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Analysis of Ballistic Characteristics of Pistol Cartridge

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Abstract:

The article deals with the ballistic analysis of pistol cartridge of calibre 9 mm Luger. As a basis of the analysis the standard cartridge with 8 g FMJ projectile was chosen. The aim of the analysis is to assess the effect of projectile mass change on basic ballistic characteristic of weapon-cartridge system during development of new types of projectiles. There is also explained possible utilisation of the obtained results with use of selected typical small arms cartridge in the end of the article.

Keywords:

Ballistics, design of ammunition, small arms cartridge, ballistic analysis

1. Introduction

Every newly developed small arms cartridge must meet a number of requirements, very often mutually antagonistic. The new cartridge must function reliably in all or nearly all types of weapons under predefined extreme conditions. The projectile's trajectory must meet customers' requirements on flatness of trajectory, time of flight to given distance, sensitivity to cross wind, etc. The projectile and its design must also ensure required effect in predefined types of targets, e.g. penetrating and wounding capabilities.

Therefore it is necessary to set general limits of both interior and exterior ballistic characteristics that are important for both the designers and also for the users of the new type of cartridge during development of a new small arms cartridge. The selected ballistic characteristics can be obtained with the use of both interior and exterior ballistic models in various iterative calculations. The methodology of calculations is demonstrated on the cartridge of calibre of 9 mm Luger (also known as 9×19) but the methodology is usable for cartridges of an arbitrary calibre.

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The ballistic analysis is divided into two follows-up parts, the interior and the exterior. The exterior ballistic analysis utilises results of the preceding interior ballistic one.

2. Interior Ballistic Analysis

The interior ballistic analysis deals with the interior ballistic characteristics and function of the cartridge in the weapon. From the interior ballistics point of view, amongst the most important characteristics of the weapon system belong:

- mass of projectile m_q,
- mass of propellant ω,
- initial velocity v_0 or muzzle velocity $v_ú$, the difference between them can be neglected,
- maximum allowed pressure of propellant gases $p_{\max}$,

and also their mutual relationships. With these characteristics are closely connected their derived characteristics that are useful for the assessment of reliable function of weapon and overall ballistic performance of the weapon system:

- initial momentum of projectile M_0,
- initial kinetic energy of projectile E_0,
- initial specific kinetic energy of projectile e_0.

The aim of the interior ballistic analysis is to find out the sensitivity of these parameters to the change of one of them (e.g. mass of projectile) under defined conditions (e.g. requirement to keep constant defined pressure $p_{\max}$, muzzle velocity $v_ú$, etc.).

During development of the new cartridge, the most important is the selection of the mass of the projectile m_q or the range of acceptable masses. To illustrate this, the extent of the mass of projectile from 4 to 20 g was chosen. The standardized maximum pressure $p_{\max}$ is defined in [1] as 260 MPa. It is assumed that the volume of the projectile remains the same and thus the initial combustion volume c_0 remains also the same.

2.1. Interior Ballistic Model

The relation between the above mentioned parameters provides the interior ballistic model. For analytic purposes, the well-known interior ballistic model with global parameters described in [2-4] has been utilised. This literature describes the same model, from the principal point of view, but with slightly different approach. The outputs from these models are fully comparable from both quantitative and qualitative point of view.

The interior ballistic model used for purposes of this analysis consists of following equations [3]:

$$
\begin{aligned}
&\psi = \kappa z + \kappa\lambda z^2 + \kappa\mu z^3 = \kappa z\left(1 + \lambda z + \mu z^2\right)\\
&p = \frac{f\,\omega\psi - \dfrac{\theta\varphi m_q v^2}{2}}{s\left(l_\psi + l\right)}\\
&\varphi\, m_q \frac{\mathrm{d}v}{\mathrm{d}t} = s\,p\\
&\frac{\mathrm{d}l}{\mathrm{d}t} = v\\
&\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{p}{I_k}\\
&l_\psi = l_0\left[1 - \frac{\Delta}{\delta} - \Delta\psi\left(\alpha - \frac{1}{\delta}\right)\right]\\
&T = T_v\left(1 - \frac{1}{\psi}\,\frac{\theta\varphi\, m_q v^2}{2\, f\,\omega}\right)
\end{aligned}
\tag{1}
$$

where:

