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Level Straight Accelerated Flights

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Abstract:

The paper introduces the analytical method derivation for flight time and flight path length in horizontal straight flight. An airplane is considered to be the mass point. Aerodynamic characteristics of the airplane are replaced by the parabolic polar curve. Engine thrust does not change with flight speed. Working formulas are determined on the base of the analytical integration of the airplane motion equation. All solutions are performed in general dimensionless shape. Determination of the real airplane characteristics requires knowledge of the optimum airspeed at operation conditions.

Keywords:

Level straight flight, equation of motion, flight performance, optimum airspeed

1. Introduction

Flight time and flight path length at changes of flying speed belong to fundamental performance in level straight flights. For an airplane they are presented usually in the form of acceleration and deceleration curves in the row of flight altitudes and required flight configurations. Known flight mechanics procedures are used for their determination on the base of the mass, aerodynamic and engine characteristics for concrete aircraft and definite flight conditions.

Performance characteristics depend on flight altitude, operating engine mode, pod occupation and air brakes position. Engine works at acceleration on maximum mode and on idling at deceleration so that accelerating or decelerating force was what biggest. Air speed changes are then reached in the shortest time and at the smallest flight distance. Deceleration intensity is possible heighten using air brakes. Special deceleration case can happen after engine failure on-the-fly, when engine thrust sags on zeros and only one force acting in the velocity direction is airplane drag.

Analytical solution of performance characteristics stems from the dimensionless equation of motion of an airplane as the mass point and from simplified airplane

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characteristics. Aircraft mass is constant, components of aerodynamic force are expressed on the basis of the parabolic polar curve and the engine thrust does not change with the airspeed. Acceleration or deceleration intensity is then expressed by the magnitude of the drag load factor.

Results of the solution are general explicit dimensionless relations for fast and sufficiently exact determination of level straight flights performance. If flight configuration does not change, only single dimensionless curves for flight time and distance are valid throughout whole flight altitudes range. Real airplane performances are obtained then by multiplying calculated data by the scales depending on optimum flight speed at flying conditions. Flight configuration change may be express by the help of percent changes of parasite drag coefficient and flight mass.

2. Equation of Aircraft Motion

Forces acting on the airplane in level straight flight are shown in Fig.1. Fixed flight altitude insures the balance of the lift L and the airplane weight W (1.a).

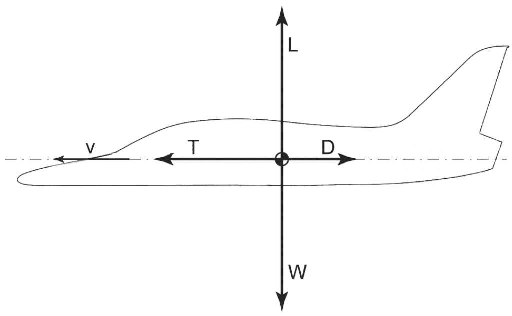

Fig. 1 Forces acting on an airplane

The tangential force R_t is expressed as the difference of the engine thrust T and the aircraft drag D (1.b). It produces the change of the airspeed. At aircraft acceleration is thrust greater than drag and accelerating force is positive, at aircraft deceleration thrust is smaller than drag and braking force is negative:

$$L = W \,, \tag{1a}$$

$$R_t = T - D \,. \tag{1b}$$

Components of aerodynamic force are expressed in usual way in aerodynamics

$$L = c_L q S, \quad D = c_D q S, \quad q = \frac{1}{2} r V^2, \tag{2}$$

where c_L and c_D are the lift and drag coefficients, S the wing surface, q the dynamic pressure and ρ is air density depending on flight altitude.

