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Parametric Adaptation as an Element of Mathematical Models Qualimetry of Complex Processes

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Abstract:

A possible approach to ensuring the necessary qualitative properties of analytical models by adapting their parameters to probable changes in the course of complex processes is considered. The approach involves the use of a polymodel description of processes with the aim of mutual compensation of the objective shortcomings of heterogeneous models, as well as the use of simulation modeling capabilities to adjust the parameters of analytical models in cases where the use of the latter is due to strict limitations on the time of obtaining calculation results and developing control influences based on them. The considered example of parametric adaptation of the Lanchester-type model reflects probable changes in the number of opposing sides during the conduct of hostilities.

Keywords:

analytical and simulation models, complex process, parametric adaptation of analytical models, qualimetry of models

1 Introduction

Model qualimetry is an applied theory in which methods of quantitative evaluation and quality assurance of mathematical models are studied and implemented. The quality of the model is defined by the property or set of its properties that determine the ability to use the model for its intended purpose. Theoretical qualimetry involves a distinction between direct and inverse problems. Direct tasks mean direct assessment of the quali-

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ty of any product, while inverse tasks are connected with quality management to ensure the necessary properties of products (in this case – mathematical models).

Since the models themselves are created both for the purpose of analysis and synthesis of already existing original objects, the inverse problems play the main role in the qualimetry of the models.

According to the authors [1], despite the constant development of methods and technologies for creating models of different classes and purposes, such tasks are often left out of consideration (in the field of information technology much more attention is paid to assessing software quality, which is reflected in relevant standards). These tasks are especially relevant when the models reflect a complex process (CP). Such processes may be unique and obtaining complete objective data on them for modelling purposes is difficult or impossible (models of accidents and disasters, hostilities, natural cataclysms and the like). The authors in [1] propose to describe any complex process using several heterogeneous mathematical models combined into a complex. This allows to increase the objectivity of the results obtained, but requires coordination between the models of the complex.

The use of complex heterogeneous models will not only increase the validity of assumptions about the probable nature of the CP, but will also provide additional opportunities to solve the inverse problems of model qualimetry. It should be noted that certain difficulties in solving these problems exist because of the lack of a well-established terminological framework in this area. As an example, we can cite the publication [2], which summarizes the definition of the key qualitative property of simulation models – their adequacy. The definitions belong to experts in the field of simulation, including Shannon, Forrester, Naylor and others. These differences in the understanding of the concept of model adequacy, different views on the fundamental possibility and methods of quantitative assessment of the adequacy of models of unique CP, do not contribute to the unambiguous establishment of such a qualitative characteristic from a qualimetric standpoint. Thus, in thematic publications it is noted that for models of unique CP, a direct quantitative assessment of adequacy is impossible, and it can only be about the usefulness of a particular model to solve a certain class of problems. In this case, it is advisable to take the following definition of the adequacy of the model: it is its ability to take into account the most significant factors arising from the course of the CP. Naturally, among heterogeneous (simulation and analytical) models, simulation models (SM) will have a much higher adequacy, in which all stages of the process are reflected while maintaining the logical sequence of their course in time. In addition, random difficulties to formalize factors are taken into account using pseudo-random values with their laws distribution [3, 4]. Thus in SM there are practically no restrictions on use of all arsenal of methods of analytical modeling. This makes it possible, after verification of the modeling algorithm and program code of the detailed SM CP, as well as validation of the original data, to use the results of simulation as a reference in relation to the analytical models (AM) of the process. This approach is to some extent forced (only for relatively unique CPs), however, it allows to solve inverse qualimetric problems using the provisions known in metrology for the evaluation of various types of measurements. It should be noted that, despite the rapid development of multiprocessor and multicore technologies in computer technology, SM remain the main consumer of CPU time due to its statistical nature. In conditions when the application of models requires consideration of the time factor, AM remain an effective means of timely obtaining the results of the necessary calculations. This applies, first of all, to the use of simulation results for management decisions (formation of control influences) in conditions of rapid changes in the situation and structural dynamics of objects of influence (typical examples – combat modeling, action management in natural disasters, man-made disasters, etc.). In this case, in order for the analytical model to remain consistent with changes in the simulated process, it is necessary to adapt its parameters to changes in the conditions of CP modeling.

