



# The Using of Kalman Filtering for Doppler Difference/Time Difference of Arrival (DD/TDOA) Systems

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## Abstract:

*This article is focused on the target localization and target velocity vector determination by the digital receiver. The target trajectory is filtered by Kalman filter whereas the target position and simultaneously the target velocity vector are input data of this filter. The verification of the mathematical solution of this type Kalman filter is performed by MATLAB simulation.*

## Keywords:

*Kalman filtering, Target localization, Digital receiver, DD/TDOA*

## 1. Introduction

The determination of target trajectory is one of the most important tasks of ELINT systems. Consequently, the Kalman filter must be a component of all modern ELINT systems. New ELINT systems, which are based on digital technology, digital signal processing etc. bring new possibilities in the area of target location methods. These methods can improve the accuracy of the target trajectory estimation. One of these new methods can be based on simultaneous measurements of frequency differences and time differences of received signals which can offer information both about target position and about target velocity vector. The Kalman filtering of position information and velocity information can bring more accurate target trajectory estimation.

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## 2. The Kalman Filter Input Data Determination

The fully digital receiver makes possible to calculate Cross Ambiguity Function (CAF) of received signals. The knowledge of CAF of received signals offers following advantages:

1. The frequency difference (Doppler difference) of received signals (on different receiving stations) can be determined. Thus, the velocity vector of the target can be calculated.
2. The knowledge of Doppler difference makes possible to improve the accuracy of target position by Kalman filtering, when the target velocity vector and its covariance matrix are used.
3. Extremely, two targets with the same position and the same radiated signal can be distinguished in case their velocity vectors are different.

Assume that the system has the digital receiver and it is able to determine the target velocity vector by Doppler difference (DD) method and the target position by Time Difference of Arrival (TDOA) method. Thus, the combined method DD/TDOA is used in this system. The operation of this method supposes the using of three stationary receiving stations in 2-D space. This situation is shown in the Figure 1.

The algorithm of DD/TDOA method enables to compute the target coordinates  $x$ ,  $y$  and the components of the target velocity vector  $v_x$ ,  $v_y$  in case the receiving station coordinates  $a$ ,  $b$ ,  $c$ , the time differences of signal arrival  $t_{CL}$ ,  $t_{CR}$  and the frequency differences of received signals  $fd_{CL}$ ,  $fd_{CR}$  are known (are measured).

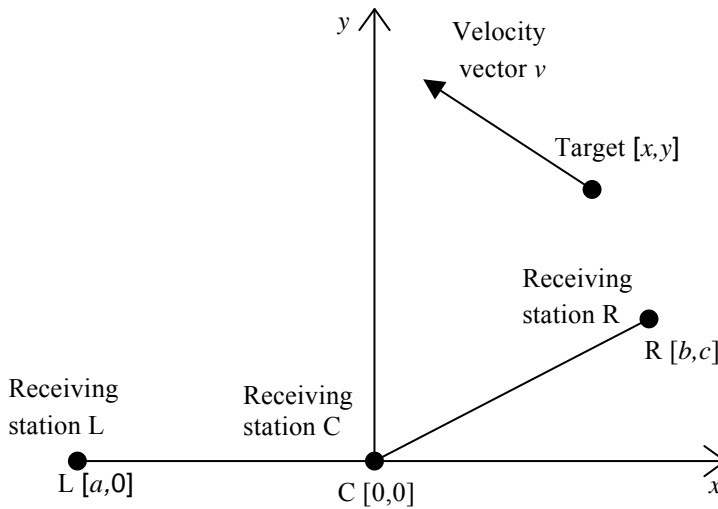


Fig. 1 The system configuration

### 3. The Kalman Filter

The Kalman filter is very frequently used as the instrument in the branch of tracking of target. The principles of operation of the general Kalman filter are well known [1, 2]. Thus, the Kalman filter can be defined by following equations:

$$H = SP \cdot M' \cdot \text{inv}(R + M \cdot SP \cdot M'), \quad (1)$$

$$X = XP + H \cdot (Y - M \cdot XP), \quad (2)$$

$$S = (I - H \cdot M) \cdot SP. \quad (3)$$

Where:

$XP$  is the matrix of the state variables that are predicted from previous measurements.

$SP$  is the matrix of the dispersion of the state variables that are predicted from previous measurements.

$Y$  is the matrix of measured values of  $x, y, v_x, v_y$ .

$R$  is covariance matrix of measured values of  $x, y, v_x, v_y$ .

$M$  is the matrix of measurement.

$I$  is the identity matrix.

$H$  is the Kalman filter gain.

$X$  is the matrix of actual filtered values of state variables.

$S$  is the matrix of the dispersion of actual values of state variables.

The prediction of the filter is defined by next equations:

$$XP = F \cdot X, \quad (4)$$

$$SP = F \cdot S \cdot F' + Q. \quad (5)$$

Where:

$F$  is the transient matrix.