In the case of parabolic polar curve the resultant drag coefficient is divided into two coefficients of parasite drag c_{D0} and induced drag (A is a parameter of induced drag). Polar curve validity is limited by the maximum value of lift coefficient $c_{L\max}$

$$c_D = c_{D0} + A c_L^2 \,, \qquad 0 < c_L \le c_{L\max} \,. \tag{3}$$

Power plant unit is characterized at given flight altitude H by fixed average value of its thrust

$$T = T(H). \tag{4}$$

Force R_t in the flight direction causes the change of flight speed on Nevton's law of motion

$$m\frac{\mathrm{d}V}{\mathrm{d}t} = R_t \ . \tag{5}$$

Equation of motion may be generalized by using drag load factor n_D [2] that is expressed as the ratio of force R_t to the airplane weight

$$n_D = \frac{R_t}{W} \ . \tag{6}$$

The equation of motion (5) when divided by the airplane weight W contains on their right-hand side term that corresponds to the drag load factor. Generalized equations of motion are very frequently used for performance calculations of general manoeuvres and also for determination of flight path [1], [2]

$$\frac{1}{g}\frac{\mathrm{d}V}{\mathrm{d}t} = n_D \ , \tag{7}$$

symbol g in equation (7) denotes gravity acceleration.

Length of the flight path is expressed as the product of the instantaneous air speed and corresponding time

$$\mathrm{d}L = V\mathrm{d}t \ . \tag{8}$$

3. Dimensionless Shape of Motion Equation

Fundamental parameter to transform equation of motion (7) and the kinematic condition (8) into the dimensionless shapes is the optimum airspeed of the level straight flight at the required flight altitude where simultaneously air density ρ is known from the ISA (International Standard Atmosphere),

$$V_{op} = \sqrt{\frac{2W}{\rho S c_{Lop}}} \ , \qquad c_{Lop} = \sqrt{\frac{c_{D0}}{A}} \ . \tag{9}$$

Optimum lift coefficient c_{Lop} is determined from the constants of the parabolic polar curve (3). The scale for the engine thrust and next forces is the magnitude of the airplane minimum drag in the optimum regime.

Reference parameters denoted as the scale factor (index asterisk) and dimensionless variables with upper dash symbol are defined both in the Tab.1:

Tab. 1 Scale factors definition

Scale factor:	Dimensionless parameters:
speed $V^* = V_{op}$, $[ms^{-1}]$,	speed $\bar{V} = \dfrac{V}{V^*}$, [1],
time $t^* = \dfrac{V_{op}}{g}$, $[s]$,	time $\bar{t} = \dfrac{t}{t^*}$, [1],
length $L^* = \dfrac{V_{op}^2}{g}$, $[m]$.	length $\bar{L} = \dfrac{L}{L^*}$, [1].

Substituting quantities from Tab.1, the equation of motion and kinematic condition are transformed into dimensionless form and may be written then in the shape

$$\frac{\mathrm{d}\bar{V}}{\mathrm{d}\bar{t}} = n_D \ , \tag{9.a}$$

$$\mathrm{d}\bar{L} = \bar{V}\mathrm{d}\bar{t} = \frac{\bar{V}\mathrm{d}\bar{V}}{n_D} \ . \tag{9.b}$$

Accelerating or braking intensity depends at any moment of a flight on the value of the drag load factor (6). The drag load factor n_D is expressed as the ratio of the difference between the engine thrust and the drag to the aircraft weight. After rearrangement may be written in the dependence on the dimensional airspeed and on the thrust parameter n_R [5]

$$n_D = \frac{T-D}{W} = \frac{1}{K_{\max}}\left[\frac{T}{D_{\min}} - \frac{D}{D_{\min}}\right] = -\frac{1}{2\cdot K_{\max}} \cdot \frac{\bar{V}^4 - 2n_R\bar{V}^2 + 1}{\bar{V}^2} \ . \tag{10.a}$$

Symbol K_{max} is so called the maximum lift-drag ratio and may be determined from the known constants of parabolic polar curve (3)

$$K_{\max} = \frac{c_{Lop}}{c_{Dop}} = \frac{1}{2\sqrt{c_{D0}A}} \ . \tag{10.b}$$

The thrust parameter is introduced as the ratio of the engine thrust to the minimum drag in horizontal straight flight [5]

$$n_R = \frac{T}{D_{\min}} = K_{\max}\frac{T}{W} \ . \tag{10.c}$$

Dimensional engine thrust includes both the engine characteristic expressed as the ratio of the engine thrust to the aircraft weight T/W (denote sometimes as thrust facilities) and the aerodynamic characteristic, the lift-drag ratio K_{max} .