The parametric adaptation of models (that is, "adjustment", correction of their parameters) is also considered in a number of thematic sources (in particular [1]) as one of the elements of solving the inverse qualimetric problem. However, these recommendations are mostly of a general theoretical nature and are limited to models of real original objects. Therefore, the main purpose of the article is to consider the applied problems of solving the inverse qualimetric problem by parametric adaptation of analytical models to probable changes in the nature of the simulated unique complex processes. We believe that such an approach will contribute to the extension of the provisions of theoretical qualimetry to analytical and simulation systems to increase confidence in the quality of analytical modeling. The approach may be of interest to crisis planning professionals, in particular combat planning.

2 Definition of Research Problem

A polymodel complex (PmC) is considered, which was designed to predict the probable variants of the course of the CP and to develop solutions on the basis of the results of calculations of control influences (CI).

PmC includes a detailed reference simulation model (SM_e) and a set of analytical models $\{AM_a\}$, $a = \overline{1,A}$ with a simplified description of the process, which provide the necessary efficiency of modeling and obtaining the whole set required for production CI calculated indicators $\overline{Y}$. If it is necessary to produce CI, as a reaction to the changes in the course of the CP, time constraints are imposed on the time of obtaining indicators $\overline{Y}$. Such limitations determine the possibility of using SM_e only at the stage of preliminary forecast calculations of probable variants of CP development, with its beginning. If it is necessary to react to changes in its course, the possibilities of an exclusively $\{AM_a\}$ complex are used. In order to ensure the adaptation of AM to changes in the conditions of CP modeling, it is necessary:

- To determine the order of comparison of the entire set f l^{th} parameters $\overline{P}_l^{(a)}$ of each a^{th} analytical model of PmC with similar parameters obtained by simulation ($\overline{P}_l^{(e)}$). The use of agreed parameters in the AM should minimize the current deviation (absolute or relative, ΔY) between the results of analytical and simulation modeling, and such deviation may not exceed some set limit value (ΔY_D):

$$\Delta Y\left[AM_a\left(\overline{P}_l^{(a)}\right);SM_e\left(\overline{P}_l^{(\mathrm{e})}\right)\right]\rightarrow \min$$

$$\overline{P}_l^{(a,\mathrm{e})}=\left\{P_1^{(a)},\ \dots,P_l^{(a)};P_1^{(\mathrm{e})},\ \dots,P_l^{(\mathrm{e})}\right\},\quad l=\overline{1,L}$$

$$\Delta Y\le \Delta Y_{\mathrm{D}}$$

e – the letter "e" (etalon) was used to emphasize that it is this simulation model (SM) that is used as a reference (generally, there may be several of them in the complex);
l = (1,L)´ – the indices of unified (for analytical and reference models) parameters,
L – the natural number,

D – the index means that a restriction should be imposed on the difference between the results of analytical and simulation modeling *Y* (as an indicator) (for example, for military models, such a restriction should be $Y_D = 15\ \%$).

- To numerically evaluate the possibility of ensuring the adaptability of the AM to changes in the conditions of the simulated CP by matching the parameters of analytical models with the parameters of the reference SM.

3 Description of the Method and Basic Mathematical Equations

Based on the above initial provisions, the presence of a detailed SM of the unique CP is a necessary condition for the parametric adaptation of analytical models of PmC. Leaving aside the issue of a comprehensive assessment of the qualitative properties of detailed SM, which will serve to obtain reference results, it is advisable to give only the general structure of such an assessment (by analogy with [3, 5]). Similar steps, with their reference to the sequence of construction of SM, can be represented as shown in Fig. 1.

