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VIRTUAL CATHODE OSCILLATOR FOR REPEATED GENERATION HPM

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A b s t r a c t :

This paper deals with the high power microwave generator with virtual cathode – vircator. There are introduced pulsed power supply method of vircator, structure of axial vircator, measurement methods and experimental results. There are discussed possible applications of vircator in the Directed Energy Weapons Microwave (DEWM) area.

1. Introduction

A wide range of HPM devices exists. Relativistic klystrons, magnetrons, slow wave devices, reflex triodes and vircators are all examples of the available technology base [1]. From the perspective of an electromagnetic weapons or warheads designer, the device of choice will be at this time the vircator. The vircator is a one shot device capable of producing a very powerful single pulse of radiation, yet it is mechanically simple, small and robust, and can operate over a relatively broad band of microwave frequencies. The physics of the vircator tube are substantially more complex than those of the preceding devices. The fundamental idea behind the vircator is that of

accelerating a high current electron beam against a foil anode. Many electrons will pass through the anode, forming a bubble of space charge behind the anode. Under the proper conditions, this space charge region will oscillate at microwave frequencies. If the space charge region is placed into a resonant cavity which is appropriately tuned, very high peak powers may be achieved. Conventional microwave engineering techniques may then be used to extract microwave power from the resonant cavity. Because the frequency of oscillation is dependent upon the electron beam parameters, vircators may be tuned or chirped in frequency, where the microwave cavity will support appropriate modes. Power levels achieved in vircator experiments ranges from 170 kW to 40 GW over frequencies spanning the decimetre and centimetre bands [1].

2.   Vircator operation fundamentals

The basic idea of the vircator is to accelerate a dense flush of an electron beam against a grid or a foil anode. Plenty of electrons pass through the anode and form a region of a space charge behind the anode called "virtual cathode". This region of a space charge at corresponding conditions can oscillate in a region of microwave frequencies. It is possible to tune the vircator in a broad band of frequencies using only a change of a space charge density. There is not necessity to have an external magnetic field to right vircator function. There is several configuration of vircator displayed in Fig. 1. In Fig. 1a there is an axial vircator. Electron beam pass through the foil anode. Microwave power is brought out axially, too. It is possible in dependence of requirements of application to bring out the microwave energy in transverse direction. This case of a vircator configuration is in Fig. 1b. Reditron is an interesting configuration of generator with virtual cathode. There is not the foil anode but there is a massive anode with big holes opposite the cathode, Fig. 1c. In Fig. 1d there is a reflective triode with the virtual cathode. In a reflective triode with the virtual cathode there is central electrode positive with respect to coaxial geometry of circuitry opposite to standard geometry when it is negative. On the central electrode there is a shank which holds anode plate and cathode is mounted to the metallic surface of generator. Frequency changes appear in vircator with standard geometry when the distance between anode and cathode get smaller due to filling the working space by plasma. Efficiency of standard geometry vircator is ordinarily about ones percent.

