\alpha = 0{,}05$, the probability of drawing a wrong conclusion for the zero hypothesis is 5%. The value of significance level selected for this test is $\alpha = 0{,}05$, unless the customer requires another level. This level represents the most commonly used acceptable risk of error for all fields of industry.

We said that the aim of this test was to find out, whether results of shooting with a clean and fouled gun would not differ with respect to position and dispersion parameters. Therefore the shooting accuracy of the particular type of ammunition is tested at the level of significance α using the conformance test of:

- the mean point of impact
- the dispersion.

The test of conformance of the mean point of impact (MPI) will provide us with an answer to the question, whether there has been statistically significant change made to the mean value of the shooting process when using a fouled gun MPI_2 compared to shooting with a clean gun MPI_1.

The conformance test of shooting dispersion (s^2) points at the statistically significant or insignificant change to the variability of shooting with a fouled gun $s^2{}_2$ compared to shooting with a clean gun $s^2{}_1$.

The dispersion test can be also conducted using the dispersion characteristics of $2R100$, i.e. a circumference containing 100% of hits. In this case we test the change of dispersion circumference $2R100_2$ for the fouled gun compared to the dispersion circumference $2R100_1$ the clean one.

From the mathematical point of view, the mean point of impact test (MPI) and the conformance test of dispersion circumference $2R100$ represent the test of compliance of two mean values, i.e. test of hypotheses about the conformance of two diameters.

The test of dispersion (variability) conformance s^2 is the test of hypothesis about the conformance of two dispersions. The test of conformance between mean values is also called the Student´s T-Test (for unknown and various dispersion values) and the test of conformance of dispersions is called the Fischer-Snedocorov F-Test.

The zero hypothesis for the mean point of impact (MPI) will be expressed in the following form:

H$_0$: $\bar{x}_1 = \bar{x}_2$

$\bar{y}_1 = \bar{y}_2$

where $$\bar{x}_1 = \frac{1}{n_1} \cdot \sum_{i=1}^{n1} x_{1i}\,, \quad \bar{x}_2 = \frac{1}{n_2} \cdot \sum_{i=1}^{n1} x_{2i}\,,$$

$$\bar{y}_1 = \frac{1}{n_1} \cdot \sum_{i=1}^{n1} y_{1i}\,; \quad \bar{y}_2 = \frac{1}{n_2} \cdot \sum_{i=1}^{n1} y_{2i}\,.$$

We will write the zero hypothesis for the dispersion characteristic of $2R100$ as follows:

H$_0$: $\overline{2R100}_1 = \overline{2R100}_2$

Where $$\overline{2R100}_1 = \frac{1}{m_1} \sum_{i=1}^{m} 2R100_{1i}\,,$$

$$\overline{2R100}_2 = \frac{1}{m_2} \sum_{i=1}^{m} 2R100_{2i}\,,$$

m –number of (ranging) hits after ten shots (i = 10), the value required is $m > 3$.

We will write the zero hypothesis for the dispersion of shooting process s^2 as follows:

H$_0$: $s_{x1}^2 = s_{x2}^2$

$s_{y1}^2 = s_{y2}^2$

where $$s_{x1}^2 = \frac{1}{n_1 - 1} \cdot \sum_{i=1}^{n1} (x_{1i} - \bar{x}_1)^2\,, \quad s_{y1}^2 = \frac{1}{n_1 - 1} \cdot \sum_{i=1}^{n1} (y_{1i} - \bar{y}_1)^2\,,$$

$$s_{x2}^2 = \frac{1}{n_2 - 1} \cdot \sum_{i=1}^{n1} (x_{2i} - \bar{x}_2)^2\,, \; s_{y2}^2 = \frac{1}{n_2 - 1} \cdot \sum_{i=1}^{n1} (y_{2i} - \bar{y}_2)^2\,.$$