$Q$  is the noise matrix.

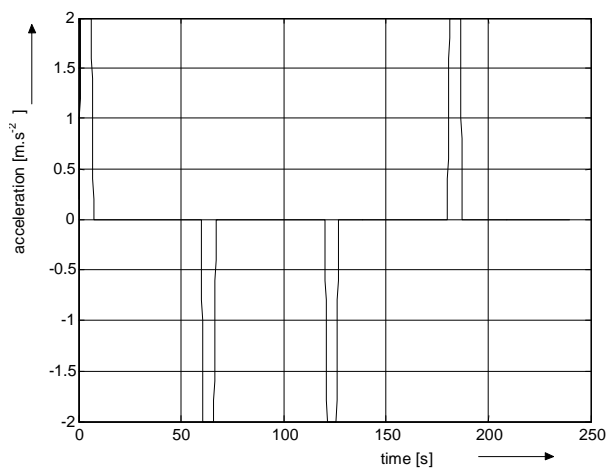
Thus, the knowledge of matrixes  $Y, R, M, F, Q$  and  $XP_1, SP_1$  (index 1 denotes an initial step of the filter operation) is necessary condition for function of the Kalman filter. Values of matrixes  $Y$  and  $R$  are given by solution of the DD/TDOA algorithm. Matrix  $M$  is expressed by equation:

$$Y = M \cdot X. \quad (6)$$

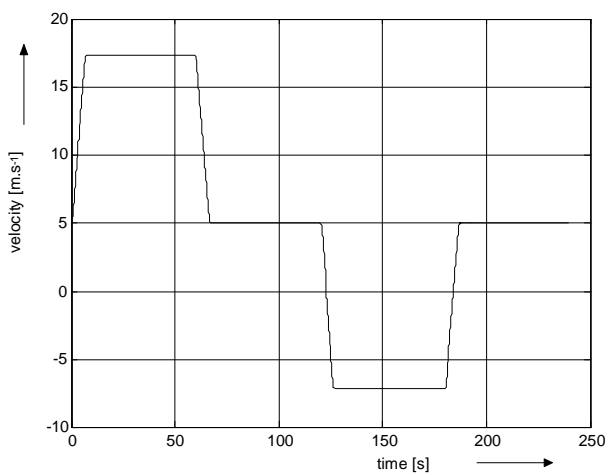
Matrixes  $F, Q, XP_1$  and  $SP_1$  are solved by the Singer target motion model.

### 4. Singer Target Motion Model

In the first step, the trajectory of target motion is generated. Thus, the target accelerations in axes  $x$  and  $y$  and target velocity are generated as the time function. These functions are shown in Figures 2, 3, 4 and 5. The overall time of target motion is 240 s (the sample rate is  $1/T = 10$  Hz, thus, the number of samples is 2400).



*Fig. 2 The x-axis target acceleration*



*Fig. 3 The x-axis target velocity*

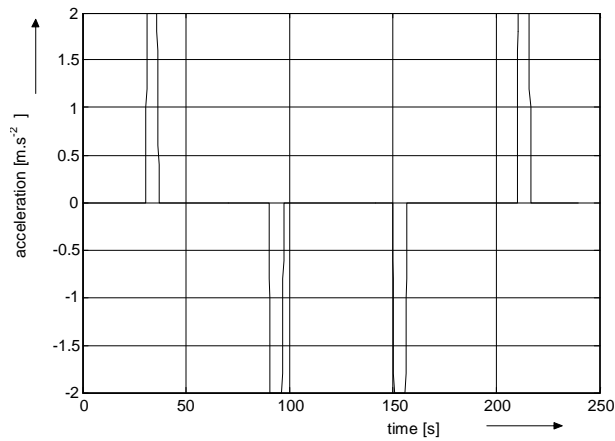


Fig. 4 The y-axis target acceleration

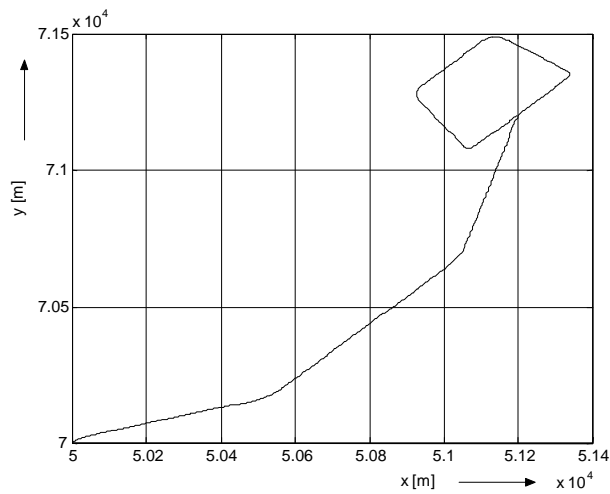


Fig. 5 The target trajectory (start point is in coordinates  $x = 50$  km,  $y = 70$  km)

In the second step, construction of Singer target motion model, the probabilities of zero target acceleration  $P_0$  and maximum target acceleration  $P_{\max}$  must be determined. Next, the value of maximum target acceleration  $A_{\max}$  must be specified. The values of  $P_0$ ,  $P_{\max}$  and  $A_{\max}$  can be determined from the acceleration histogram. This histogram is shown in Figure 6.