Drag load factor sign tells about acceleration sense. It will be negative in such flight regime in which thrust deficit comes on and positive at thrust excess. Drag load factor (10.a) may be divided in two components

$$n_D = -\frac{1}{2K_{\max}}\left(\overline{V}^2 + \frac{1}{\overline{V}^2}\right) + \frac{n_R}{K_{\max}} = -n_D^A + n_D^T \quad . \tag{10.d}$$

$$n_D^A = \frac{1}{2K_{\max}}\left(\overline{V}^2 + \frac{1}{\overline{V}^2}\right), \qquad n_D^T = \frac{n_R}{K_{\max}} .$$

The first component belongs to the airframe (aerodynamic part) and expresses aircraft drag contribution. It is always negative. The second part expresses influence of power unit thrust (engine part) and it is always positive as far as engine produces thrust. At zero trust is zero too.

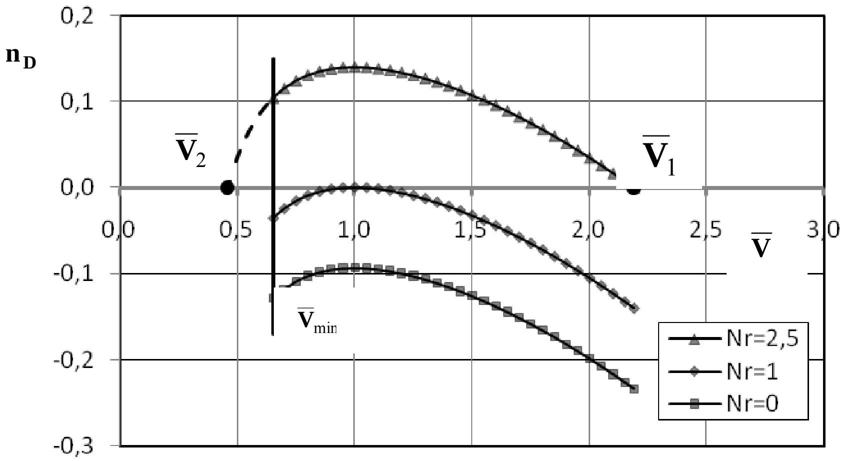

Fig. 2 Drag load factor

Drag load factor curves in dependence on dimensionless speed for three different thrust parameters are stated in Fig.2. At n_R < 1 thrust deficit causes aircraft deceleration in the whole range of speeds. Situation similar is at n_R = 1 except zero value of drag load factor when the dimensional speed is equal unity. At n_R >1 thrust excess exists and aircraft is accelerated in the range (V_1,V_2). Both boundary airspeeds may be determined from condition of zero drag load factor on the base of solution of the biquadratic equation (10.a or 10.b):

$$\overline{V}_1 = \sqrt{n_R + \sqrt{n_R^2 - 1}} \ , \qquad \overline{V}_2 = \sqrt{n_R - \sqrt{n_R^2 - 1}} \ . \tag{11}$$

Speeds $\overline{V}_1$ and $\overline{V}_2$ represent maximum and minimum dimensional airspeed of a level straight flight at thrust limitation. Speed dependences on the thrust parameter are presented in the Fig.3. If the calculated speed $\overline{V}_2$ is less than stall speed then this value is not real and corresponding flight regime is on the polar curve over the maximum lift coefficient. In such causes minimum speed $\overline{V}_2$ must be identify with the stall speed

$$\overline{V}_2 \equiv \overline{V}_{\min} = \frac{V_{\min}}{V^*} = \left(\frac{c_{x0}}{A c_{L\max}^2} \right)^{1/4} . \qquad (12)$$

