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Radio Signal Source Position from Measured Azimuths

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Abstract:

The determination of moving object position is quite frequent task of armed forces. There are several techniques which use active or passive systems. The passive systems are based on capture, identification and analysis of any radio signal emitted by fast moving object (airplane, rocket etc.). The position of object can be determinate from this information. One possibility how to determinate the object position is a usage the measured geodetic azimuths of the moving object from several mobile devices (initial points). The principles of spherical trigonometry and geodesy are applied. The employed solution comprises the following key steps: calculation of azimuths and the connecting line of the initial points, calculation of unknown distances to the target, determination of geographic coordinates of the target, transformation of geographic coordinates into planar right-angled ones in the projected coordinate system. The above described technique was tested in the area of the Europe and final results and position differences were evaluated. Several recommendations for effective usage of described technique were done with respect of results of passed tests.

Keywords:

Target position determination, direct/inverse geodetic problem, chord method, numerical integration, radio direction finder

1. Introduction

Practical military activities compel armies to tackle a number of tasks connected to the determination of various objects' location. These tasks differ both in input conditions and in requirements for resulting values. The key factors which affect the selection of

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appropriate methods encompass the extent of area where given activities are conducted, accuracy of the input data as well as the required accuracy of results. However, the speed and effectiveness of a solution combined with a corresponding degree of information technology involvement represent a significant factor as well [1-3].

2. Problem Definition

Geographical coordinates of a minimum of two (possibly up to eight) initial points are given. Geodetic azimuths of a moving source of a radio signal are measured at the initial points with accuracy of $1° \div 2°$. The task is to determine coordinates of a radio signal in the World Geodetic System 84 (WGS84) and Universal Transverse Mercator projection (UTM). The task is calculated on an ellipsoid surface.

3. Points of Departure

There are a number of ellipsoids (parameters of which have been determined by various methods) which can easily be found in literature. In this particular case, the WGS84 ellipsoid, whose parameters were determined through satellite measurements, was applied.

When solving the task, a number of methods can be used to solve direct geodetic problem (to calculate coordinates) and inverse geodetic problem (to calculate distance and azimuth). These methods differ in the following key factors:

- used reference surface,
- difference between the initial point and the target,
- given and determined quantities,
- accuracy of given values and required accuracy of solution.

When calculating an unknown point position, it is necessary to solve both direct and inverse geodetic problems.

Direct geodetic problem (hereinafter DGP) is used for determining the geodetic coordinates of an unknown point on condition that coordinates of the initial point, azimuth and the distance to the target are known. Inverse geodetic problem (hereinafter IGP) is used to calculate the distance and azimuth of the connecting line of two points whose coordinates are known.

Based on task analysis, a combined solution was selected. For part of calculations it uses the spherical reference surface (calculation of the connecting line of the initial points and the distance between direction finder and source – source distances), while for calculations of geodetic coordinates of the target it uses the reference surface of an ellipsoid.

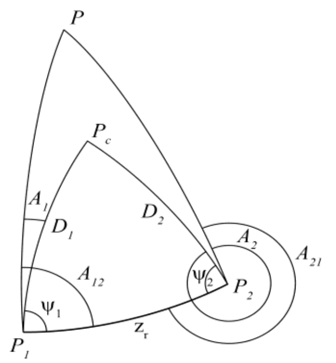
4. Procedure

The employed solution comprises the following key steps:

1. Calculation of azimuths and the connecting line of the initial points.
2. Calculation of unknown distances to the target.
3. Determination of geodetic coordinates of the target.
4. Transformation of geographic coordinates into planar right-angled ones in the projected coordinate system.

4.1. Calculation of Azimuths and the Length of the Connecting Line of the Initial Points

In order to determine the coordinates of the target, it is essential to know its coordinates and the measured azimuth as well as one more variable – the distance between the initial point and the target. The distance can be calculated from the spherical triangle (see Fig. 1).



where are:

- PP_1, PP_2 … meridian,
- P … Pole,
- P_c … Target,
- P_1, P_2 … initial points,
- A_1, A_2 … azimuths of target,
- A_{12}, A_{21} … azimuths of the link of initial points,
- ψ_1, ψ_2 … angles of the spherical triangle,
- z_r ... length of normal section,
- D_1, D_2 … distances between initial points and the target.