Despite of its low efficiency vircator is very attractive for army applications because it is very simple to make it and it is a compact device and there it is not necessary to have an external magnetic field. For more information about construction and properties of various types of vircator you can see e. g. [1] [2] [3] [4] [5].
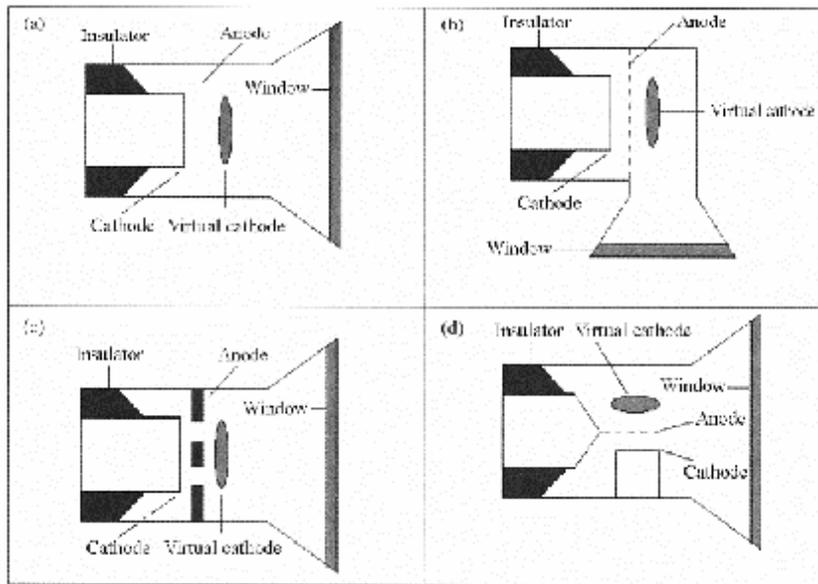
Fig. 1. Schematic representation of vircator configuration
a) axial, b) transverse, c) reditron, d) reflective triode

Technical issues in vircator design are output pulse duration, which is typically of the order of a microsecond and is limited by anode melting, stability of oscillation frequency, often compromised by cavity mode hopping, conversion efficiency and total power output. Coupling power efficiently from the vircator cavity in modes suitable for a chosen antenna type may also be an issue, given the high power levels involved and thus the potential for electrical breakdown in insulators.

3.   Mathematical description of vircator operation

For microwave frequencies generation it is necessary to meet number of conditions relevant to power supply and electrode geometric proportions of vircator. Determination of this conditions results from [6] and we use simplified geometry displayed in Fig. 2 to derive mathematical characterization.

The most common case is that a pulse duration τ is much longer than a beam transit time across cavity it $t >> L/C$.

We assume that an external axial magnetic field obstruct transverse electron motion. The space charge of beam makes a negative potential energy eF in drift space which breaks the electrons. If the space charge potential reaches the value of accelerating voltage (electrons are stopped in beam) beam with bigger current can not expand.

There is a value of current which cause stopping of electrons. This value is called vacuum critical current. There have to be an above critical current to vircator generate microwave oscillation.
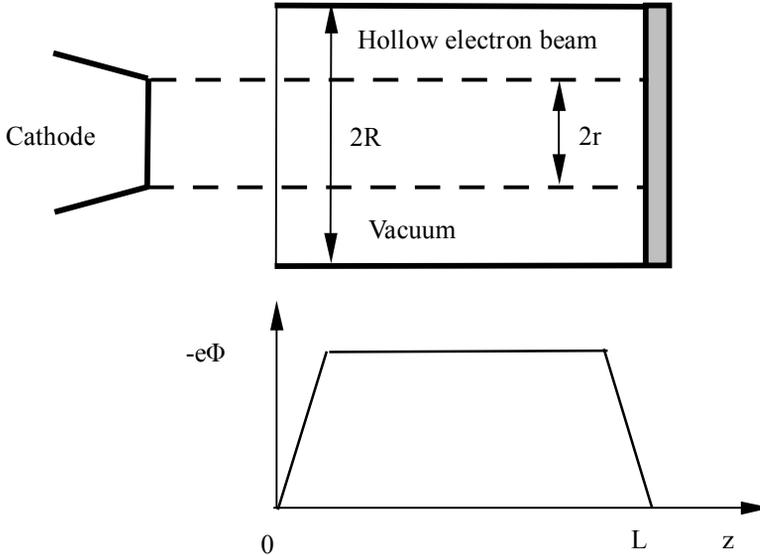
Fig. 2. Simplified geometry of axial vircator

We suppose a hollow beam; its charge is concentrated in thin layer with radius *r*. Behind transition space (which is comparable with radius of chamber – backward conductor) there is nearly homogenous potential. It is possible to determine potential quantity with consideration that it is a coaxial capacitor and its inner cylinder is saturated by beam charge.