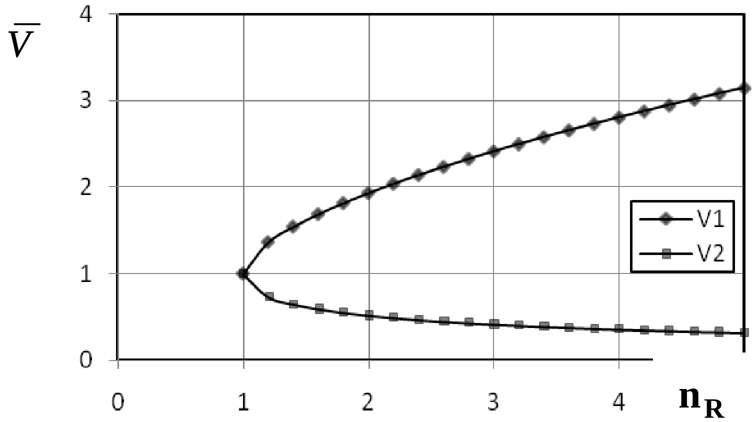

Fig.3 Boundary airspeeds

This dimensionless minimum speed of level straight flight $\overline{V}_{\min}$ is not only very important flight characteristics but also concentrates all constants of the parabolic polar curve.

4. Performance Characteristics

Level straight flights performance is characterized by flight time a flight distance. Initial manoeuvre speed is denoted V_I and manoeuvre final speed V_F. Flight time is determined from equation motion (9.a) after substitution for drag load factor from Eq.(10.a) and following integration in the range of manoeuvre speeds

$$\bar{t} = \int_{\overline{V}_I}^{\overline{V}_F} \frac{\mathrm{d}\overline{V}}{n_D} = 2K_{\max} \int_{\overline{V}_F}^{\overline{V}_I} \frac{\overline{V}^2 \mathrm{d}\overline{V}}{\overline{V}^4 - 2n_R \overline{V}^2 + 1} . \qquad (13)$$

Flight length is expressed accordingly using Eq. (9.b)

$$\overline{L} = \int_{\overline{V}_I}^{\overline{V}_F} \frac{\overline{V} \mathrm{d}\overline{V}}{n_D} = 2K_{\max} \int_{\overline{V}_F}^{\overline{V}_I} \frac{\overline{V}^3 \mathrm{d}\overline{V}}{\overline{V}^4 - 2n_R \overline{V}^2 + 1} . \qquad (14)$$

Tab. 2 Primitive functions

Thrust parameter	Primitive functions F_t, F_L	Region
$n_R < 1$	$F_t = \dfrac{1}{2\sqrt{2(1+n_R)}}\left[\dfrac{1}{2}\ln\left\|\dfrac{\overline{V}^2 - \sqrt{2(1+n_R)}\cdot\overline{V}+1}{\overline{V}^2 + \sqrt{2(1+n_R)}\cdot\overline{V}+1}\right\| + \sqrt{\dfrac{1+n_R}{1-n_R}}\cdot arctg\left(\dfrac{\sqrt{2(1-n_R)}\cdot\overline{V}}{1-\overline{V}^2}\right)\right]$	deceleration for all $\overline{V} \geq \overline{V}_{\min}$
	$F_L = \dfrac{1}{4}\left[\ln\left\|\overline{V}^4 - 2n_R\overline{V}^2 + 1\right\| + \dfrac{2n_R}{\sqrt{1-n_R^2}}\, arctg\left(\dfrac{\overline{V}^2 - n_R}{\sqrt{1-n_R^2}}\right)\right]$	
$n_R = 1$	$F_t(\overline{V}) = \dfrac{1}{4}\ln\left\|\dfrac{1-\overline{V}}{1+\overline{V}}\right\| + \dfrac{1}{2}\cdot\dfrac{\overline{V}}{1-\overline{V}^2}$	deceleration for $\overline{V} \neq 1$
	$F_L = \dfrac{1}{2}\left[\ln\left\|1-\overline{V}^2\right\| + \dfrac{1}{1-\overline{V}^2}\right]$	
$n_R > 1$	$F_t = \dfrac{1}{4\sqrt{n_R^2 - 1}}\left[\overline{V}_1\cdot\ln\left\|\dfrac{\overline{V}_1 - \overline{V}}{\overline{V}_1 + \overline{V}}\right\| - \overline{V}_2\cdot\ln\left\|\dfrac{\overline{V}_2 - \overline{V}}{\overline{V}_2 + \overline{V}}\right\|\right]$	acceleration for $\overline{V} \in (\overline{V}_2, \overline{V}_1)$
	$F_L = \dfrac{1}{4}\left[\ln\left\|\overline{V}^4 - 2n_R\overline{V}^2 + 1\right\| + \dfrac{n_R}{\sqrt{n_R^2-1}}\ln\left\|\dfrac{\overline{V}^2 - n_R - \sqrt{n_R^2-1}}{\overline{V}^2 - n_R + \sqrt{n_R^2-1}}\right\|\right]$	
	$F_L = \dfrac{1}{4}\left[\ln\left\|\overline{V}^4 - 2n_R\overline{V}^2 + 1\right\| + \dfrac{n_R}{\sqrt{n_R^2-1}}\ln\left\|\dfrac{\overline{V}^2 - \overline{V}_1^2}{\overline{V}^2 - \overline{V}_2^2}\right\|\right]$	deceleration for $\overline{V} \geq \overline{V}_{\min} \cap \overline{V} \notin (\overline{V}_2, \overline{V}_1)$