Fig. 1 Spherical triangle

This triangle is defined by the distance between initial points, lengths of connecting lines between P_1, P_2, P_c and the corresponding angles. In order to obtain these data, the first step involves calculating the length of the connecting line of the initial points and its direct and backwards azimuth which allow us to establish the respective angles of the triangle.

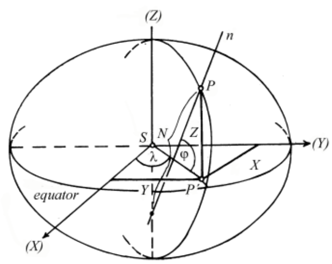
Calculation of length and the connecting line's azimuths is encompassed in IGP and can be solved either on a sphere or an ellipsoid. When trying to find out the optimum method, both possibilities were looked into.

Direct/inverse geodetic problems on a sphere may be calculated in geographic coordinates or in right-angled planar coordinates. The solution method encompassing IGP in geographical coordinates was selected for the given task. It draws on input values, i.e. geographical coordinates φ_1, λ_1 and φ_2, λ_2 of points P_1 and P_2 and azimuths A_1 and A_2 in these points (see Fig. 1). The computing procedure is shown e.g. in [5].

From results obtained through this method of calculation we may conclude that for shorter distances – up to 100 km – this procedure is sufficient. For distances exceeding this limit, however, inaccuracies in the calculated distances and azimuths cause unacceptable mistakes in localization.

For medium and long distances it is therefore necessary to opt for an ellipsoid when calculating IGP. For the above mentioned reasons, two methods were tested – the so called ellipsoidal chord method and method of numerical integration of differential equations of a geodetic curve adjusted for IGP.

The *ellipsoidal chord method* is based on calculations of ellipsoidal chord and on establishing the length of the ellipsoid's normal sections. Calculations of two initial points, P_1 and P_2 position take the advantage of spatial right-angle coordinates (see Fig. 2).



where are:
φ, λ … geographic coordinates,
X, Y, Z … spatial right-angle coordinates,
n … normal,
N … radius of the curvature in the prime vertical for point P.

Fig. 2 Spatial right-angle coordinates

$$
\begin{aligned}
X_1 &= N_1 \cos\varphi_1 \cos\lambda_1 & X_2 &= N_2 \cos\varphi_2 \cos\lambda_2 \\
Y_1 &= N_1 \cos\varphi_1 \sin\lambda_1 & Y_2 &= N_2 \cos\varphi_2 \sin\lambda_2 \\
Z_1 &= N_1 (1-e^2) \sin\varphi_1 & Z_2 &= N_2 (1-e^2) \sin\varphi_2
\end{aligned} \tag{1}
$$

where e^2 is a square of eccentricity calculated from semiaxes a, b of ellipsoid as follows:

$$e^2 = \frac{a^2 + b^2}{a^2}.$$

According to [5] the spatial coordinates of two initial points are defined after adjustment of equations (1) by the following relations:

$$
\begin{aligned}
X_1 &= N_1 \cos\varphi_1 & X_2 &= N_2 \cos\varphi_2 \cos(\lambda_2 - \lambda_1) \\
Y_1 &= 0 & Y_2 &= N_2 \cos\varphi_2 \sin(\lambda_2 - \lambda_1) \\
Z_1 &= N_1 (1-e^2) \sin\varphi_1 & Z_2 &= N_2 (1-e^2) \sin\varphi_2
\end{aligned} \tag{2}
$$

The length of connecting line between points P_1 and P_2, i.e. ellipsoidal chord t_1, is defined by the formula:

$$t_1 = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2}\,. \tag{3}$$

For azimuth of a direct normal section in point P_1 the following holds:

$$\cot A_{12} = \frac{-(X_2 - X_1)\sin\varphi_1 + (Z_2 - Z_1)\cos\varphi_1}{Y_2 - Y_1}\,. \tag{4}$$

After substituting equations (2) we get:

$$A_{12} = \operatorname{arccot}\left[\frac{(1-e^2)(N_2 \sin\varphi_2 - N_1 \sin\varphi_1)\cos\varphi_1 - (N_2 \cos\varphi_2 \cos\Delta\lambda - N_1 \cos\varphi_1)\sin\varphi_1}{N_2 \cos\varphi_2 \sin\Delta\lambda}\right] \tag{5}$$

