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Theoretical Criterion for Evaluation of the Frangibility Factor

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Abstract:

The capability of frangible bullet to disintegrate hitting the surface of a hard target can be expressed according to the [1] by so called Frangibility Factor. At the time there are several procedures serving for evaluation of the frangibility of the frangible bullet. As far as it is known to me, all of these methods are experimental. This article deals with the theoretical criterion of the evaluation of the Frangibility Factor. This criterion is based on the evaluation of the static strength of frangible bullet and on the theoretical calculation of the limit kinetic energy of the bullet hitting the hard target of known mechanical properties.

Keywords:

Frangible ammunition, frangibility, ballistics

1. The Frangibility of Bullet

From the terminal ballistics point of view is the **frangibility** the most important ballistic feature of the frangible bullet. Frangibility of the bullet is its capability to disintegrate into pieces by hitting a hard target. The easier bullet disintegrates hitting a hard target, the higher is the frangibility of this bullet.

The frangibility is mostly influenced by following three factors:

- characteristics of the bullet,
- characteristics of the target,
- conditions of the impact.

Regarding the **bullet's characteristics** the frangibility of the bullet is affected by the mechanical properties of the bullet, by the bullet's dimensions and by the shape of the bullet. The frangibility is increasing with decreasing strength of the bullet's

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material and with the decreasing material's toughness. The shape of bullet is affecting the frangibility as well. The higher is the radius of bullet's ogive, the higher is the bullet's frangibility.

From **characteristics of hard target** is the frangibility most influenced by the stiffness of the target and by the strength of its material. At the impact of bullet upon a target is the force acting against the bullet's motion increasing with the increasing stiffness and deformation of the target. The higher are stiffness and deformation of the target, the higher is the increase of the force acting against the motion of the bullet and in the shorter time is the bullet's material strength exceeded.

Conditions at the impact are significantly influencing the bullet's frangibility. The kinetic energy, angular velocity of bullet's rotation and the geometry of an impact have the highest influence on the bullet's frangibility.

In order to be able to define a theoretical criterion for evaluation of the bullet's frangibility we can use an experimentally found phenomenon that for any frangible bullet and any hard target there is a unique value of the **limit velocity of the bullet** $\boldsymbol{v_{lim}}$. This velocity is the highest velocity of bullet hitting given target at which the bullet still does not fragment into pieces. To this velocity corresponding kinetic energy of the bullet is called **limit kinetic energy of bullet** $\boldsymbol{E_{k,lim}}$. The results of experiments show, that when the limit kinetic energy is exceeded, the amount of bullet fragments raises with increasing kinetic energy of bullet. The amount of fragments can be expressed in dependence on the surplus of kinetic energy of bullet E_k. Based on this experience there is the **Frangibility Factor (FF)** defined. The Frangibility Factor is defined as a ratio between the kinetic energy of a bullet hitting specific hard target and the bullet's limit kinetic energy for this target.

Frangibility Factor is defined for the **standard hard target** and for the perpendicular impact on the hard target's surface by formula

$$FF = \frac{E_K}{E_{K,\lim}} . \quad (1)$$

The frangibility factor expresses the surplus of kinetic energy of the bullet, which is used for damage of the bullet and its fragmentation, for the damage of the target and for the acceleration of the projectile fragments. For the value of FF lower than 1 the bullet doesn't fragment into pieces at the impact on the hard target, for value of FF higher that 1 the bullet fragments at least into pieces hitting the hard target.

The expected relationship between the amount of bullet's fragments and the kinetic energy of the bullet is depicted on the figure 1. With increasing kinetic energy of projectile the amount of bullet's fragments grows as well. For very high levels of kinetic energy of bullet is the amount of fragments theoretically approaching the amount of metal powder particles used for the bullet manufacturing (N_0).

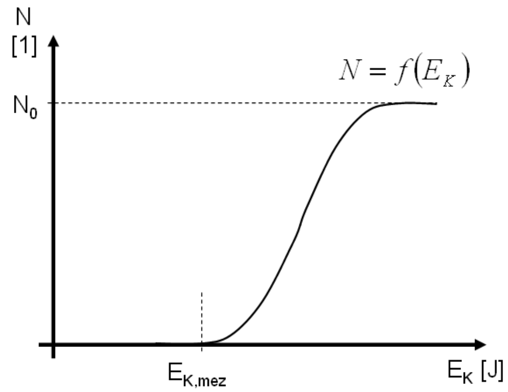

Fig. 1 Relationship between the amount of bullet fragments and bullet's kinetic energy for specific hard target

Frangibility Factor can be evaluated **experimentally** and **theoretically**. For the theoretical description of the frangibility factor it is necessary to describe mathematically dynamic load of the frangible bullet during the interaction with target. The model is to be based on the experimentally found characteristics of the bullet, on the known characteristics of the hard target and on specified conditions of the interaction. For the derivation of the mathematical model describing the dynamic load of the frangible bullet it is necessary to create:

- model of the frangible bullet,
- model of the frangible bullet's material,
- model of the hard target,
- model of the bullet's load at the interaction with a hard target.