Infinite integrals in the integral parts for dimensional time (13) and length (14) can be expressed by the help of the elementary functions [3]. Primitive functions of these integrals for determination of time F_t and flight path length F_L are presented in the Tab.2. Their shapes are distinguished for all three relative values of the thrust parameter n_R.

Dimensionless deceleration time and dimensionless flight path length can be then express as depending on primitive functions for the initial and final manoeuvre speeds

$$\bar{t} = 2K_{\max}\left[F_t(\bar{V}_I) - F_t(\bar{V}_F)\right], \tag{15}$$

$$\bar{L} = 2K_{\max}\left[F_L(\bar{V}_I) - F_L(V_F)\right]. \tag{16}$$

Computational relations (15) and (16) hold generally for determination performance characteristics in horizontal rectilinear unsteady flights. Computational terms will be considerably simplified at deceleration with insignificant engine thrust or if engine failure occurs during flight (n_R=0). Relations of the primitive functions are presented separately in this case in Tab.3.

Tab. 3 Primitive functions at zero thrust parameter

Thrust parameter	Primitive function F_t, F_L	Region
$n_R = 0$	$F_t = \frac{1}{2\sqrt{2}}\left[\frac{1}{2}\ln\left\lvert\frac{\bar{V}^2 - \sqrt{2}\bar{V} + 1}{\bar{V}^2 + \sqrt{2}\bar{V} + 1}\right\rvert + arctg\left(\frac{\sqrt{2}\bar{V}}{1 - \bar{V}^2}\right)\right]$	deceleration for all $\bar{V} \geq \bar{V}_{\min}$
	$F_L = \frac{1}{4}\ln\left\lvert\bar{V}^4 + 1\right\rvert$	

Dimensionless acceleration and deceleration curves are presented together in the Fig.4. Acceleration (deceleration) curves give relations between required time, fly through distance and flight speed. Acceleration begins from minimum dimensionless speed, deceleration from maximum dimensionless speed. Real time and corresponding distance at general manoeuvre will be determined from reading off initial and final primitive function and multiplication their differences by the scales from Tab.1.

Deceleration curves hold for zero thrust parameter n_R=0, acceleration curves for thrust parameter n_R=2.5. Polar curve is given by the constants: c_{D0}=0.026, A=0.084, c_{Lmax}=1.3. These values match maximum lift-drag ration K_{max}=10.70 and dimensionless stall sped $\bar{V}_{\min} = 0.654$.