The alternate hypothesis H_1 for the mean point of impact (MPI) will be expressed as follows:

$$H_1: \quad \overline{x}_1 \neq \overline{x}_2$$

$$\overline{y}_1 \neq \overline{y}_2 .$$

The interpretation for dispersion circumference 2*R*100 will be:

$$H_1: \quad \overline{2R100}_1 \neq \overline{2R100}_2 .$$

The interpretation for dispersion of shots s^2 will be:

$$H_1: \quad s_{x1}^2 \neq s_{x2}^2 \quad .$$

$$s_{y1}^2 \neq s_{y2}^2 .$$

In case of testing the mean point of impact (MPI), the testing criterion is determined by the relationship:

$$t_x = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\frac{s_{x1}^2}{n_1} + \frac{s_{x2}^2}{n_2}}} \qquad \text{for the } x \text{ axis,}$$

$$t_y = \frac{\overline{y}_1 - \overline{y}_2}{\sqrt{\frac{s_{y1}^2}{n_1} + \frac{s_{y2}^2}{n_2}}} \qquad \text{for the } y \text{ axis.}$$

This testing criterion features the Student´s *t*- distribution with (υ) degrees of variance [1]. The number of variance degrees can be calculated using the formula:

$$\upsilon_x = \frac{\left(\frac{s_{x1}^2}{n_1} + \frac{s_{x2}^2}{n_2} \right)^2}{\frac{s_{x1}^4}{n_1^2(n_1 - 1)} + \frac{s_{x2}^4}{n_2^2(n_2 - 1)}} \qquad \text{for the } x \text{ axis,}$$

$$\upsilon_y = \frac{\left(\frac{s_{y1}^2}{n_1} + \frac{s_{y2}^2}{n_2} \right)^2}{\frac{s_{y1}^4}{n_1^2 (n_1 - 1)} + \frac{s_{y2}^4}{n_2^2 (n_2 - 1)}}$$ for the *y* axis.

The testing criterion for dispersion characteristic $2R100$ is defined by the formula:

$$t = \frac{\overline{2R100}_1 - \overline{2R100}_2}{\sqrt{\frac{s_{2R100_1}^2}{m_1} + \frac{s_{2R100_2}^2}{m_2}}}$$

where

$$s_{2R100_1}^2 = \frac{1}{m_1 - 1} \cdot \sum_{i=1}^{m1} (2R100_{1i} - \overline{2R100}_1)^2 ,$$

$$s_{2R100_2}^2 = \frac{1}{m_2 - 1} \cdot \sum_{i=1}^{m2} (2R100_{2i} - \overline{2R100}_2)^2 .$$

This testing criterion features the Student´s *t*-distribution with (υ) degrees of variance. The number of variance degrees can be calculated using the formula:

$$\upsilon = \frac{\left(\frac{s_{2R100_1}^2}{m_1} + \frac{s_{2R100_2}^2}{m_2} \right)^2}{\frac{s_{2R100_1}^4}{m_1^2 (m_1 - 1)} + \frac{s_{2R100_2}^4}{m_2^2 (m_2 - 1)}} .$$

In order for the application of zero hypothesis H_0 saying that the mean point of impact (MPI) has not changed, the values of testing criteria (t_x *a* t_y) must fall within the region of non-refusal (the interval of acceptance):

$(t_{\alpha/2} (\upsilon) ; \; t_{1-\alpha/2} (\upsilon))$, where $(t_{\alpha/2} (\upsilon)$ and $t_{1-\alpha/2} (\upsilon)$ are quantiles of the Student´ distribution for the level of significance α and the number of degrees of variance (υ), while the following applies $t_{\alpha/2} = - t_{1-\alpha/2}$ (see *Figure 3*).

The level of significance α = 0,05 is associated with the region of non-refusal of the zero hypothesis H_0 $(t_{0,025} ; t_{0,975})$, where $t_{0,025}$ a $t_{0,975}$ are quantiles of the Student´

distribution for the number of degrees of variance (υ), while the following applies $t_{0,025} = - t_{0,975}$.