Backward azimuth A_{21} in point P_2 can be considered as the initial normal section azimuth in point P_2 and is calculated in the following way:

$$A_{21} = \operatorname{arccot}\left[\frac{(1-e^2)(N_2 \sin\varphi_2 - N_1 \sin\varphi_1)\cos\varphi_2 - (N_2 \cos\varphi_2 - N_1 \cos\varphi_1 \cos\Delta\lambda)\sin\varphi_2}{N_1 \cos\varphi_1 \sin\Delta\lambda}\right] \tag{6}$$

The length of ellipsoidal chord is then transformed into normal section length (see Fig. 3). With respect to other possible solutions of this navigation task, the elliptical arc of normal section can be replaced with an arc of a circle of the radius $R = (2a + b)/3$, where a, b are semiaxes of ellipsoid. For the length z_r of normal section we have:

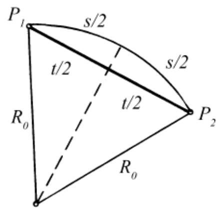


Fig. 3 The length of ellipsoidal chord

$$z_r = \left(t_1 + \frac{t_1^{\,3}}{24R^2} + \frac{3t_1^{\,5}}{640R^4} \right) \frac{\rho}{R},$$

where $\rho = 180°/\pi$. Distances calculated in this way are sufficiently accurate and can be applied in such cases when distances between points exceed 1000 km. Further specification can be achieved by using the obtained distances and azimuth as IGP input values to calculate distances of the initial points and their azimuth of connecting line on the site of the direction finder through numerical integration method. This method is described below.

4.2. Calculation of Distances Between Initial Points and the Point Defined by the Solution of a Spherical Triangle

By calculating azimuths and the arc length of the geodetic line we obtain required quantities for further solution of a spherical triangle. Angles of the spherical triangle (see Fig.1) and distances to the target are determined by the following equations:

$$\psi_1 = A_{12} - A_1, \qquad \psi_2 = A_2 - A_{21}, \tag{7}$$

$$D_{r1} = \arctan\left[\frac{\cos\left(\frac{\psi_2 - \psi_1}{2}\right)}{\cos\left(\frac{\psi_1 + \psi_2}{2}\right)} \tan\left(\frac{z_r}{2}\right) \right], \qquad D_{r2} = \arctan\left[\frac{\sin\left(\frac{\psi_2 - \psi_1}{2}\right)}{\sin\left(\frac{\psi_1 + \psi_2}{2}\right)} \tan\left(\frac{z_r}{2}\right) \right], \tag{8}$$

$$D_1 = (D_{r1} + D_{r2}) \frac{R}{\rho}, \qquad D_2 = (D_{r1} - D_{r2}) \frac{R}{\rho}. \tag{9}$$

4.3. Determining Geodetic Coordinates of the Target

Geographical coordinates of the target are calculated with the help of DGP. The given quantities in this case are geographical coordinates of the initial point, the azimuth and distance of the target. This task can be solved with the help of several methods, with respect to corresponding reference surfaces, coordinate systems, distances between initial points and targets as well as the required accuracy.

The method of numerical integration of differential equation of the geodesic line seems to be appropriate. Its main advantage is simplicity of solution and the possibility to programme the calculation's cyclical repetition. The differential equations of a geodesic line are as follows:

$$\frac{\mathrm{d}\varphi}{\mathrm{d}s}=\frac{\cos\alpha}{M}, \qquad \frac{\mathrm{d}\lambda}{\mathrm{d}s}=\frac{\sin\alpha}{N\cos\varphi}, \qquad \frac{\mathrm{d}\alpha}{\mathrm{d}s}=\frac{\sin\alpha}{N\cot\varphi} \tag{10}$$

where M is the radius of meridian and N is radius of the curvature in the prime vertical. The Runge-Kutta method [7] is suitable for numerical integration. In terms of DGP, the calculations can be conducted in the following order (see Fig. 4):

1. Calculated distance to the target is divided into identical segments h.
2. From the initial values φ_1, λ_1, α_1 on point P_1, values $\varphi_{(1)}$, $\lambda_{(1)}$, $\alpha_{(1)}$ at the end of the first segment are calculated (in point $P_{(1)}$). These values become initial values for the following segment whose end point has coordinates $\varphi_{(2)}$, $\lambda_{(2)}$, $\alpha_{(2)}$. Calculations continue in this way until the end point of the geodetic line P_2 whose coordinates will be φ_2, λ_2, α_2 .