ψ – relative burnt propellant mass,

z – relative burnt thickness of grain,

κ, λ, μ – form function coefficients,

f – force,

φ – coefficient of fictivity,

α – covolume of propellant gases,

I_k – total impulse of propellant gases,

l_0 – reduced length of initial combustion volume,

l_ψ – reduced length of free barrel bore volume,

l – projectile's trajectory,

Δ – density of propellant charge,

δ – density of propellant mass,

T – temperature of propellant gases,

T_v – adiabatic flame temperature,

θ – parameter of propellant gases expansionSimulations and Their Results

From the designer point of view, the previously mentioned interior ballistic model was utilised in two different ways. In the first case, the mass of propellant was kept constant; its value was the same as in the case of standard cartridge, i.e. 0.3 g. In the second case, the mass of propellant charge was changed to reach the chosen maximum allowed pressure p_{max} for the particular projectile mass; in this showcase it was 260 MPa.

Constant Mass of Propellant

The calculations have been carried out for the standard cartridge and only the mass of projectile was changed in the above mentioned range. This case is presented rather for the comparison and to show how sensitive the weapon system is on to the change of the projectile mass.

Results of interior ballistic modelling are summarised in Table 1. The dark grey part of the table highlights the unsafe combinations of mass of projectile and mass of propellant because the maximum allowed pressure p_{max} was exceeded; projectiles heavier than 10.0 g. The light grey part contains projectile – propellant charge combinations unsuitable from the reliability of weapon function point of view;

projectiles of mass lower than 7.0 g. For the reliable function of the semiautomatic handgun the minimal impulse of 2.5 Ns is required.

The same colour scheme is used in Table 3 and Table 4, where the results of exterior ballistic analysis are presented.

Table 1 Results of interior ballistic modelling – constant mass of propellant ω

	Mass of projectile [g]								
	4.0	**6.0**	**8.0**	**10.0**	**12.0**	**14.0**	**16.0**	**18.0**	**20.0**
$v_{ú}$ **[m/s]**	453.1	389.0	344.8	312.5	287.7	268.0	251.8	238.2	226.6
$t_{ú}$ **[ms]**	0.42	0.48	0.52	0.57	0.61	0.65	0.68	0.71	0.75
p_{max} **[MPa]**	151.2	193.7	234.0	272.9	310.7	347.4	379.2	406.1	429.0
$p_{ú}$ **[MPa]**	21.0	19.5	18.8	18.4	18.2	18.0	17.8	17.7	17.6
$m_q \cdot v_{ú}$ **[Ns]**	1.81	2.33	2.76	3.13	3.45	3.75	4.03	4.29	4.53
ω **[g]**	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
Δ **[kg/m³]**	447.5	447.5	447.5	447.5	447.5	447.5	447.5	447.5	447.5

Constant Maximum Pressure

In this case of interior ballistic analysis, the mass of propellant charge with which it is possible to reach the required maximum pressure p_{max} has been iteratively searched for; mass of projectile m_q is a parameter. The secant method was used for finding the appropriate mass of propellant charge ω. Results of modelling are summarised in Table 2. All solutions are obviously in the safe region from the maximum allowed pressure point of view, but the projectiles of the mass lower than 5 g do not ensure reliable semiautomatic function of handguns.

Table 2 Results of interior ballistic modelling – constant maximum pressure p_{max}

	Mass of projectile [g]								
	4.0	**6.0**	**8.0**	**10.0**	**12.0**	**14.0**	**16.0**	**18.0**	**20.0**
$v_{ú}$ **[m/s]**	552.2	429.9	356.8	307.7	272.2	245.2	223.9	206.8	192.8
$t_{ú}$ **[ms]**	0.34	0.43	0.50	0.58	0.64	0.71	0.77	0.83	0.89
p_{max} **[MPa]**	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0
$p_{ú}$ **[MPa]**	26.9	22.4	19.8	18.0	16.7	15.7	14.9	14.2	13.6
$m_q \cdot v_{ú}$	2.2	2.5	2.8	3.0	3.2	3.4	3.5	3.7	3.8