Capacity of coaxial capacitor is given by equation

$$C = \frac{2 \cdot p \cdot e_0}{\lg\left(\dfrac{R}{r}\right)} \tag{1}$$

where
    ε_0 is the vacuum electric permitivity [Fm^{-1}],
    R is chamber radius [m],
    r is beam radius [m].

Potential is given by equation

$$\Phi = \frac{Q}{C} \tag{2}$$

where

Q is the charge linear density [Cm^{-1}],

C is the capacity per length unit [Fm^{-1}].

We can define charge linear density

$$Q = \frac{I}{v} \tag{3}$$

where

v is an electrons module velocity [ms^{-1}] and current is defined by equation

$$I = e \cdot n \cdot v \cdot S \tag{4}$$

where

e is the charge of electron, e = 1.602x10^{-19} C,

n is the linear electron density [m^{-3}],

S is a profile of electron beam [m^2].

When substitute Eq. (1) and (3) into Eq. (2) we can obtain expression for potential in this form

$$\Phi = \frac{Q}{C} = \frac{I \cdot \lg\left(\frac{R}{r}\right)}{2 \cdot p \cdot e_0 \cdot v} \tag{5}$$

The electron velocity v is connected with their initial energy and potential by energy preservation law

$$m_e \cdot g_0 \cdot c^2 + e\Phi = m_e \cdot g \cdot c^2 \tag{6}$$

where

γ_0 is initial relativistic factor of beam,

γ is relativistic factor inside a system reduced due to potential.

Relativistic factor inside a system is possible to express by equation

$$g = g_0 + \frac{e\Phi}{m_e \cdot c^2} \tag{7}$$

where

m_e is electron mass, m$_e$ = 9.108x10^{-31} kg,

c is speed of light [ms^{-1}].

Relativistic factor is possible to express also by equation

$$g = \frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}} \tag{8}$$

Electrons velocity in drift space v is possible to specify from following equations

$$g^2 = \left(g_0 + \frac{e\Phi}{m_e \cdot c^2}\right)^2 \tag{9}$$

$$v = c \cdot \sqrt{1 - \frac{1}{\left(g_0 - \dfrac{e\Phi}{m_e \cdot c^2}\right)^2}} \tag{10}$$

Substitute Eq. (10) into Eq. (3) and multiple by $\dfrac{e \cdot v}{m \cdot c^3}$ we can find relation

$$F(\Phi) = \frac{e \cdot \Phi}{m \cdot c^2} \cdot \sqrt{1 - \frac{1}{\left(g_0 - \dfrac{e\Phi}{m_e \cdot c^2}\right)^2}} = \frac{e \cdot I \cdot \lg\left(\dfrac{R}{r}\right)}{2 \cdot p \cdot e_0 \cdot m \cdot c^3} \tag{11}$$

Function $F(\Phi)$ has peak value in the conditions describes with relation

$$\frac{e \cdot \Phi}{m \cdot c^2} = g_0 - g^{\frac{1}{3}} \tag{12}$$

The evaluation is written in the form

$$F(\Phi) = \sqrt{\left(g^{\frac{2}{3}} - 1\right)^3} = \frac{e \cdot I \cdot \lg\left(\dfrac{R}{r}\right)}{2 \cdot p \cdot e_0 \cdot m \cdot c^3} \tag{13}$$

There is not possibility to find solution of relation (11) for bigger values of function $F(F)$. According to relation (13) the critical vacuum current is

$$I_{cr} = \frac{2 \cdot p \cdot e_0 \cdot m \cdot c^3}{e \cdot I \cdot \lg\left(\dfrac{R}{r}\right)} \sqrt{\left(g^{\frac{2}{3}} - 1\right)^3} \tag{14}$$