The region of refusal of the zero hypothesis, i.e. acceptation of the alternate hypothesis H_1 saying that the dispersion circumference 2*R*100 has changed after shooting with a fouled gun (barrel) is determined by the so called critical region, i.e. a set of values greater than $t_{1-\alpha/2}(\upsilon)$ or lower than $t_{\alpha/2}(\upsilon)$ (see *Figure 4*). For $\alpha = 0{,}05$ the value of testing criterion must be (t) greater than $t_{0,975}(\upsilon)$ or lower than $t_{0,025}(\upsilon)$.

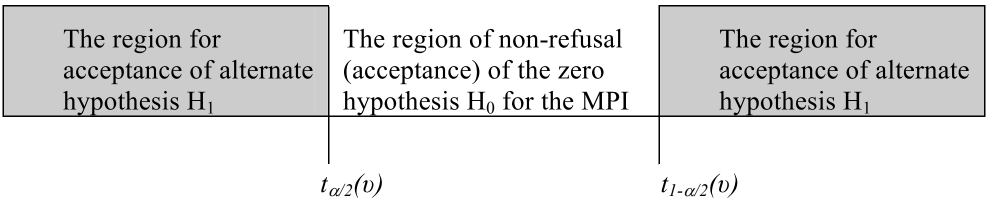

Figure 3

In order for the zero hypothesis to be true, saying that the dispersion circumference 2*R*100 has not changed, the value of testing criterion (t) must fall within the non-refusal region (the interval of acceptance) : $(t_{\alpha/2} (\upsilon) ; t_{1-\alpha/2} (\upsilon))$, where $t_{\alpha/2} (\upsilon)$ and $t_{1-\alpha/2} (\upsilon)$ The quantiles of Student´s distribution are once again α for the level of significance and (υ) for the number of degrees of variance, while the following applies $t_{\alpha/2} = - t_{1-\alpha/2}$ (see *Figure 4*).

The level of significance of $\alpha = 0{,}05$ is associated with the region of non-refusal of the zero hypothesis H_0 $(t_{0,025} ; t_{0,975})$, where $t_{0,025}$ and $t_{0,975}$ are quantiles of the Student´s distribution for the number of degrees of variance (υ), whereas the following applies $t_{0,025} = - t_{0,975}$.

The region of refusal of the zero hypothesis, i.e. acceptation of the alternate hypothesis H_1 saying that the dispersion circumference 2*R*100 has changed during shooting with a fouled gun (barrel), is defined by the so called critical region, i.e. a set of values greater than $t_{1-\alpha/2}(\upsilon)$ or lower than $t_{\alpha/2}(\upsilon)$ (see *Figure 4*). For $\alpha = 0{,}05$ the value of testing criterion (t) has to be greater than $t_{0,975}(\upsilon)$ or lower than $t_{0,025}(\upsilon)$.

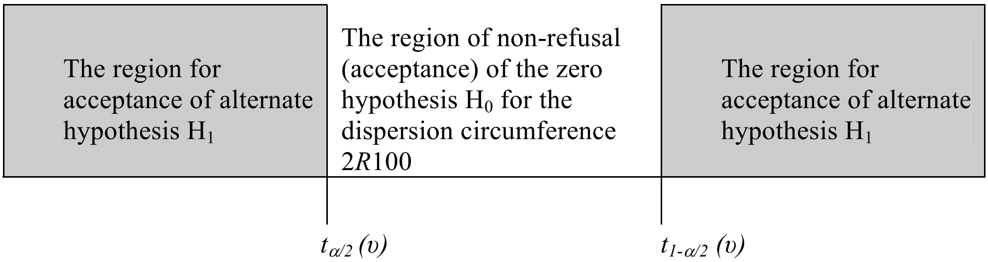

Figure 4

In case of the shooting dispersion conformance test the testing criterion is defined by the formula:

$$F = \frac{s_1^2}{s_2^2}$$

i.e. for the *x* axis: $F_x = \frac{s_{x1}^2}{s_{x2}^2}$ and for the *y* axis: $F_y = \frac{s_{y1}^2}{s_{y2}^2}$.