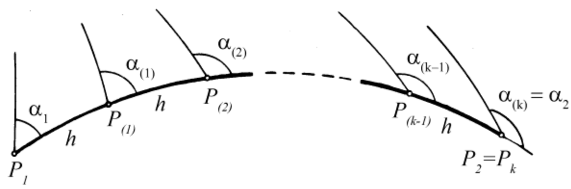


Fig. 4 The Runge-Kutta method

The following formulas are used:

$$\varphi_{i+1}=\varphi_i+\frac{1}{6}(\varphi_{k1}+2\varphi_{k2}+2\varphi_{k3}+\varphi_{k4}),\ \lambda_{i+1}=\lambda_i+\frac{1}{6}(\lambda_{k1}+2\lambda_{k2}+2\lambda_{k3}+\lambda_{k4}),$$

$\alpha_{i+1}=\alpha_i+\frac{1}{6}(\alpha_{k1}+2\alpha_{k2}+2\alpha_{k3}+\alpha_{k4})$. Symbols φ_{kj}, λ_{kj}, α_{kj} (j = 1,..., 4) are products of individual derivations and the integration step, they are calculated as follows:

$$\lambda_{kj}=\frac{\mathrm{d}\lambda}{\mathrm{d}s}h,\ \varphi_{kj}=\frac{\mathrm{d}\varphi}{\mathrm{d}s}h,\ \alpha_{kj}=\frac{\mathrm{d}\alpha}{\mathrm{d}s}h.$$

Individual products are calculated according to the following table:

Table 1: Individual products

j	φ	λ	α
1	φ_i	λ_i	α_i
2	$\varphi_i + 0.5\varphi_{k1}$	$\lambda_i + 0.5\lambda_{k1}$	$\alpha_i + 0.5\alpha_{k1}$
3	$\varphi_i + 0.5\varphi_{k2}$	$\lambda_i + 0.5\lambda_{k2}$	$\alpha_i + 0.5\alpha_{k2}$
4	$\varphi_i + \varphi_{k3}$	$\lambda_i + \lambda_{k3}$	$\alpha_i + \alpha_{k3}$

The accuracy of the algorithm of the procedure algorithm depends on the integration step which is in turn conditioned by the position of the geodesic line on the ellipsoid. For distances of up to 100 km we can choose step h which is identical with the distance to the target. For longer distances, the following procedure may be applied. For the first calculation the value of initial step between $100 \div 200$ km is selected. Further calculation is done with a half-step and both results are then compared. If the difference is within set limits, the second result is considered final.

Generally, it can be said that the accuracy of determining the signal source position depends on the accuracy of the location of the direction finder (φ_{ZAM}, λ_{ZAM}) and the accuracy of the direction finder-source azimuth bearings. Apart from this, accuracy is also affected by the approximation of the Earth to the reference surface (radius R_{REF}), i.e. the accuracy with which the reference sphere or ellipsoid attaches to the geoid. Applying the Error Propagation Law, we get:

$$\varphi_{ZDR} = f_1(\varphi_{ZAM}, \lambda_{ZAM}, D_{ZAM-ZDR}, R_{REF}, A_{ZAM-ZDR})$$

$$m_{\varphi ZDR} = \left\{ \mathrm{E}\left(\begin{aligned} &\frac{\partial \varphi_{ZDR}}{\partial \varphi_{ZAM}} \delta\varphi_{ZAM} + \frac{\partial \varphi_{ZDR}}{\partial \lambda_{ZAM}} \delta\lambda_{ZAM} + \frac{\partial \varphi_{ZDR}}{\partial D_{ZAM-ZDR}} \delta D_{ZAM-ZDR} + \frac{\partial \varphi_{ZDR}}{\partial R_{REF}} \delta R_{REF} \\ &+ \frac{\partial \varphi_{ZDR}}{\partial A_{ZAM-ZDR}} \delta A_{ZAM-ZDR} \end{aligned} \right)^2 \right\}^{\frac{1}{2}}.$$

The attached tables illustrate the accuracy of the method. Generally, we can state that the accuracy of source localization is primarily affected by errors in azimuth determination ($A_{ZAM\text{-}ZDR}$) and in distance determination ($D_{ZAM\text{-}ZDR}$).