When we substitute known constant (like $\pi = 3.1415$) we obtain equation

$$I_{cr} = \frac{8.5}{\lg\left(\dfrac{R}{r}\right)} \sqrt{\left(g^{\frac{2}{3}} - 1\right)^3} \tag{15}$$

where

    I_{cr} is critical vacuum current [kA]

For vircator implementation it is considered energy of electrons about 500 keV which respond to relativistic factor of $\gamma 0 = 2$. Thus planar diode current limited by space charge follows three-half Child-Langmuir law

$$I \approx \frac{U^{\frac{3}{2}} \cdot S_k}{d_{ka}^{2}} \tag{16}$$

where

    U is voltage between anode and cathode [V],

    S_k is surface of cathode [m^2],

    d_{ka} is distance between anode and cathode [m].

Important characteristic of vacuum planar diode is its impedance which is given by equation

$$Z = 1.36 \cdot 10^5 \cdot U^{-\frac{1}{2}} \cdot \frac{d_{ka}^{2}}{r_k^{2}} \tag{17}$$

where

    Z is impedance of vacuum planar diode [Ω]

    r_k is radius of cathode [m].

Vacuum critical current is a maximal current can distribute itself through vacuum. It means it is possible to measure the same current until to distance bigger than diameter of beam. It is just the same value when potential of space charge of electrons beam is equal to accelerating voltage. There is the limit when the virtual cathode is generated and vircator start working. The generated frequency depends on a plasma frequency of

electron beam. The plasma frequency is determined by the electron beam current density. The plasma frequency is frequency of electron in space charge field oscillation. The electrons can oscillate due to influence of repulsive force which takes effect between particles with the same charge. The plasma frequency depends on concentration of electrons in beam. The current density is given by number of charges which are able to pass through the surface 1 cm^2 per one second. Thus we can calculate the current density i [Am^{-2}]

$$i = e \cdot n \cdot c_e \tag{18}$$

where

e is electron charge e = 1.602x10^{-19} C,

n is electron volume density [m^{-3}],

c_e electron velocity [ms^{-1}].

Plasma frequency of electron beam is given by equation

$$f_{pe} = 9 \cdot 10^7 \cdot \sqrt{i} \tag{19}$$

4.   Pulsed power supply

For some long pulse applications it is desirable to couple the Marx generator directly to the vacuum diode; however, the pulse rise time is then limited by the Marx inductance and capacitance, and the generator impedance is greater than typically several tens of ohms. In order to produce short, fast-rising, low-impedance beam outputs it is customary to use the Marx to charge a pulse-forming line (PFL). Although the PFLs may be constructed in a variety of shapes (strip, coaxial, radial, etc.), they are typically used in only two types of circuits – the simple transmission line, and the double, or Blumlein line. In contrast to the simple transmission line, an alternate circuit invented by A. D. Blumlein is capable of producing an output pulse into a matched load that equals the charge voltage. A cylindrical version of the Blumlein circuit fabricated by team is represented in Fig. 3. It consists of three coaxial cylinders with the intermediate cylinder being charged by the Marx generator (Fig. 4). The inner cylinder is connected to the outer grounded cylinder by means of an inductor. The inductor acts as a short circuit during the charge cycle, and then as an open circuit for the short duration of the output pulse [2].

Pulsed power supply parameters:

Marx generator		Pulse forming line	
Number of stage	$n = 18$	Impedance	9.12 Ω
Capacity of 1 stage	100 nF	Length	1 m
Input voltage	$U_{in} = 25$ kV	Pulse duration	60 ns
Output voltage	$U_{out} = 450\ kV$	Working medium	*water*
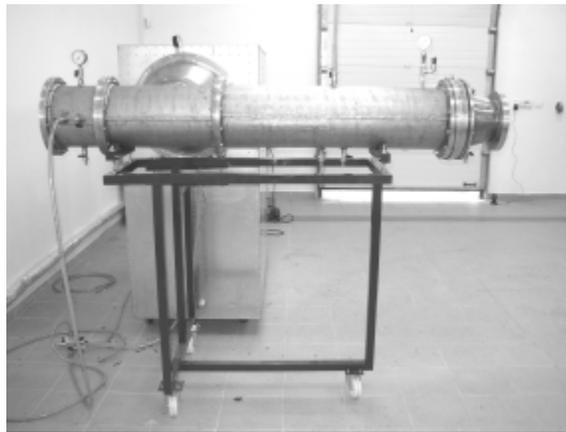
Fig. 3. A cylindrical version of the Blumlein circuit
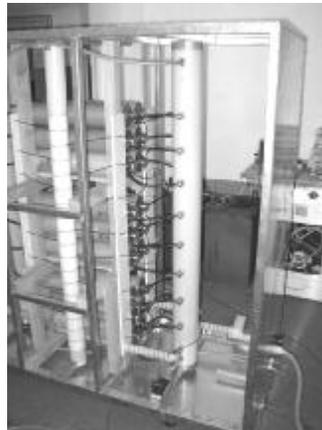
Fig. 4. Marx generator

5.   Vircator design

Following theoretical assume given above the axial vircator with grid anode was fabricated. Structure of grid anode enables repeated operations in contradistinction to vircator with foil anode. The cathode diameter r_k = 17.5 mm and working chamber radius R = 40 mm were choice by reason of decrease value of vacuum current. The critical vacuum current is I_{cv} = 4.63 kA of this configuration. The vircator impedance match to the forming line is very important requirement of vircator correct working. Because of that there is fabricated cathode with possibility to change distance d_{ka} between cathode and anode.

In Fig. 5 there is section picture of built-up axial vircator wit possibility to change distance between anode and cathode. In Fig. 6 there is a view of fabricated vircator.
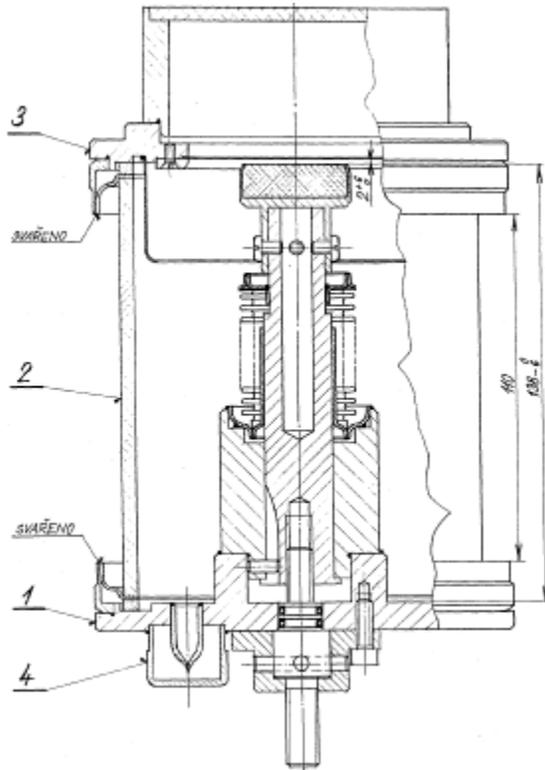
Fig. 5. Section picture of built-up axial vircator wit possibility to change distance
between anode and cathode:
1 cathode contact ring, 2 ceramic insulator, 3 anode contact ring,
4 pipe for vacuum pumping shelter

Fig. 6. View of fabricated axial vircator with adjustable distance between anode and cathode

6. Measured characteristics

For the right function guarantee, the working point setting and the properties inquiry of vircator it is necessary to measure some fundamental characteristics on the vircator input and output. This chapter is concerned to the detailed description of the vircator fundamental characteristics measurement.