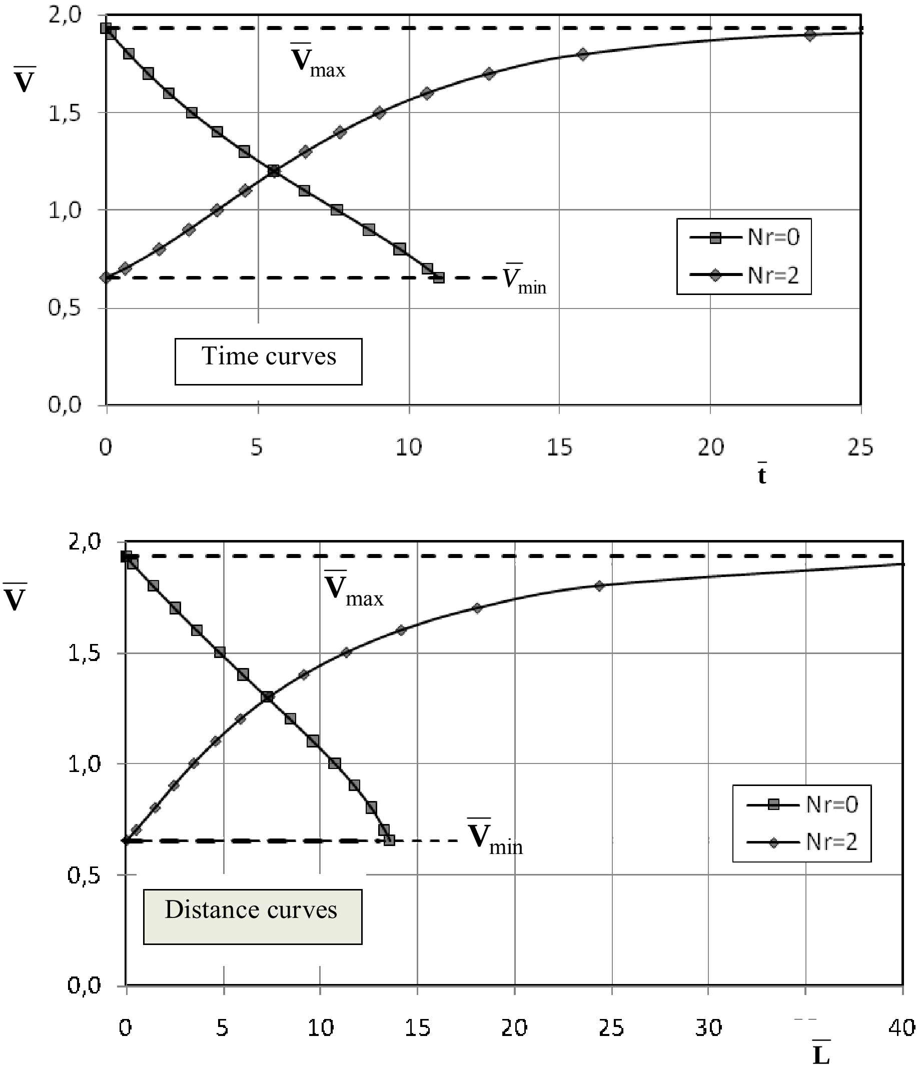

Fig.4 Acceleration a deceleration curves.

5. Drag Load Factor at Configuration Changes

Change of flight configuration, it means mass difference, pod load or using air brakes, happen to changes flight mass about the value Δm and changes of the parasite drag coefficient about the value Δc_{x0}. New variables are denoted by lower subscript "N",

$$m_N = m + \Delta m\,, \qquad\qquad (c_{D0})_N = c_{D0} + \Delta c_{D0}\,.$$

Flight mass enhancement evokes at the same real flying speed consonant lift coefficient change

$$\left(c_L\right)_N = c_L(1+\frac{\Delta m}{m}).$$

Flight configuration change impresses also drag load factor magnitude and acting tangential force. If the influences of mass and parasite drag changes are investigated it is profitable to keep the same scale in dimensionless speed corresponding to original configuration (Tab.2). Relation for drag load factor it is possible then step by step adjust

$$\left(n_D\right)_N = \frac{T-D}{W} = -\frac{\left(C_D\right)_N}{\left(c_L\right)_N} - \left(n_R\right)_N = -\frac{c_{D0}\left(1+\frac{\Delta c_{D0}}{c_{D0}}\right)}{Ac_L\left(1+\frac{\Delta m}{m}\right)} - c_L\left(1+\frac{\Delta m}{m}\right) + \frac{n_R}{\left(1+\frac{\Delta m}{m}\right)K_{\max}}$$