If the distances within the triangle exceed 100 km, errors in unknown target coordinate determination reach higher values. The values are affected by azimuth determination and by the distances between individual points. It is therefore necessary to switch from circular arc length to geodetic arc length on an ellipsoid. Owing to the fact that highly accurate calculation geodetic methods are relatively complicated and unsuitable for navigation tasks and due to possible errors in determination of geodetic azimuths also inefficient, numerical integration method was selected again, this time adjusted for solutions of IGP.

Computations use the above mentioned principle, whose first step encompasses determination of approximate azimuth α'_1 and distance s' with the help of the following equations:

$$\Delta\varphi = \varphi_2 - \varphi_1, \qquad \Delta\lambda = \lambda_2 - \lambda_1, \qquad \varphi = (\varphi_1 + \varphi_2)/2, \tag{18}$$

$$\alpha = \arctan\left(\frac{N\cos\varphi\Delta\lambda}{M\Delta\varphi}\right), \qquad s' = \frac{M\Delta\varphi}{\cos\alpha}, \qquad \alpha_1' = \alpha - \frac{\Delta\lambda}{2}\sin\varphi \tag{19}$$

Approximate or input values can also be find out with the help of the ellipsoidal chord method (mentioned in the paragraph 4.1), which will further specify the acquired values of normal section distance and its azimuths.

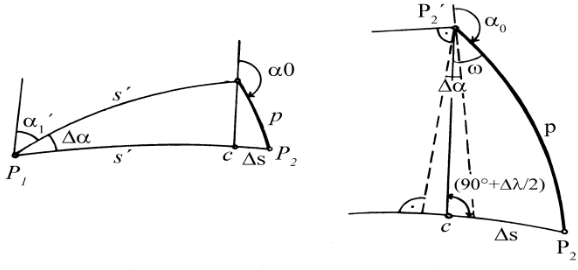


Fig. 5 Calculation of approximate values of point P2

Calculation of approximate values of point P_2 coordinates according to relations for DGP follows. The calculated coordinates φ'_2, λ'_2 and azimuth α'_2 are generally not identical with point P_2 coordinates (see Fig. 5). Based on the differences between P_2 and P'_2 coordinates, azimuth α_0 of geodesic line p which connects both points is calculated.

$$\tan\alpha_0 = \frac{(\lambda_2 - \lambda'_2)\cos\varphi_2}{\varphi_2 - \varphi'_2}, \qquad p = \sqrt{(\varphi_2 - \varphi'_2)^2 + [(\lambda_2 - \lambda'_2)\cos\varphi_2]^2}, \qquad \Delta\alpha' = \frac{p\cos\omega}{\sin\dfrac{s'}{R}},$$

$$\Delta\alpha = p\frac{\cos\omega}{\sin\left(\dfrac{s'}{R} + \Delta s'\right)}, \qquad \Delta s' = p\sin\left(\omega + \frac{\Delta\alpha}{2}\right), \quad \Delta s = R\Delta s', \qquad \omega = \alpha_2' - \alpha_0 + \frac{\pi}{2}$$

The above listed formulas can be used to calculate corrected values α'_1 and s', which are then used for the second iteration of DGP calculation.

The calculation is repeated, as it is done in determining unknown point coordinates, until corrections $\Delta\alpha$ and Δs fall within limits of the set accuracy, which means identical values of the calculated and given end points. The computing procedure is shown e.g. in [5].

4.4. Transformation of Geographic Coordinates Into Planar Right-angled ones in the Projected Coordinate System

The obtained results are in the WGS84 geographic coordinate system. For further use, it is necessary to convert geographic coordinates into Cartesian ones. The conversion procedure accompanied by an example is described e.g. in [7].

5. Accuracy of Determination of Unknown Point Coordinates

For the purposes of the task, a small number of measured values will usually be available (the worst-case scenario involves the lowest necessary number of measurements, i.e. two measured azimuths) [7, 8]. The procedure of results improvement will therefore be based on simpler methods.

In the case of minimum number of measurements is available, accuracy depends on mean errors of input values, i.e. m_A, m_D as well as on m_φ , m_λ , m_{REF} .

If the target is localized from more than two direction finders, its most probable location will be defined by arithmetic means of the calculated coordinates, i.e. φ_{ZDR} and λ_{ZDR}.