6.1. Feeding pulsed voltage

The pulsed voltage incoming from the shaping line is measured using two stage voltage divider. The first stage is design from liquid divider with a fusion of water and bluestone. The divider is a part of shaping line. Its configuration is in the Fig. 7. The second stage is constructed from a conventional low inductivity resistance divider. The output impedance of the second divider is 50 Ω because of matching to a coaxial cable.
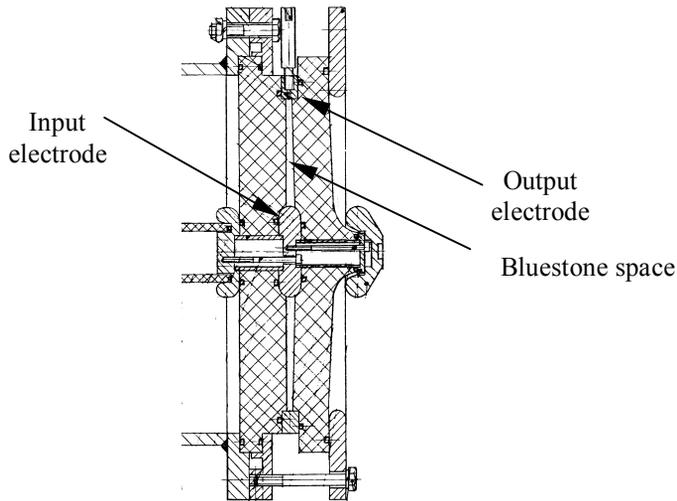
Fig. 7. Sectional view of the liquid divider.

6.2. Vircator current

The Rogowski coil was used for vircator flowing current measurement. Theoretical description of function and construction of the Rogowski coil is written in [7]. We can find the Rogowski coil voltage as

$$U_R = k_R \cdot \frac{\mathrm{d}I}{\mathrm{d}t} \tag{20}$$

where

U_R is Rogowski coil voltage [V],

k_r is Rogowski coil sensitivity is given by Eq. (21),

$\dfrac{\mathrm{d}I}{\mathrm{d}t}$ is differentiation of current flow through space of Rogowski coil,

$$k_R = m_0 \cdot \frac{N}{l} \cdot S \tag{21}$$

where

N is number of Rogowski coil turns,

l is length of Rogowski coil [m],

S is profile of Rogowski coil [m^2].

From the equations written above it is obvious that the Rogowski coil voltage is proportional to a differentiation of current flow through space of Rogowski coil. It is necessary to make a coil voltage integration to get a linear relation between the current sampled by Rogowski coil and the voltage on this coil. For the parameters and the time characteristics of the current flow through the vircator determination we can use the Rogowski coil inductivity L_{rog} and the resistor R_s as an integrator. We can measure the voltage time behaviour on the resistor R_s. We can determinate the equation for the vircator output current calculation using the integrator substitution diagram with the inductivity as it is shown in Fig. 8. The resistor R_{rog} reflects a Rogowski coil internal resistance. The voltage V reflects the Rogowski coil generated voltage which is induced by a current flow through vircator. The voltage U is a real measured voltage on resistor R_s.
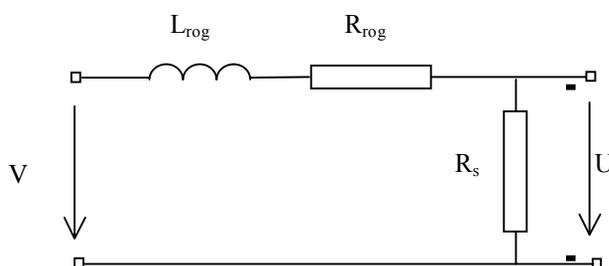
Fig. 8. Substitution diagram of integrator for vircator flowing current measurement.

We can describe the integrator according to Fig. 8. by following relation

$$U + t \cdot \frac{\mathrm{d}U}{\mathrm{d}t} = bV \tag{22}$$

where

b is an attenuation factor which is described by equation

$$b = \frac{R_s}{R_{rog} + R_s} \tag{23}$$

where

R_{rog} is an internal resistance of Rogowski coil.