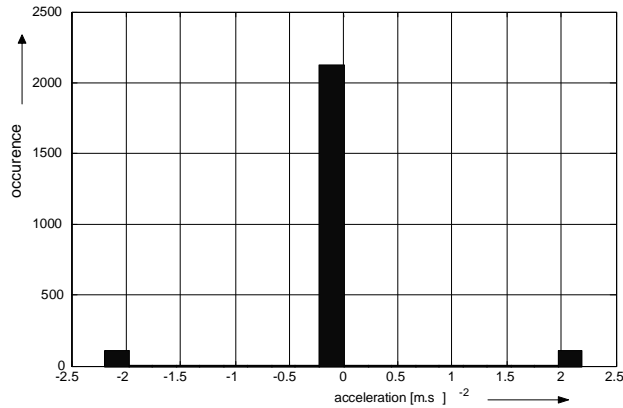


Fig. 6 The acceleration histogram

If the overall time of target motion is 240 s and sample rate is 10 Hz then (from Fig.6.) we can evaluate:

$$\begin{aligned} A_{\max} &= 2 \text{ m/s}^2, \\ P_0 &= 2120/2400 = 0.8833, \\ P_{\max} &= 104/2400 = 0.0433. \end{aligned}$$

The knowledge of values of  $P_0$ ,  $P_{\max}$  and  $A_{\max}$  enables numeration of elements of the  $Q$  matrix with using the equation:

$$S_a^2 = \frac{A_{\max}^2}{3} (1 + 4 \cdot P_{\max} - P_0). \quad (7)$$

(For this target trajectory is  $\sigma_a^2 = 0.3865 \text{ m}^2/\text{s}^4$ ).

The parameter  $t$  generally describes the time interval of main lobe of correlation function of target acceleration [1]. We can see (from Fig. 3, 4), that the target manoeuvre takes approximately 5 s, thus,  $t = 5 \text{ s}$  and  $a = 1/t = 0.2 \text{ Hz}$ .

Then, the transcient matrix  $F$  can be solved by equation:

$$F(T, t) = \begin{bmatrix} 1 & T & t^2 \left[ -1 + \frac{T}{t} + \exp\left(-\frac{T}{t}\right) \right] \\ 0 & 1 & t \left[ 1 - \exp\left(-\frac{T}{t}\right) \right] \\ 0 & 0 & \exp\left(-\frac{T}{t}\right) \end{bmatrix} \quad (8)$$

and matrix  $Q$  by equation :

$$Q = 2 \cdot a \cdot s_a^2 \cdot \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{12} & q_{22} & q_{23} \\ q_{13} & q_{23} & q_{33} \end{bmatrix}. \quad (9)$$

$$q_{11} = \frac{1}{2a^5} \left( 1 - e^{-2aT} + 2aT + \frac{2a^3T^3}{3} - 2a^2T^2 - 4aTe^{-aT} \right),$$

$$q_{12} = \frac{1}{2a^4} \left( e^{-2aT} + 1 - 2e^{-aT} + 2aTe^{-aT} - 2aT + a^2T^2 \right),$$

$$q_{13} = \frac{1}{2a^3} \left( 1 - e^{-2aT} - 2aTe^{-aT} \right),$$

$$q_{22} = \frac{1}{2a^3} \left( 4e^{-aT} - 3 - e^{-2aT} + 2aT \right),$$

$$q_{23} = \frac{1}{2a^2} \left( e^{-2aT} + 1 - 2e^{-aT} \right),$$

$$q_{33} = \frac{1}{2a} \left( 1 - e^{-2aT} \right).$$

## 5. The Simulation of The Kalman Filter Operation

The trade-off of target trajectories obtained by Kalman filtering is the main aim of this chapter. In the first case, the target position was measured only by TDOA method and, in the second case, one was measured by DD/TDOA method.

The situation when the target trajectory is calculated only from output data of TDOA method is shown in Figure 7. The symbols '\*' denote measured target positions, symbols '.' denote points of true target trajectory and symbols '+' denote points of target trajectory that is calculated by Kalman filter. In this the measurement error of time difference is  $s_t = 50 \times 10^{-9}$  s.