For the description of the bullet's dynamic load at the interaction with target a simplified **model of frangible bullet** was used. Considering usual flat shape of the point of most of the frangible bullets and neglecting typical small radius of ogive, simplified bullet's cylindrical shape is assumed. The diameter of cylinder base and the length of the cylinder equal to the bullet's diameter and bullet's length. The influence of the front ogive on the bullet's load and on the value of limit velocity and limit kinetic energy of the real frangible bullet is given by using the features measured for the real frangible bullets (this will be shown in further parts of this article).

As it has been shown in the previous paragraphs, the frangible bullet's material is neither homogeneous nor isotropic. The description of mechanical properties of such kind of material is possible by means of 21 material constants. All these constants are describing different material features in three different directions and would have to be experimentally found.

For the definition of the **model of frangible bullet's material** homogeneity and isotropy of these materials is assumed. This assumption is taken with regard to small dimensions of pistol frangible bullets. The investigations published in [1] show that the optimal way of describing the properties of these materials is using the axial compression test of material specimens or of frangible bullets. The character of load at this test is close to the character of load at the impact on target and this test is not too demanding for the laboratory equipment.

On the figure 2 there is an introduction of relationship between the stress and relative compression of the tested specimen of material of bullet SR Frangible*. Based on measured material characteristics at the compression test a material model can be defined. In a simplified way an **ideal elastic material model** for a brittle material of frangible bullets can be assumed. The behaviour of material is then given by an elastic modulus in compression K and the material strength in compression σ_{dB}, therefore with the **maximal compression at break** ε_{dmax}. The figure 2 also shows an example of application of an ideal elastic material model for material of frangible bullet SR Frangible and its material's compression test.

The ideal elastic material model is suitable for materials of frangible bullets which are based on a composite with metal matrix. In case of composites with a polymer matrix the large plastic deformation before rupture of material cannot be neglected.

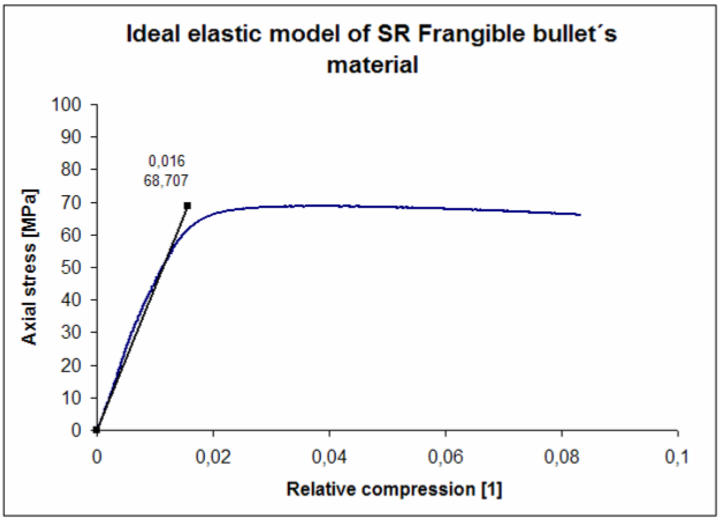

Fig. 2 Ideal elastic material model for material of bullet SR Frangible

The material behaviour is different for its static and dynamic loading. The material strength is increased at the impact loading of the material. At the same time the fracture toughness of material is decreased (the brittleness of material is increasing at the impact loading). The change of material properties at the dynamic loading is neglected in following contemplations. Mechanical features of material defined by the static tests are assumed for the derivation of the mathematical model of dynamic load of the frangible bullet. This simplification can be taken into the consideration in regard of low material strength of the materials of frangible bullets and considering low limit velocity of frangible bullets on a hard target (approximately 30 – 80 $m.s^{-1}$).

* *Bullet SR Frangible is a prototype developed by the company Svachouček (the abbreviation SR is made of the first letters of names of the company and the bullets designer – Svachouček, Rydlo)*

Next important step is to define a **failure criterion** suitable for fragile materials of frangible bullets. The criterion has to be based on the compression test of materials and frangible bullets. In [1] there is the failure criterion based on the **maximal relative compression** ε_{dmax} of material defined.