The time constant τ is defined by equation

$$t = b \cdot \frac{L_{rog}}{R_s} \tag{24}$$

When we integrate relation (22) we can find for a voltage U

$$U = \frac{b}{t} \cdot \int V \cdot \mathrm{d}t - \frac{1}{t} \cdot \int U \cdot \mathrm{d}t \qquad (25)$$

The final equation for the current flow through the vircator I we can get using a substitution V from Eq. (20) and modification of this Eq:

$$I = \frac{L_{rog}}{k_r \cdot R_s} \left[U + \frac{R_s}{b \cdot L_{rog}} \cdot \int\limits_0^t U \cdot \mathrm{d}t \right] \qquad (26)$$

It is a common that the RL-integrator time constant is usually choosen ten times greater than the pulse duration. If the pulse duration and the RL-integrator constant do not keep to this term it is necessary to calculate the current flow through vircator using Eq. (26).

The Rogowski coil connection to the recording device which is a digital oscilloscope is usually made by the coaxial cable with a characteristic impedance of $Z_0 = 50\ \Omega$ according to Fig. 9.
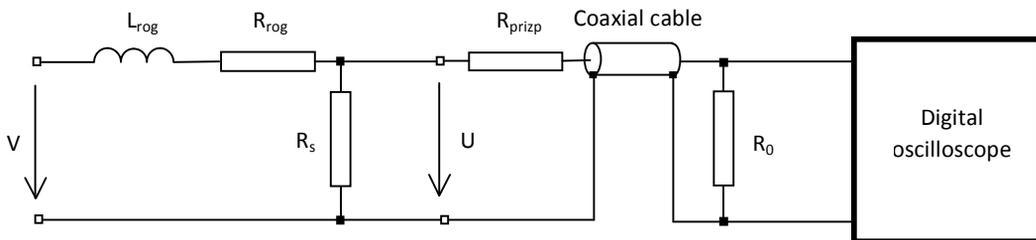
Fig. 9. Diagram of connection the Rogowski coil to the recording device.

We choose the value of the resistor $R_s = 1\ \Omega$. Because of matching to the coaxial cable we choose the value of resistor $R_{priz} = 49\ \Omega$. On the side of oscilloscope there is the matching resistor $R_0 = 50\ \Omega$. Because the duration of current pulse is near the same as the RL-integrator time constant is, for obtaining the right value of output current it is necessary to make a numerical integration of a digital oscilloscope data using Eq. (26).

Another possibility to measure the heavy current is using method based on a Faraday magneto optical effect. For more information you can see [8] [9] [10] [11] [12].

6.3. Output energy

The calorimetric energy sensor Gentek is used for the output energy measurement. This sensor is placed in to the cylindrical waveguide. The time dependence of the energy is displayed on the screen of the digital oscilloscope. The scale of displayed energy is 2.5 V/J (marked curve in the Fig. 10).
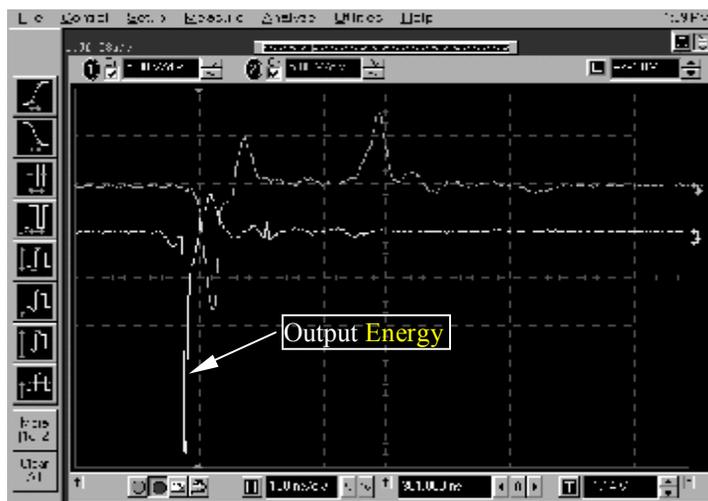
Fig. 10. Time dependence of output energy