6. Testing Conditions

In order to test the selected method, three areas were designated. Their size and shape of patterns defined by the initial and target points reflected possible conditions of actual measurements.

The first area roughly corresponds to the area of the Czech Republic. The values of distances between initial and target points are in hundreds of kilometres, which means that PGT for short and medium distances are addressed. The distribution of initial and target points is shown in Fig. 6 and Fig. 7.

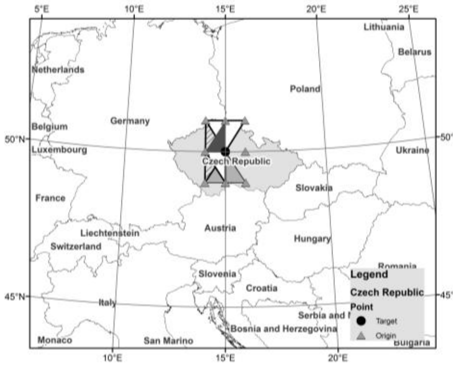


Fig. 6 Origins and Target in CR

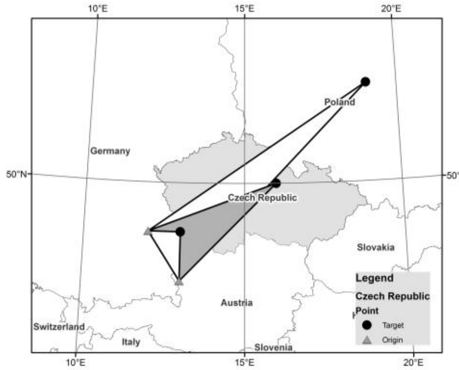


Fig. 7 Origins and Targets around CR

The second area corresponds to the area of Europe. Distances within this area reach several thousand kilometres. In this case we refer to long distance PGT. The tested points are shown in Fig. 8 and Fig. 9.

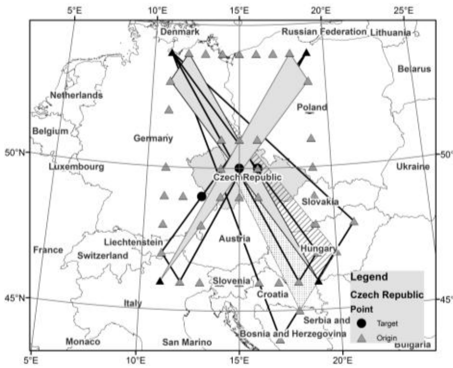


Fig. 8 Origins and Targets in Europe

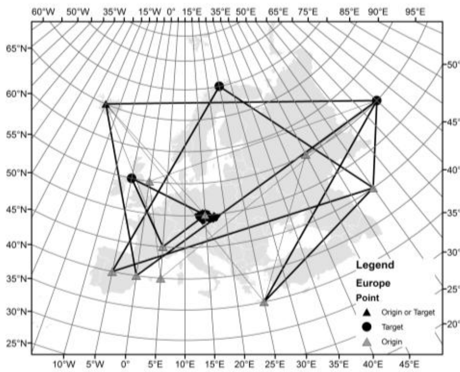


Fig. 9 Origins and Targets around Europe

The third area is the entire Earth and individual points are selected to simulate sites both on the Eastern and Western Hemispheres, north and south of the equator.

Based on conveniently selected coordinates of initial points and the signal source, accurate geodetic azimuths were calculated. Consequently, the coordinates of signal source were determined from the initial points coordinates and calculated azimuths by applying the proposed method, so the directly verifiable results were obtained.

Owing to the fact that two reference surfaces, a sphere and an ellipsoid, were used, the application of different radiuses of the spherical reference surface was tested as well to establish their influence on the resulting calculation. A single radius was used for calculations of the ellipsoidal chord method. The calculations were done in MathCad2000 software.

7. Selected Results of Computing Procedures Testing

We have got several results from our calculations. First one was calculated using points located according Fig. 6. There are 9 combinations of measurements with maximum error 0.0001 decimal degree, which is approximately 12 m in system of projected coordinates.

Points according Fig. 8 were used for the following test calculation and results obtained are in Table 2 and Table 3. The maximum error reaches approximately 400 m, respectively 200 m. It is up to conditions of calculation.