Maximal relative compression ε_{dmax} is evaluated from results of the compression test of material and is given by formula

$$\varepsilon_{d,\max} = \frac{\sigma_{dB}}{K} \quad . \tag{2}$$

The bullet is damaged at the interaction with hard target if the highest relative compression of bullet during the interaction exceeds the maximal relative compression ε_{dmax}.

For the definition of theoretical criterion for the bullet's frangibility evaluation it is necessary to define and to describe a **standard hard target**. The definition of standard hard target has to be representative to real targets, for example the steel barriers used on the shooting ranges that are supposed to stop the bullet and not to sustain any damage. When the standard hard target defined, the limit kinetic energy of bullet specific for this target is a measure of the bullet's frangibility.

Standard hard target in a form of round steel plate of diameter D and thickness h has been defined. The steel plate is restrained around its circumference. The mechanic features of steel are given by the elastic modulus E_c, the strength of steel is supposed to be much higher than the strength of materials of frangible bullets. After theoretical contemplations and after evaluation of several shooting experiments were following features of the standard hard target defined:

- $D = 0.5$ m,
- $h = 10$ mm,
- $E_c = 210$ GPa.

The stiffness of the standard hard target was calculated for given geometrical and material features (c = 6055.3 kN.m^{-1}).

The bullet's frangibility is also influenced by the stress in the bullet's material before the impact on the target. During the bullet's motion in the atmosphere is the bullet's motion decelerated by acting of a drug force on the tip of bullet. This force is causing an additional axial stress in the bullet's material, but its effect is negligible compared to the effect of brake force acting on the bullet at the interaction with hard target. For this reason there is the drug force neglected in following considerations.

The bullet that moves with velocity v and rotates with angular velocity ω is loaded by centrifugal force, which causes radial σ_r and tangential σ_t stress in the bullet's material. In case of a rotational symmetrical bullet's body the radial and the tangential stresses are equal in their values. Using formulas published in [2] the radial stress in bullet's body can be described in the relationship with the diameter of bullet d, material density ρ, angular velocity of bullet's rotation ω and the Poisson constant of bullet's material μ. It can be shown that for the bullet SR Frangible is the value of equivalent stress from 3.2 MPa till 9 MPa at the impact velocity 300 – 500 m.s^{-1}. This effect cannot be neglected at high impact velocities of frangible bullets, because it has a high influence on the bullet's frangibility.

According to [3] is the tensile strength of brittle materials and particle composites approximately five times lower than the strength of corresponding material in compression. For the measured strength of SR Frangible bullet material in compression (68.7 MPa) the level of strength in tension approximately $\sigma_m \sim 14$ MPa can be expected. The bullet SR Frangible would be under this assumption on the limit of material strength under the load of centrifugal force at the initial velocity ~ 600 m.s^{-1}. Theoretical assessments and experimental results show that with regards to the material strength of bullet SR Frangible the load caused by the centrifugal force can be neglected up to the initial velocity of bullet about 100 m.s^{-1}. Maximal radial stress in the SR Frangible bullet's material at the initial velocity 100 m.s^{-1} is just 0.36 MPa (this value corresponds approximately to 2.6% of its material strength). This fact has a big influence on the aforementioned definition of the standard hard target. Among other conditions this target has been optimized in order to enable reaching limit velocities of frangible bullets lower than 100 m.s^{-1}.

2. Loading of Frangible Bullet at the Interaction with Hard Target

During the interaction of a frangible bullet with hard target the brake force F_B acts against the motion of bullet. This force is given by stiffness of the target c and the actual deformation f of the target according to the formula

$$F_B = cf \ . \tag{3}$$

The brake force acting against the bullet's motion can be also expressed as a result of the bullet's mass m_q and the bullet's deceleration a, that is caused by the acting brake force. The effect of the brake force is depicted on the figure 3. The brake force causes the compressive axial stress in the whole volume of the bullet. The maximal value of this stress (σ_B) is in the section at the tip of the bullet, in the direction to the bullet's bottom is the intensity of stress decreasing, in the section at the bottom of the bullet the axial compressive stress equals zero. The brake force F_B on the top of the bullet can be expressed using known material density ρ and its known volume V according to the formula

$$F_B = \rho V a \ . \tag{4}$$

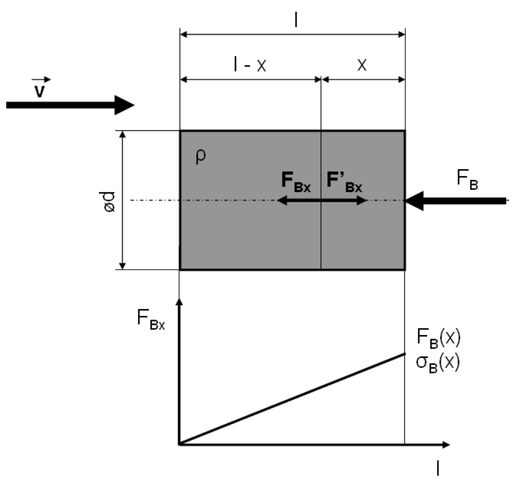

Fig. 3 Axial compressive stress in the body of frangible bullet under the load of brake force