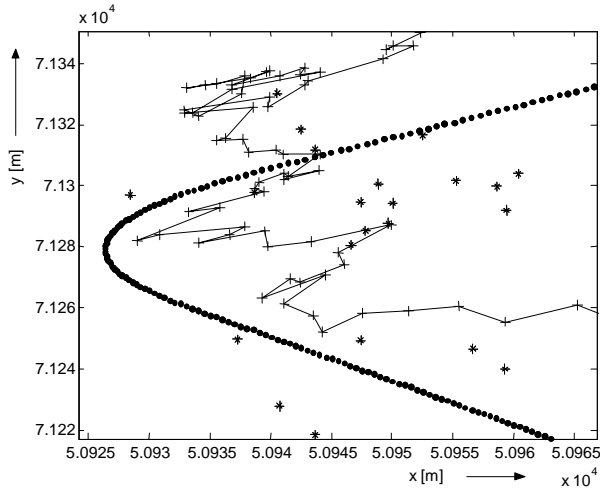


Fig. 7 Target trajectory; Filtering only target positions obtained by TDOA method

The situations when the target trajectory is calculated from output data of DD/TDOA method (the value of target velocity vector is used) are shown in Figure 8, 9. The symbols '\*' denote measured target positions, symbols '.' denote points of true target trajectory and symbols '+' denote points of target trajectory that are calculated by Kalman filter. The measurement error of time difference is  $S_t = 50 \times 10^{-9}$  s and the measurement error of frequency difference is  $S_f = 50$  Hz and  $S_f = 10$  Hz.

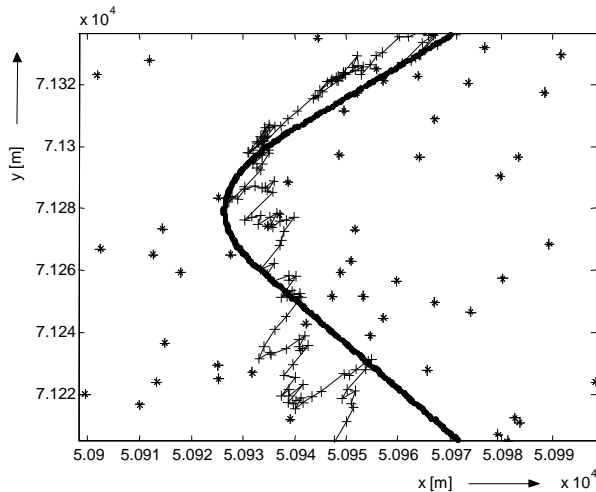


Fig. 8 Target trajectory; Filtering target positions obtained by DD/TDOA method; the error in the frequency difference measure is  $S_f = 50$  Hz



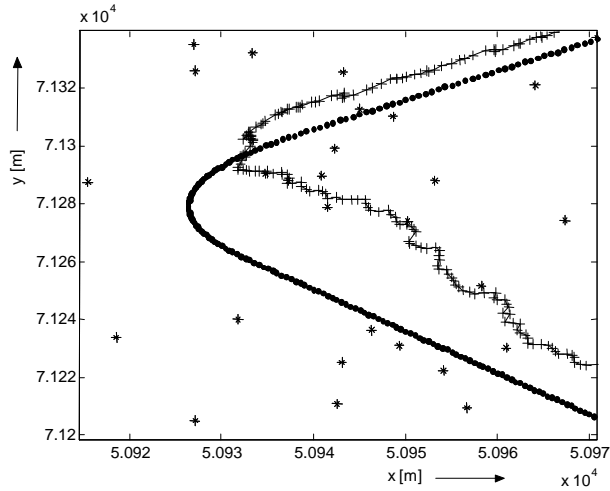


Fig. 9 Target trajectory; Filtering target positions obtained by DD/TDOA method; the error in the frequency difference measure is  $S_f = 10$  Hz

## 6. Conclusion

The time-difference (of arrival of signals to separated position) accuracy  $S_t$  and the Doppler-difference measuring accuracy  $S_f$  should be balanced together. When the  $S_t$  is higher than  $S_f$ , the target's trajectory is tracked very well, but it is shifted to real trajectory.

The effect of DD/TDOA method for various  $S_t$  to  $S_f$  ratios is clearly visible in the figures. Nevertheless, this ratio shouldn't be generalized due to its dependency on target's position and velocity vector (carrier frequency, receivers' positions, etc.).

Thus, it is adequate to reach the lowest possible  $S_t$  and  $S_f$ , but with respect to the Kalman filtering on complex estimated target position accuracy represented by  $S$  covariance matrix.

The final product of DD/TDOA algorithm in current time are  $X$  and  $S$  matrices, resp.  $XP$  and  $SP$  matrices if the predicted position for given time is requested. These matrices represent the target's position, velocity and acceleration vectors and the corresponding error-covariance matrix.

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