Symbols φ_1, λ_1, φ_2, λ_2 designate coordinates of direction finders, φ_c and λ_c represent the calculated source coordinates and φ, λ are predefined coordinates of the signal source.

Table 2: Geographical coordinates of signal source location (results after 4 iterations for 2 sites, R = sqrt(M,N)), application of the IGP on a sphere

P_1		P_2		P_c							
φ_1	λ_1	φ_2	λ_2	φ_c	λ_c	φ	λ	$\Delta\varphi$	$\Delta\lambda$	x	y
49	12	48	13	49.000 49.001	12.997 13.000	49	13	0.0000	0.0015	5429382.9 5429385.9	–146280.9 –146390.6
49	12	48	13	49.999 50.001	16.002 16.002	50	16	0.0000	–0.0020	5539109.8 5539111.7	71666.4 71809.8
49	12	48	13	52.001 52.002	19.006 19.006	52	16	-0.0015	–0.0060	5762926.8 5763082.3	–137294.4 –136880.9

Table 3: Geographical coordinates of signal source location (4 iterations, 2 sites, R = 6 371 008 m), ellipsoidal chord method

P_1		P_2		P_c							
φ_1	λ_1	φ_2	λ_2	φ_c	λ_c	φ	λ	$\Delta\varphi$	$\Delta\lambda$	x	y
49	12	48	13	49.000	13.000	49	13	0.0000	0.0000	5429382.9 5429382.9	–146280.9 –146280.9
49	12	48	13	50.000	16.002	50	16	0.0000	–0.0016	5539109.8 5539111.3	71666.4 71781.1
49	12	48	13	52.000	19.0032	52	19	-0.0002	–0.0031	5762926.8 5762943.2	–137294.4 –137081.0

Third set of tests was calculated from points located in area of Europe, however first group of calculation was performed from points distributed around the Czech Republic according Fig. 8.

Thirty different combinations of calculations were done with mean error value 0.001 Decimal Degrees (DD) which corresponds to distance of 111 m and 0.0014 DD which corresponds to distance of 155 m as maximum error. The position of centroid of this area (50.0 lat and 15.0 long) was determined by these calculations.

In further test the coordinates of corner points of these area were calculated in eight combinations with max error e.g. 0.0062 DD, standard position error 0.005 (approximately 590 m).

The results of last test group are in Table 4, where points are located according Fig. 9. In the Table 5 are given the results of calculation of coordinates of the source signal. In this calculation the differences in the determination of azimuth of source from one or two given points were applied. The maximum variance is in the latitude direction $\Delta\varphi = 0.0143°$, and in the longitude direction $\Delta\lambda = 0.0294°$.

Table 4: Geographical coordinates of signal source location (100 iterations, 3 sites, R = 6 371 008 m), ellipsoidal chord method

P_1		P_2		P_c							
φ_1	λ_1	φ_2	λ_2	φ_c	λ_c	φ	λ	$\Delta\varphi$	$\Delta\lambda$	x	y
45	5	50	15	54.9996	–4.9995	55	–5			6124104	–511176
				54.9999	–4.9992						
				54.9998	–4.9994			0.0002	0.0006	6124077	–511140
45	5	50	15	64.9968	–19.9917	65	–20			7217779	–235591
				64.9982	–19.9894						
				64.9975	–19.9906			0.0025	0.0094	7217466	–235171
40	–5	45	55	69.9970	24.9910	70	25			7767125	–76331
				69.9985	25.0053						
				69.9978	24.9982			0.0022	0.0018	7766882	–76407
65	–20	40	0	55.0271	69.9620	55	70			6095249	63967
				55.0077	69.9389						
				55.0174	69.9504			–0.0174	0.0496	6097141	60768
55	0	40	5	55.0122	69.9631	55	70			6095249	63967
				55.0037	69.9581						
				55.0080	69.9606			–0.0080	0.0394	6096104	61435
55	–5	35	25	55.0073	69.9799	55	70			6095249	63967
				54.9961	69.9714						
				55.0017	69.9756			–0.0017	0.0244	6095416	62404
35	25	45	55	55.0006	70.0026	55	70			6095249	63967
				55.0008	70.0015						
				55.0007	70.0020			–0.0007	–0.0020	6095328	64094
35	25	55	45	54.9988	69.9904	55	70			6095249	63967
				55.0012	69.9885						
				55.0000	69.9894			0.0000	0.0106	6095239	63289
55	–5	35	25	55.0073	69.9799	55	70			6095249	63967
				54.9961	69.9714						
				55.0017	69.9756			–0.0017	0.0244	6095416	62404