For the calculation of force acting in an arbitrary transverse cross section of the bullet is necessary to express this force depending on the length of bullet l by the means of formula

$$F_B = \rho a S l\,, \tag{5}$$

where S is the bullet's transverse section area.
In an arbitrary section in the bullet in the x distance to the tip of the bullet can be the brake force $F_B(x)$ described as

$$F_B(x) = \rho a S(l - x)\,. \tag{6}$$

Then the equation for axial compressive stress in any section of the bullet $\sigma_x(x)$ can be written in the form of the formula

$$\sigma_x(x) = \rho a(l - x)\,. \tag{7}$$

From the relationship between the axial stress at the deceleration the dependence of axial contraction of the bullet on the distance x from the top of the bullet can be expressed by following formula

$$u_x(x) = \int \varepsilon(x)dx = \int \frac{\sigma(x)}{K}dx = \int \frac{1}{K}\rho a(1 - x)dx\,. \tag{8}$$

After the integration is the bullet's contraction in certain section given as a function of the distance x like

$$u_x(x) = \frac{\rho a}{K}\left(lx - \frac{x^2}{2}\right) + C\,, \tag{9}$$

where the constant of integration is determined from the initial condition u(x = 0) = 0 and equals zero. Then the complete bullet's axial contraction equals to

$$u_x(l) = \frac{\rho a l^2}{2K}\,. \tag{10}$$

A hypothesis of maximal relative compression is used for calculation of maximal axial compressive load of the bullet, at which the rupture in the bullet's material occurs. For using this hypothesis is a simplification accepted, that the relative compression of the bullet is equal to the axial contraction of the bullet divided by the original length of the bullet (mean value of the relative compression of the bullet is assumed)

$$\varepsilon_x = \frac{u_x(l)}{l}\,. \tag{11}$$

Then following relationship for the relative compression of the bullet can be written

$$\varepsilon_x = \frac{\rho a l}{2K}\,. \tag{12}$$

Based on the hypothesis of the maximal relative compression the **maximal deceleration of the bullet a_{max}** can be defined. The maximal relative compression of material is reached at the maximal deceleration of the bullet and the bullet is stressed on the maximal limit of its material strength. Therefore the maximal deceleration that the bullet can withstand is a measure of its dynamic strength. Following relationship can be derived from formula 12.

$$\varepsilon_{x\max} = \frac{\rho a_{\max} l}{2K} \tag{13}$$

Quasi-static theory of impact can applied on the description of interaction of the frangible bullet with a hard target. The kinetic energy of bullet hitting a hard target ($E_k < E_{k,lim}$) is changed in the deformational energy of the hard target W_c, deformation energy of the bullet W and the thermal energy. Because of low level of kinetic energy of the bullet ($E_k < E_{k,lim}$) the change of part of kinetic energy of the bullet into the thermal energy is very small and is neglected in further considerations. Then for the conservation of energy is valid

$$E_K = \frac{1}{2} m_q v_d^2 = W + W_c\,. \tag{14}$$

In agreement with [2], where the formula for calculation of deformation work of axially loaded rod is published, the deformation work of the bullet W in moment when the maximal brake force acts on the bullet can be expressed following way

$$W = \frac{F_{B,\max}^2 l}{6KS}\,. \tag{15}$$

The deformation work of the hard target W_c could be calculated based on the maximal acting brake force at the moment of bullet's rupture and the known stiffness of the hard target using relationship

$$W_c = \frac{1}{2}\frac{F_{B,\max}^2}{c}\,. \tag{16}$$

According to the law of the energy conservation can be written

$$\frac{1}{2} m_q v_{\max}^2 = \frac{F_{B,\max}^2 l}{6KS} + \frac{F_{B,\max}^2}{2c}\,. \tag{17}$$