If the zero hypothesis is valid: $s_1^2 = s_2^2$, saying that the dispersion has not changed, the testing criterion will feature the F distribution (Fischer-Snedocorov distribution) with $\upsilon_1 = n_1 - 1$ and $\upsilon_2 = n_2 - 1$ degree of variance [1].

In order for the zero hypothesis H_0 to be true, saying that the accuracy of shooting with a fouled gun (barrel) has not changed, the values of testing criteria (F_x a F_y) must fall within the region (interval of acceptance) : ($F_{\alpha/2}\ (\upsilon_1, \upsilon_2)$; $F_{1-\alpha/2}\ (\upsilon_1, \upsilon_2)$), where $F_{\alpha/2}\ (\upsilon_1, \upsilon_2)$ and $F_{1-\alpha/2}\ (\upsilon_1, \upsilon_2)$ and the quantiles of *F*-distribution for the value of significance α and the number of degrees of variance $\upsilon_1 = n_1 - 1$ a $\upsilon_2 = n_2 - 1$, while the following applies: $F_{\alpha/2}\ (\upsilon_1, \upsilon_2) = 1/F_{1-\alpha/2}\ (\upsilon_1, \upsilon_2)$ (see *Figure 5*).

The region of non-refusal of the zero hypothesis H_0 for the level of significance α = 0,05 is $(F_{0,025}\ ; F_{0,975})$, where $F_{0,025}$ and $F_{0,975}$ are quantiles of the *F*-distribution for (υ_1, υ_2) degrees of variance, while the following applies: $F_{0,025} = 1/F_{0,975}$

The region of refusal of the zero hypothesis and the acceptation of alternate hypothesis H_1, saying that the dispersion of shooting has changed after gun (barrel) fouling, defines the so called critical region, i.e. a set of values greater than $F_{1-\alpha/2}(\upsilon_1, \upsilon_2)$ or lower than $F_{\alpha/2}(\upsilon_1, \upsilon_2)$ (see *Figure 5*).

For $\alpha = 0{,}05$ the values of testing criteria (F_x a F_y) must be greater than $F_{0,975}(\upsilon_1, \upsilon_2)$ or lower than $F_{0,025}(\upsilon_1, \upsilon_2)$.

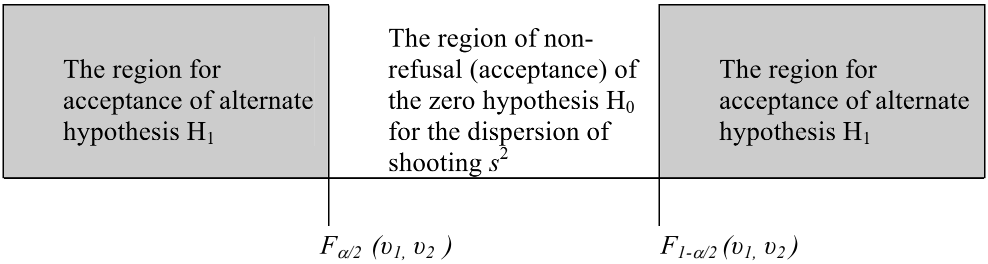

Figure 5

Decisions on zero hypothesis H_0 referring to individual characteristics of the shooting process are listed exactly in the table below.