	Mass of projectile [g]								
	4.0	**6.0**	**8.0**	**10. 0**	**12. 0**	**14. 0**	**16. 0**	**18. 0**	**20. 0**
[Ns]	1	8	5	8	7	3	8	2	6
***ω* [kg]**	0.4 1	0.3 5	0.3 2	0.2 9	0.2 7	0.2 6	0.2 4	0.2 3	0.2 2
***Δ* [kg/m³]**	61 0.1	52 8.0	47 4.3	43 5.6	40 7.2	38 3.4	36 4.0	34 9.0	33 5.6

The muzzle velocities obtained during the solution of both cases are presented in dependency on the projectile mass in Fig. 1. The dark circles on light grey line represent the muzzle velocities achieved with breeching the maximum allowed pressure. Dark triangles represent the cartridges that do not ensure the reliable semiautomatic function of weapon.

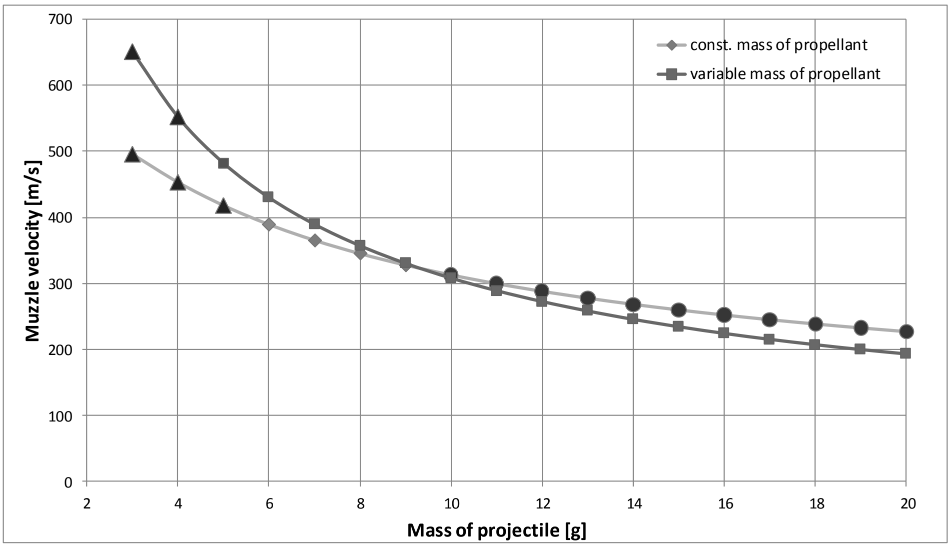

Fig. 1 Muzzle velocity vs. mass of projectile

3. Exterior Ballistic Analysis

The aim of the exterior ballistic analysis is to evaluate the calculated exterior ballistic characteristics of individual versions of the cartridge and provide sufficient information to select the most suitable version or range of versions. The characteristics of interest are:

- vertex height Y_{max},
- angle of departure θ_0,
- time of flight to target range t_{30},
- impact velocity v_{30},
- impact and specific impact kinetic energy E_{30}, e_{30}.

The introduced analysis is based on the assumption that the size and shape of the projectile remains the same, i.e. shape coefficient $i_{43} = 1.432$ (derived from standard

FMJ projectile) is the same for all projectile and the ballistic coefficient c_{43} changes only with the changing mass of projectile. The change of mass of the projectiles is achieved only by the change of material of the projectile; variable density of the projectile material. The target distance 30 m has been chosen as a typical range of fire for handguns and thus all exterior ballistic characteristics are related to this distance.

3.1. Exterior Ballistic Model

For the purposes of the exterior analysis, the point mass trajectory model (3 DoF model) described in [4-8] is used. This model is sufficiently accurate for the purposes of the ballistic analysis of small arms projectile and also firing at short ranges.