By the substitution $F_{B,max} = m_q a_{max}$ is following relationship gained

$$v_{\max}^2 = \frac{m_q a_{\max}^2 l}{3KS} + \frac{m_q a_{\max}^2}{c}\,, \tag{18}$$

where a_{max} is a characteristic of a shape and material features of the frangible bullet. From the formula (13) can be derived

$$a_{\max} = \frac{\varepsilon_{x,\max} 2K}{\rho l} . \tag{19}$$

Than with the substitution for a_{max} and simplification of the formula, maximal impact velocity of the frangible bullet can be calculated, at which the bullet still does not damage at the interaction with a hard target. The limit velocity of bullet equals to

$$v_{\lim} = \frac{\varepsilon_{x,\max} 2K}{\rho l} \sqrt{m_q \left(\frac{4l}{3K\pi d^2} + \frac{1}{c} \right)} . \tag{20}$$

Limit velocity of the bullet v_{lim} is a theoretical maximal impact velocity of the bullet, with which the bullet still doesn't fragment at the impact on a hard target of the stiffness c. The inertia of the hard target is in this contemplation neglected.

The derivation of the formula is based on the quasi-static theory of impact. This theory is assumed to be correct with regards to relatively low limit velocities v_{lim} of usual frangible bullets. The formula (20) shows, that the limit velocity of frangible bullet is a function of the stiffness of a hard target c, mechanical features and the geometry of the frangible bullet. All mechanical features of the bullet's material necessary for a calculation of the limit velocity of the bullet are gained by the axial compression test of the bullet. Additionally, as it's been shown in former parts of this article, in case the impact velocity of the bullet is lower then 100 m.s^{-1}, the influence of the centrifugal force can be neglected.

The value of limit velocity of bullet can be used for calculation of the limit kinetic energy of the bullet $E_{k,lim}$ using formula

$$E_{k,\lim} = \frac{1}{2} m_q v_{mez}^2 . \tag{21}$$

Limit kinetic energy of the bullet is used for calculation of the frangibility factor according to the formula (1).

3. Frangibility Factor of Real Frangible Bullets

Mathematical model developed for calculation of the limit kinetic energy of frangible bullet was derived with assumption of cylindrical shape of frangible bullet. The influence of the shape of bullet is evaluated at the axial compression test and the resulting values are compared with results of compression test accomplished with cylindrical testing specimens.

At the evaluation of the limit kinetic energy of real bullets we are using the mechanic characteristics of real frangible bullets, which have been found by evaluation of the compression test of these bullets. Maximal relative compression of real bullet ε_{dsmax} and the elastic modulus in compression of the real bullet K_s are used in the formula (20).

Results of the shooting experiments and results of calculations for the bullets Sinterfire and SR Frangible are summarized in the table 1. For the purpose of this article just two types of frangible bullets are mentioned. The current trend in the development of frangible ammunition shows a preference in using the composite materials based on the polymer matrix. For these materials the aforementioned model for the frangibility factor calculation cannot be used, because for these materials the linear elastic material model is not valid.

Tab. 1 Characteristics of frangible bullets

Frangible Bullet	Sinterfire	SR
Caliber d [m]	0.009	0.009
Length of bullet l_s [m]	0.0161	0.01435
Mass of bullet m_s [kg]	0.00648	0.0055
Density of bullet ρ_s [kg.m^{-3}]	7443	6763
Elastic modulus in compression K_s [MPa]	10597	3999
Maximal relative compression of the bullet $\varepsilon_{sx,lim}$ [1]	0.01172	0.01255
Limit velocity of the bullet $v_{s,lim}$ [m.s^{-1}]	69.4	32.9
Limit kinetic energy of the bullet $E_{ks,lim}$ [J]	15.6	2.9
Kinetic energy of the bullet in the distance 2 m from barrel muzzle E_{k2} [J]	470.3	366.4
Frangibility Factor of the bullet FF [1]	30.1	126.3

The resulting frangibility factor of bullets in the table 2 is calculated for the velocity of bullet v_2. The limit velocity $v_{s,lim}$ of the bullet Sinterfire is 2.1 times higher than the limit velocity of the bullet SR Frangible. The limit kinetic energy of the bullet Sinterfire is then 5.3 times higher than the limit energy of bullet SR Frangible. The resulting frangibility factor of the bullet SR Frangible is 4.2 times higher than the frangibility factor of the bullet Sinterfire.

These results confirm diametrical difference in terminal ballistic behaviour of both types of bullets, which has been confirmed by several shooting experiments.

4. Conclusion

The capability of frangible bullets to disintegrate hitting a hard target is usually described based on a ballistic experiment. The evaluation of the experiment is performed considering amount of bullet fragments gained by interaction of bullet and defined target.

Frangibility Factor introduced in this paper can be evaluated experimentally and theoretically. The model introduced in this paper enables an estimation of frangibility factor using the bullet's material features gained by the material static laboratory testing and enables optimisation of material and geometrical features of frangible bullet.

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