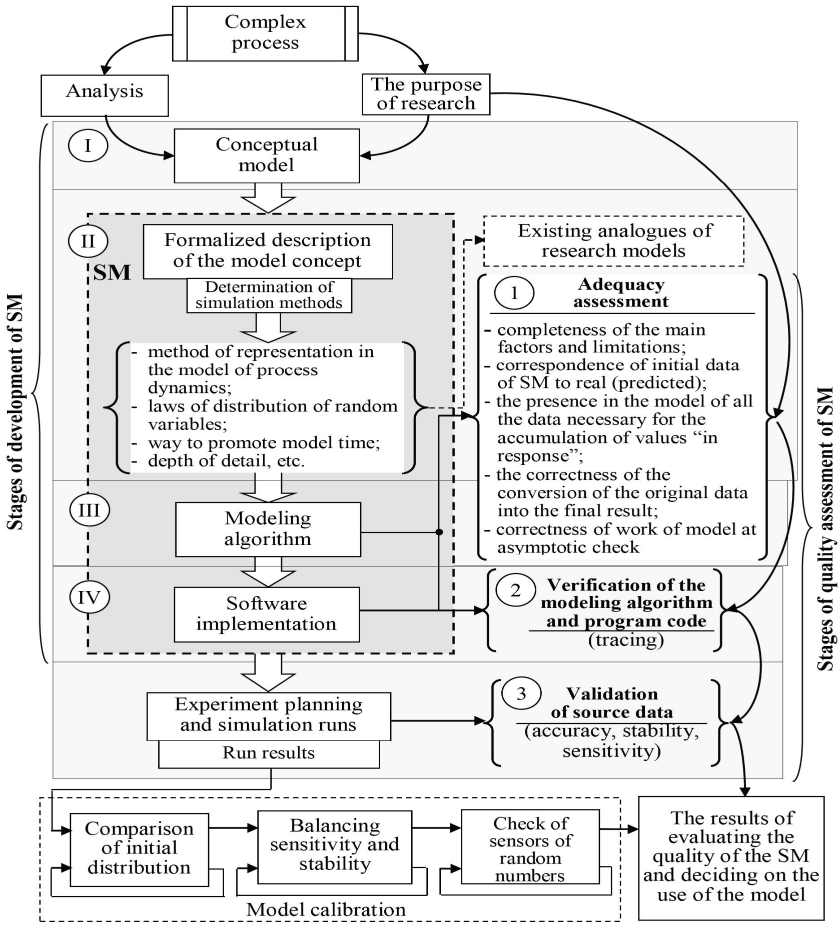

Fig. 1 Structure of comprehensive assessment of quality of SM by stages of construction of SM complex process

The presence of the tested SM in the composition of PmC allows to use wide possibilities of simulation modeling for early specification of all sets of parameters ($\overline{P}_l^{(a)}$) concerning the most probable variants of development of CP. In this case, according to the results of the required (or specified) number of runs, SMe parameters ($\overline{P}_l^{(e)}$) will be obtained in the form of statistical distribution *Validation* ns. Depending on the class of analytical models included in the PmC, the use of the obtained distributions makes it possible to use them in the AM in different ways. For clarity, we assume that the analytical calculation of indicators ($\overline{Y}$) is carried out using models as systems of differential equations. In this case, as a set of parameters ($\overline{P}_l^{(a)}$) will be the coefficients for phase variables and free members of the equations, and the adjusted parameters ($\overline{P}_l^{(a)}$) can be represented in the form:

- estimates of mathematical expectations of statistical distributions (constants),
- time variables,
- stochastic quantities.

The choice of a certain form of parameters is determined by the analysis of obtained statistical distributions and physical essence of desired indicator. An explanation of the procedure for refining the parameters of AM can be demonstrated by the following practical example.

Let the PmC simulate the course and results of hostilities, then one of the necessary indicators of the population ($\overline{Y}$) is the number of opposing groups at certain points in time and such indicators are determined analytically using modifications of Lanchester models. It should be noted that in modern PmC combat operations (for example, Joint Conflict and Tactical Simulation (JCATS) or Joint Theater Level Simulation (JTLS) complexes – [6, 7]) the combined use of simulation models with Lanchester-type models is quite successfully implemented. Suppose also that under the conditions of the problems, such AM must take into account the restoration of the number of parties (instead of lost) and has the form of the following system of equations:

$$\left.\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= (-c-b)\,y + d\left[x(0)-x\right] \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= (-g-f)\,x + h\left[y(0)-y\right] \end{aligned}\right\} \tag{1}$$

where

x, y – the number of groups of opposing parties at the time $t \geq 0$; $x(0)$,
$y(0)$ – initial conditions for the number of groups at the time of the action ($t = 0$),
c, g, b, f – the intensity of losses of the parties due to the influence of different types of weapons,
d, h – the intensity of the recovery of the number of parties.

Despite the uniqueness of each implementation of plans for hostilities, general trends in the development of such a CP, based, in particular, on previous experience for similar conditions, are tentatively known. That is, some average values of the AM parameters are also known, assuming, for example, that the parameters c, g and b, f characterize the losses from the action of planar and point means of damage, respectively. The variant of such parameters can take the following values (Tab. 1).