The exterior ballistic model consists of the following equations [7]:

$$
\begin{aligned}
\frac{\mathrm{d}v_x}{\mathrm{d}t} &= -c_{43}\left(\frac{p_N}{p_{0N}}\right)\left(v_x - w_x\right)\left(\frac{\tau_{0N}}{\tau_N}\right)^{0.5} G_{43} \\
\frac{\mathrm{d}x}{\mathrm{d}t} &= v_x \\
\frac{\mathrm{d}v_y}{\mathrm{d}t} &= -c_{43}\left(\frac{p_N}{p_{0N}}\right)\left(v_y - w_y\right)\left(\frac{\tau_{0N}}{\tau_N}\right)^{0.5} G_{43} - g \\
\frac{\mathrm{d}y}{\mathrm{d}t} &= v_y \\
\frac{\mathrm{d}v_z}{\mathrm{d}t} &= -c_{43}\left(\frac{p_N}{p_{0N}}\right)\left(v_z - w_z\right)\left(\frac{\tau_{0N}}{\tau_N}\right)^{0.5} G_{43} \\
\frac{\mathrm{d}z}{\mathrm{d}t} &= v_z
\end{aligned}
\tag{2}
$$

where:

c_{43} – ballistic coefficient (drag law 1943),

p_N – standard atmospheric pressure,

p_{0N} – standard atmospheric pressure at sea level,

w – wind speed,

τ_N – standard virtual air temperature,

τ_{0N} – standard virtual air temperature at sea level,

G_{43} – drag function (drag law 1943).

For all the exterior ballistic calculations the standard artillery atmosphere [8] was utilised.

3.2. Results of Modelling

In this part of work the angle of departure θ_0 for the individual versions of the cartridges and the range of fire 30 m were calculated. The value of initial (muzzle) velocity $v_ú$ was taken from the results of the interior ballistic analysis, and the value of ballistic coefficient c_{43} was calculated with the use of shape coefficient i_{43} = 1.432 (derived from the shape of FMJ projectile) and the mass of projectile m_q. Consequently, the corresponding trajectories and their significant characteristics were calculated. For the angle of departure and trajectory calculations, a verified program was used. The verification was carried out by comparison of calculated results for number of weapon systems with corresponding data from firing tables. Results of these calculations are summarised in Table 3 and Table 4.

The following figures, Figs 2 and 3, show the drop in projectile kinetic energy after traveling from muzzle to the target (30 m). There are shown the initial E_0 and the impact E_{30} kinetic energies of individual projectiles in the figures. This drop in kinetic energy (or ratios E_0/E_{30}, and e_0/e_{30}) can be utilised for selection of the cartridge when the minimum impact energy at given distance is required.

Table 3 Results of exterior ballistic modelling – constant mass of propellant ω

	Mass of projectile [g]								
	4.0	**6.0**	**8.0**	**10.0**	**12.0**	**14.0**	**16.0**	**18.0**	**20.0**
c_{43} [m^2/kg]	29.00	19.33	14.50	11.60	9.67	8.29	7.25	6.44	5.80
v_{30} [m/s]	387.0	351.2	324.2	303.1	281.3	263.1	247.8	234.8	223.7
t_{30} [s]	0.08	0.09	0.09	0.10	0.11	0.12	0.13	0.13	0.14
Y_{max} [m]	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.03
E_0 [J]	410.6	454.0	475.5	488.3	496.6	502.8	507.2	510.7	513.5
e_0 [MJ/m^2]	6.56	7.25	7.60	7.80	7.93	8.03	8.10	8.16	8.20
E_{30} [J]	299.6	370.0	420.4	459.4	474.8	484.4	491.1	496.2	500.4
e_{30} [MJ/m^2]	4.79	5.91	6.71	7.34	7.58	7.74	7.84	7.92	7.99
E_0/E_{30} [1]	1.37	1.23	1.13	1.06	1.05	1.04	1.03	1.03	1.03
θ_0 [°]	0.046	0.060	0.074	0.088	0.103	0.119	0.134	0.150	0.166

Table 4 Results of exterior ballistic modelling – constant maximum pressure p_{max}