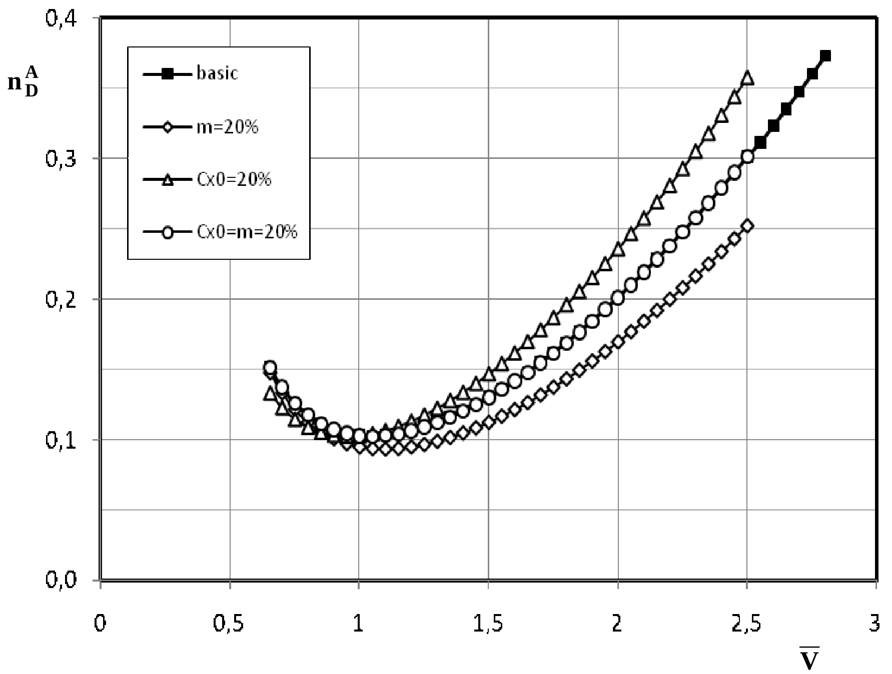

Fig.5 Influence of configuration change

Introducing original dimensionless speed the resultant relation for new drag load factor is

$$\left(n_D\right)_N = -\frac{1}{2K_{\max}}\left[\left(1+\frac{\Delta c_{D0}}{c_{D0}}\right)\left(1+\frac{\Delta m}{m}\right)^{-1}\overline{V}^2 + \left(1+\frac{\Delta m}{m}\right)\frac{1}{\overline{V}^2}\right] + \frac{n_R}{K_{\max}}\left(1+\frac{\Delta m}{m}\right)^{-1}$$

and similarly for its magnitude change

$$\Delta n_D = (n_D)_N - n_D = -\frac{1}{2K_{\max}}\left[\left(\frac{\Delta c_{D0}}{c_{D0}} - \frac{\Delta m}{m}\right)\left(1+\frac{\Delta m}{m}\right)^{-1}\bar{V}^2 + \frac{\Delta m}{m}\frac{1}{\bar{V}^2}\right] +$$

$$+\frac{n_R}{K_{\max}}\frac{\Delta m}{m}\left(1+\frac{\Delta m}{m}\right)^{-1} .$$

Influences of mass and parasite drag coefficient changes on aerodynamic part of drag load factor (10.d) are presented in Fig.5