Shooting characteristic	Testing criterion	Zero hypothesis not refused	Zero hypothesis refused
Mean point of impact $(\bar{x}, \bar{y})$	t	$t_{\alpha/2}(\upsilon) < t < t_{1-\alpha/2}(\upsilon)$	$t < t_{\alpha/2}(\upsilon)$ $t > t_{1-\alpha/2}(\upsilon)$
Dispersion circumference 2R100	t	$t_{\alpha/2}(\upsilon) < t < t_{1-\alpha/2}(\upsilon)$	$t < t_{\alpha/2}(\upsilon)$ $t > t_{1-\alpha/2}(\upsilon)$
Shooting dispersion (s^2)	F	$F_{\alpha/2}(\upsilon_1, \upsilon_2) < F < F_{1-\alpha/2}(\upsilon_1, \upsilon_2)$	$F < F_{\alpha/2}(\upsilon_1, \upsilon_2)$ $F > F_{1-\alpha/2}(\upsilon_1, \upsilon_2)$

Example

The accuracy test conducted with 9x19 mm cartridge generated values of dispersion circumferences 2R100 from four ranging hits after ten shots, which are listed in the table below. gun fouling was ensured by firing more than 100 rounds for the particular type of ammunition.

Hit No. (a´10 rounds)	2*R*100 clean gun	2*R*100 fouled gun
1	31,4 mm	37,2 mm
2	41,6 mm	41,0 mm
3	40,0 mm	34,9 mm
4	33,0 mm	43,3 mm

With the 5% level of probability, we want to prove the hypothesis stating that the fouling of gun did not cause any change to the accuracy of the particular type of ammunition.

The data from this table will help us determine the mean value of dispersion circumference when shooting with a clean gun.

$$\overline{2R100_1} = \frac{1}{4}\sum_{i=1}^{m} 2R100_{1i} = 36{,}5 \text{ mm}$$

and the dispersion $s^2_{2R100_1} = \frac{1}{m_1 - 1}\cdot\sum_{i=1}^{m1}(2R100_{1i} - \overline{2R100_1})^2 = 25{,}50 \text{ mm}^2$.

Further we will determine the mean value of dispersion circumference when shooting with a fouled gun

$$\overline{2R100_2} = \frac{1}{4}\sum_{i=1}^{m} 2R100_{2i} = 39{,}1 \text{ mm}$$

and the dispersion

$$s^2_{2R100_2} = \frac{1}{m_2 - 1}\cdot\sum_{i=1}^{m2}(2R100_{2i} - \overline{2R100_2})^2 = 13{,}91 \text{ mm}^2.$$

The value of testing criterion is: $t = \frac{\overline{2R100_1} - \overline{2R100_2}}{\sqrt{\frac{s^2_{2R100_1}}{m_1} + \frac{s^2_{2R100_2}}{m_2}}} = -0{,}828.$

The number of degrees of variance is: $$\upsilon = \frac{\left(\frac{s^2_{2R100_1}}{m_1} + \frac{s^2_{2R100_2}}{m_2} \right)^2}{\frac{s^4_{2R100_1}}{m_1^2(m_1-1)} + \frac{s^4_{2R100_2}}{m_2^2(m_2-1)}} = 5{,}5 \cong 5.$$

In the table with Student´s distribution [1] we will seek for the critical value $t_{0,975}$ = 2,57 for the υ=5 degrees of variance. The second quantile $t_{0,025}$ has the value $t_{0,025} = - t_{0,975}$ = -2,57.

As the value of testing criterion t= -0,828 lies within the interval of (-2,57, 2,57) and the condition $t_{\alpha/2}(\upsilon)$=-2,57 < t=-0828 < $t_{1-\alpha/2}(\upsilon)$=2,57 has been met, we do not refuse the zero hypothesis on the level of significance 5 %, i.e. the change of dispersion circumference 2R100 in connection with gun fouling has not been proven. The test did no prove the difference between accuracy of shooting with the particular type of ammunition using a clean or a fouled gun (barrel). This means that tested results of accuracy of shooting with a clean or fouled gun are identical from statistical point of view and the differences between ranging hits must be considered statistically insignificant. Fouling of a barrel bore with particular type of ammunition is not significant and does not affect the accuracy if shooting.

R e f e r e n c e s

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