Tab. 1 A variant of the averaged values of the parameters $\overline{P}_l^{(a)}$ *of the system of* Eq. (1)

side *x*	*c*	*b*	*d*
	0.0087	0.0121	0.0179
side *y*	*g*	*f*	*h*
	0.0107	0.0058	0.0264

For specified conditions of hostilities and for the purpose of parametric adaptation of AM type (1) according to the results of simulation, the distributions of the specified parameters and values of the required indicator of CP (in this case it is the number of opposing parties $x_{\mathrm{SM_e}}$ and $y_{\mathrm{SM_e}}$ at a certain time of hostilities) are obtained. In accordance with the task, it is necessary to minimize the discrepancy between the indicators of the CP, obtained by analytical modeling and simulation, while complying with the condition $\Delta Y \leq \Delta Y_{\mathrm{D}}$. For the considered type of models, the limits of admissible difference make 15 % and such a difference can be estimated based on the known expression for establishment of relative error of calculations (concerning certain indicators):

$$\delta_x = \frac{\left| x_{\mathrm{SM_e}} - x^*_{\mathrm{AM}} \right|}{x_{\mathrm{SM_e}}} 100; \quad \delta_y = \frac{\left| y_{\mathrm{SM_e}} - y^*_{\mathrm{AM}} \right|}{y_{\mathrm{SM_e}}} 100 \tag{2}$$

where δ_x, δ_y – the results of estimating relative deviations,
$x_{\mathrm{SM_e}}$, $y_{\mathrm{SM_e}}$ – the number of opposing parties,
x^*_{AM}, y^*_{AM} – the number of opposing parties at a certain point in time, established by the results of analytical modeling with adjusted parameters.

The choice of the form of the parameters is determined by the type of the obtained statistical distributions and the results of estimating the relative deviations δ_x, δ_y. In the future, they can be used as adjusted in the AM ($\overline{P}_l^{(a)}$).

In the simplest case, estimates of mathematical expectations (ME) of the obtained distributions are used as refined parameters, that is model (1) will be a system of linear inhomogeneous equations. As an example, the results of the simulation modeling show the following estimates of the mathematical expectation of the statistical distributions of the parameters of the system (1) (Tab. 2).

Tab. 2 Estimates of mathematical expectations of the parameters $\overline{P}_l^{(\mathrm{e})}$ *of the system of equations obtained by simulation* (1)

side *x*	*c*	*b*	*d*
	0.0141	0.0152	0.0302
side *y*	*g*	*f*	*h*
	0.0137	0.0078	0.0264

The results of simulation calculations for the indicators of the CP (obtained using SM type JCATS or JTLS) are also known. For certainty, we assume that the analysis of actions is performed on a time interval up to $t = 5$ (units of time) with arbitrarily taken initial conditions $x(0) = 30\,000$; $y(0) = 25\,000$. The values of the obtained indicators SM_e are as follows: $x_{SM_e}(5) = 23\,173$; $y_{SM_e}(5) = 19\,051$. The results of numerical integration of system (1) for certain initial conditions are presented in Fig. 2a.

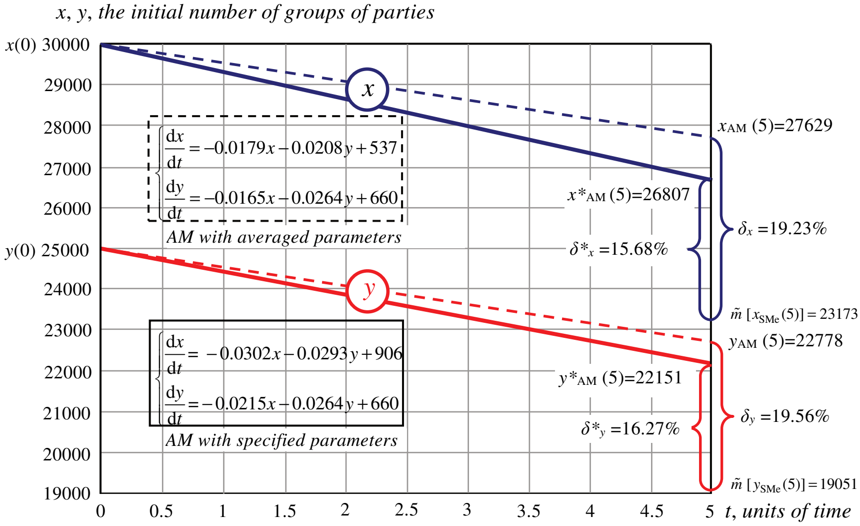

Fig. 2a Results of solution of system (1) *for the determined initial conditions*

It is also clear that the change (adjustment) of the parameters of this type of AM necessitates the assessment of the stability (in the sense of Lyapunov) of the obtained solutions, for the parameters of Tab. 2 and systems of type (1), the phase portrait will be a stable node (Fig. 2b).