6. Calculation at Changed Configuration

At practical utilization of integral formulas for calculation flight time (15) and flight distance (16) with changed flight configuration it is necessary respect new reference values corresponding to the new configuration. Configuration change varies generally airplane shape. Thereby changing the polar curve also change characteristic points on it. Under assumption of the same value of induced parameter new value of the optimum lift coefficient (9) and lift-drag ratio may be expressed as relations

$$(c_{Lop})_N = \sqrt{\frac{c_{D0}+\Delta c_{D0}}{A}} = c_{Lop}\left(1+\frac{\Delta c_{D0}}{c_{D0}}\right)^{1/2} ,$$

$$(K_{\max})_N = \frac{1}{2\sqrt{(c_{D0}+\Delta c_{D0})A}} = K_{\max}\left(1+\frac{\Delta c_{D0}}{c_{D0}}\right)^{-1/2} .$$

Thereby referential speed changes as well (9) and its new value will be

$$V_N^* = \sqrt{\frac{2m_N g}{rS(c_{Lop})_N}} = V^*\left(1+\frac{\Delta m}{m}\right)^{1/2}\left(1+\frac{\Delta c_{D0}}{c_{D0}}\right)^{-1/4} .$$

Minimum dimensionless speed (12) is effected only by parasite drag change and therefore

$$(\bar{V}_{\min})_N = \left(\frac{c_{D0}+\Delta c_{D0}}{Ac_{L\max}^2}\right)^{1/4} = \bar{V}_{\min}\left(1+\frac{\Delta c_{D0}}{c_{D0}}\right)^{1/4} .$$

The trust parameter is also altering in spite of preservation original value of the engine thrust

$$(n_R)_N = \frac{T}{gm_N}(K_{\max})_N = n_R\left(1+\frac{\Delta m}{m}\right)^{-1}\left(1+\frac{\Delta c_{D0}}{c_{D0}}\right)^{-1/2} .$$

Introducing new referential values formal relations for calculation time (15) and distance (16) stay unchangeable. Nevertheless real dimensionless initial and final manoeuvre speeds must be related to the new referential values.

7. Conclusion

Aircraft performance in horizontal straight flights with acceleration (positive or negative) belongs to fundamental aeroplane characteristics. In flight manuals are usually presented in forms of acceleration and deceleration curves in a row of flight altitudes for choice flight configurations.

Mathematical model of an airplane in this paper is considered to be a mass point with constant mass. Such model is commonly used for performance analyses. Airplane is characterized by two points of parabolic polar curve (parasite drag coefficient, induced drag parameter) limited by value of maximum lift coefficient, engine thrust magnitude at flying altitude and flying mass. Formulas for calculation performance characteristics are derived on basis analytical integration of the equation of motion.

Manoeuvre characteristics, acceleration or deceleration time and corresponding flight distance at flight speed change are in the form of relatively simple formulas depending on thrust deficit or excess and on initial and final speeds. All derivations are consistently performed on generalized dimensionless shape and results of solution are expressed in dimensionless shape, too.

Introducing dimensionless formulation generalizes achieved results without detail knowledge aircraft characteristics and operation height. Only one final curve is valid if configuration stays fixed. To recalculate dimensionless results on real (dimensional) values just knowledge of optimum airspeed is requisite at required altitude. Utilizing presented procedure for other flight configuration consists in descending reduction of the optimum speed, the maximum lift-drag ratio and the engine thrust parameter depending on percentage changes of the mass and the parasite drag coefficient regarding to original configuration.

Working formulas make possible quickly pass judgment on airplane acceleration and deceleration. They may be completed about fuel consumption during manoeuvre. Meaningful information is also possible to determinate a maximum time to keep aircraft on rectilinear horizontal flight after engine failure. This statement presents important information about remaining time reserve, which pilot can employ for his decision in given situation.

Analytical solution is valid for acceptable assumptions on aircraft aerodynamic characteristics. At higher airspeeds when aerodynamic characteristics of aircraft depend significantly on Mach number, assumptions of the solution are not fulfilled. It is necessary to determine flight course by numerical procedures - modelling of flight path. This approach requires code elaboration and airplane characteristic approximation. Nevertheless, even in these cases the presented results offer valuable information about mentioned characteristics. The accuracy of results is effected in these cases by a compensation manner of real polar curves with compressibility influence by means of only one parabolic polar.

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