8. Analysis of Results and Recommendations for the Proposed Solution Application

Several fundamental conclusions can be drawn from the overall results:

1. In order to calculate the location of a signal source for distances of approximately 100 km, both methods for target coordinates determination can be applied. However, owing to the fact that application of the ellipsoidal chord method is not complicated, it is more convenient to use this method both for long and short distances.
2. Should the ellipsoidal chord method be applied, the accuracy of determination of distance to given points is comparable for differently selected radiuses of the reference sphere. Since accuracy analysis revealed that the influence of errors in the length of ellipsoidal chord is significantly lower than the influence of

errors in calculated direction finder-target distance, a radius determined from an ellipsoid by one of the three known methods – both solids have either identical volume, surface or the sphere radius equals the arithmetic mean of all three ellipsoid semi axes – can thus be applied. When rounded to 0.1 km, thus determined radiuses are identical. In this case, the third method of sphere radius calculation was applied. This method is sufficiently accurate for the given purposes.

Table 5: Errors in geographical coordinates upon changes of azimuth of 0.5° ÷ 2.0° (2sites, 4 iterations, R = 6371008 m), ellipsoidal chord method

ΔA_1 / ΔA_2		**0.500**	**1.000**	**1.500**	**2.000**
–2.000	$\Delta\varphi$	0.0091	0.0109	0.0126	0.0143
	$\Delta\lambda$	0.0215	0.0243	0.0269	0.0294
–1.500	$\Delta\varphi$	0.0076	0.0095	0.0114	0.0131
	$\Delta\lambda$	0.0177	0.0207	0.0235	0.0262
–1.000	$\Delta\varphi$	0.0060	0.0081	0.0100	0.0118
	$\Delta\lambda$	0.0135	0.0168	0.0199	0.0227
–0.500	$\Delta\varphi$	0.0043	0.0065	0.0086	0.0105
	$\Delta\lambda$	0.009	0.0126	0.0160	0.0191
0.000	$\Delta\varphi$	0.0025	0.0049	0.007	0.0091
	$\Delta\lambda$	0.0042	0.0081	0.0118	0.0151
0.500	$\Delta\varphi$	0.0006	0.0031	0.0054	0.0076
	$\Delta\lambda$	–0.0010	0.0033	0.0072	0.0109
1.000	$\Delta\varphi$	–0.0015	0.0012	0.0037	0.0060
	$\Delta\lambda$	–0.0067	–0.0020	0.0024	0.0064
1.500	$\Delta\varphi$	–0.0037	–0.0008	0.0018	0.0043
	$\Delta\lambda$	–0.0129	–0.0077	–0.0029	0.0015
2.000	$\Delta\varphi$	–0.0061	–0.0030	–0.0002	0.0024
	$\Delta\lambda$	–0.0198	–0.0140	–0.0087	–0.0039

2. Should the ellipsoidal chord method be applied, the accuracy of determination of distance to given points is comparable for differently selected radiuses of the reference sphere. Since accuracy analysis revealed that the influence of errors in the length of ellipsoidal chord is significantly lower than the influence of errors in calculated direction finder-target distance, a radius determined from an ellipsoid by one of the three known methods – both solids have either identical volume, surface or the sphere radius equals the arithmetic mean of all three ellipsoid semi axes – can thus be applied. When rounded to 0.1 km, thus determined radiuses are identical. In this case, the third method of sphere radius calculation was applied. This method is sufficiently accurate for the given purposes.

3. Upon distances of up to 100 km between the direction finder and target, iteration step which equals the corresponding distance can be used. Upon longer distances it is necessary to follow the general description of the iteration method and to take into account mistakes due to rounding as well as the influence of the initial and target points location.

4. All calculations were applied on an ellipsoid. Should it be deemed necessary, ellipsoidal heights of the initial points must be included in calculations of spatial coordinates and height angles must be determined in the course of localizing the target.

5. Finally, the accuracy of results depends on the distance between initial points, their number and position on the reference plane, the distance to the target and number of iterations. But it is a theoretical problem. In practice the accuracy of several hundred meters is acceptable for the fast moving target, where the initial position and direction of the flight path is more important.

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