6.4. Output microwave power

For measurement output microwave power of the vircator there were designed and made two versions of measuring device which are using calorimetric method to power measurement. The first measuring device was established for measurement microwave power in a circular waveguide, the second for measurement in free space. The calorimetric sensor was of the disc design. The carbon with changed crystal lattice was designed for use as one of the thin layer types. The prototype of the sensor was designed for the measurement of vircator with output power of $P_{max} = 250$ MW, length of pulse $t_p = <10, 60>$ ns. In the Fig. 11 there is function chart of calorimetric measuring device for measurement microwave power in circular waveguide and in the Fig. 12 there is implementation of this device and principle scheme of the sensor circuit. In the Fig. 13 there is view of calorimetric measuring device for measurement microwave power in free space.

For the detailed description of function and calorimetric measuring device implementation for microwave power measurement you can see [8] [9] [10] [12].
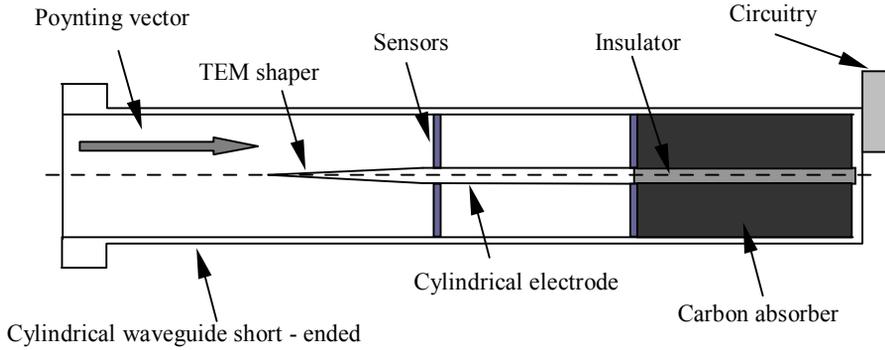
Fig. 11. Function chart of calorimetric measuring device for measurement microwave power in circular waveguide
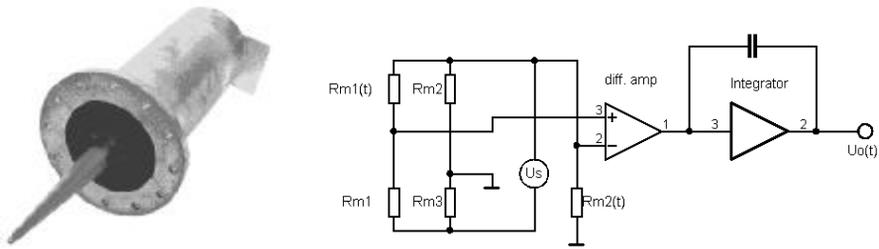
Fig. 12. Waveguide-fitted calorimetric sensor and principle scheme of the sensor circuit

Fig. 13. Implementation of free space calorimetric measuring device

7. Conclusion

During testing of vircator was reach energy value **10.2 J** for pulse width **44 ns**. The energy to agree with the microwave output power value **233 MW**.

The anode structure allows compared to usual published solutions repeated operation because there is not a destruction of anode. It happen high-frequency discharge on output window at some tests along vircator development and testing period. This problem will have to be solved in further period of research work.

It is possible to find two basic sphere of vircator in military technology practical apply:

- The vircator can be used directly as the high power generator in Directed Energy Weapons Microwave (DEWM).

- Vircator can be used like High Power Microwave (HPM) generator during immunity of electronic devices testing in Electromagnetic Compatibility (EMC) area.

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