	Mass of projectile [g]								
	4.0	**6.0**	**8.0**	**10.0**	**12.0**	**14.0**	**16.0**	**18.0**	**20.0**
c_{43} [m^2/kg]	29.00	19.33	14.50	11.60	9.67	8.29	7.25	6.44	5.80
v_{30} [m/s]	476.0	386.9	333.1	298.8	266.3	240.7	220.3	203.9	190.3
t_{30} [s]	0.06	0.08	0.09	0.10	0.12	0.13	0.14	0.15	0.16
Y_{max} [m]	0.01	0.01	0.01	0.02	0.02	0.02	0.03	0.03	0.04
E_0 [J]	609.8	554.4	509.2	473.4	444.6	420.9	401.0	384.9	371.7
e_0 [MJ/m^2]	9.74	8.86	8.13	7.56	7.10	6.72	6.41	6.15	5.94
E_{30} [J]	453.1	449.1	443.9	446.4	425.6	405.6	388.3	374.0	362.2
e_{30} [MJ/m^2]	7.24	7.17	7.09	7.13	6.80	6.48	6.20	5.97	5.79
E_0/E_{30} [1]	1.35	1.23	1.15	1.06	1.04	1.04	1.03	1.03	1.03
θ_0 [°]	0.030	0.049	0.069	0.091	0.115	0.142	0.170	0.199	0.229

4. Discussion of Results

The interior ballistic analysis shows that the use of projectiles of different mass without modification of the propellant charge is neither possible nor suitable way for the development of a new cartridge. It can be seen from Table 1 that in this case only small changes of the projectile mass are possible due to safety concerns.

From Table 2 it is apparent that it is possible to set up a cartridge with both light and heavy projectiles with appropriate propellant charge. In case of light projectiles, the raising density of propellant charge *Δ* must be taken into account.

In case of heavy projectiles, problems appear with higher impulses which are not comfortable for the shooter and which could also affect accuracy and the real rate of fire. The velocity drops by on third in case of heaviest 20 g projectile.

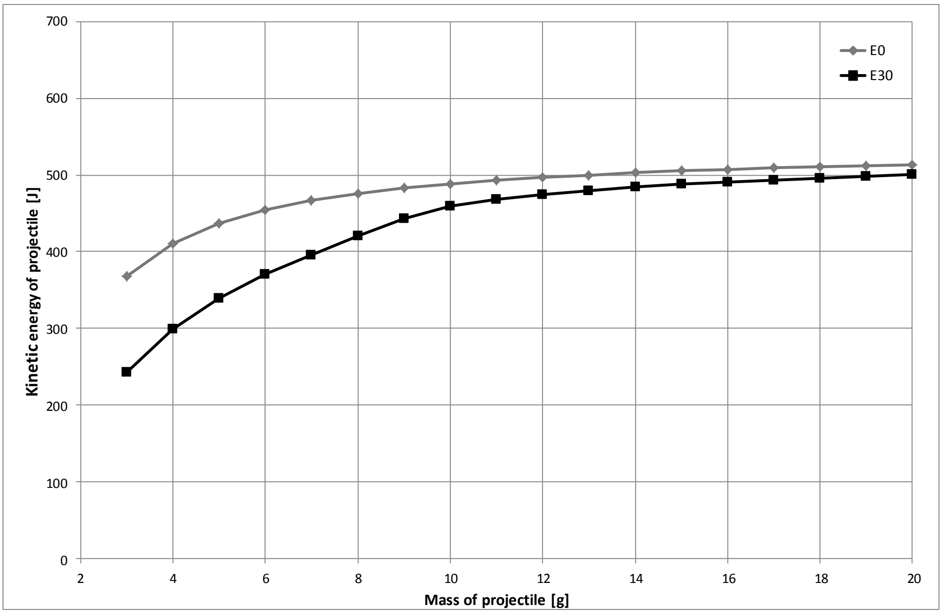

Fig. 2 Kinetic energy of projectile vs. mass of projectile – constant mass of propellant

From the exterior ballistic analysis, data concerning the in-flight behaviour of the assessed projectiles and the terminal ballistic characteristics were obtained. For the practical evaluation of individual cartridges and their use against moving targets, the time of flight to the target range t_{30} is important. The heavier projectiles have longer times of flight and in this case they exceed 0.14 s. For the introduction of a new cartridge, the magnitude of the angle of departure (the angle of departure is directly connected with the aiming angle) can be important, because standard weapons are equipped with solid sights and a big change in aiming angles (standard vs. new round) is undesirable. For the use of new cartridge in tight spaces (e.g. buildings, transportation means, etc.), the height of vertex should be kept as low as possible (in other words, the projectile trajectory must be as flat as possible). This requirement is also important for the use of standard weapons without need to adjust the sights.

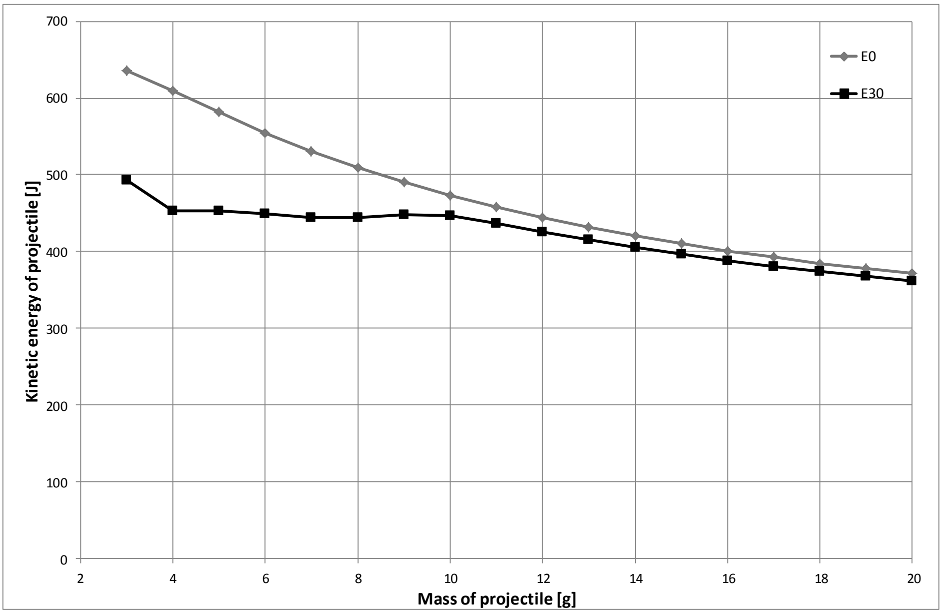

Fig. 3 Kinetic energy of projectile vs. mass of projectile – variable mass of propellant

From the projectile's terminal effect point of view, the most important characteristics are impact kinetic energy E_{30} (or specific impact kinetic energy e_{30}). Their dependencies on the mass of projectile are shown in Figs 2 and 3. When comparing Figs 2 and 3, it is necessary to take into account the difference between the two cases; constant propellant charge (pressure raises with raising projectile mass) and constant maximum pressure. In case of constant maximum pressure (variable mass of propellant charge), the lighter projectiles move for significant time in the transitional area between subsonic and supersonic velocities and therefore the curve of E_{30} in Fig. 3 could be interpreted as unusual or illogical. For shorter ranges of fire (e.g. 10 m), the drop in kinetic energy is smaller and the curve has a similar shape as the curve of energy $E_{0,}$ because the velocity of projectile does not go through the transitional region where the drag sharply changes.

5. Conclusion

This approach to the setting of technical limits during development of new types of small arms cartridges has proved its suitability. The obtained results contain interesting and complex data about possible solutions of a new cartridge. The method can be utilised with different parameters and also with different required parameters. The article introduced only one of possible application with variable mass of propellant and required value of maximum pressure p_{max}.

This approach can be also utilised in a reverse way, when the requirements are set on the performance of the projectile in the target (e.g. impact energy) and the suitable range of combinations of muzzle velocity, mass of projectile, mass of propellant, and ranges of fire is obtained.

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