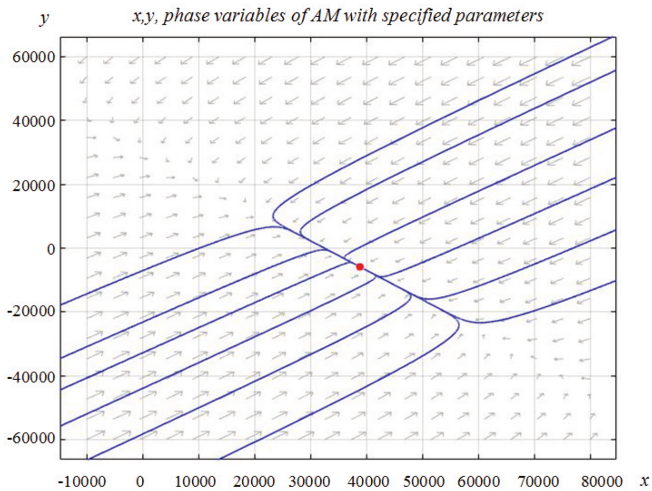

Fig. 2b Evaluation of stability of solution for parameters of Tab. 1

The assessment of the established constraint $\Delta Y \leq 15$ % was carried out in accordance with (2). It can be stated that the use of both averaged (Tab. 1) and refined (Tab. 2) parameters does not provide a permissible deviation of the results of simulation and analytical modeling. Although the use of refined parameters provides a reduction in the deviation of $\delta_{x,y}$ from 19 % to 16 % (denoted as $\delta_{x,y}$ and $\delta_{x,y}^*$ in Fig. 2a, respectively), the resulting discrepancy remains unsatisfactory.

If the physical essence of CP and the simulation results indicate the need to take into account the influence of numerous random factors difficult to predict, the reflection of a complex process is possible in the form of AM as systems of stochastic differential equations (SDE). For the considered conditions for 40 independent runs of IM_e, the following data concerning parameters b and d, grouped in 10 digits of their values, are received (Tab. 3).

Tab. 3 Statistical series of parameters b and d of the system (1), *which is obtained from results of simulation (conditional variant)*

Parameter *b*										
	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$	$i = 7$	$i = 8$	$i = 9$	$i = 10$
I_i	0.0125 0.0131	0.0131 0.0137	0.0137 0.0143	0.0143 0.0149	0.0149 0.0155	0.0155 0.0161	0.0161 0.0167	0.0167 0.0173	0.0173 0.0179	0.0179 0.0185
m_i	1	3	4	5	11	7	5	1.5	1.5	1
P_i^*	0.025	0.075	0.1	0.125	0.275	0.175	0.125	0.0375	0.0375	0.025
Parameter *d*										
	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$	$i = 7$	$i = 8$	$i = 9$	$i = 10$
I_i	0.0286 0.0290	0.0290 0.0294	0.0294 0.0298	0.0298 0.0302	0.0302 0.0306	0.0306 0.0310	0.0310 0.0314	0.0314 0.0318	0.0318 0.0322	0.0322 0.0326
m_i	4	3	4	6	5	4	5	4	2	3
P_i^*	0.1	0.075	0.1	0.15	0.125	0.1	0.125	0.1	0.05	0.075

* In Tab. 3 the following notations are accepted: I_i – the digits of the received values of parameters; m_i – the number of values obtained within the category; $P_i^* = m_i/n$ – the statistical frequencies, $m = 40$.

Histograms showing the data of Tab. 3 that is the statistical series of parameters *b* and *d* shown in Figs 3a and 3b.

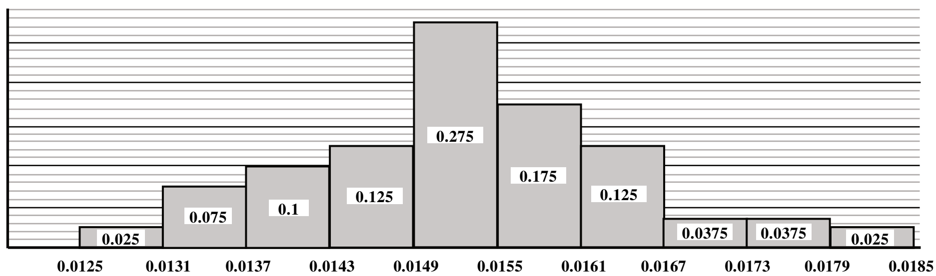

Fig. 3a Histogram of statistical series of parameter b of the system (1)

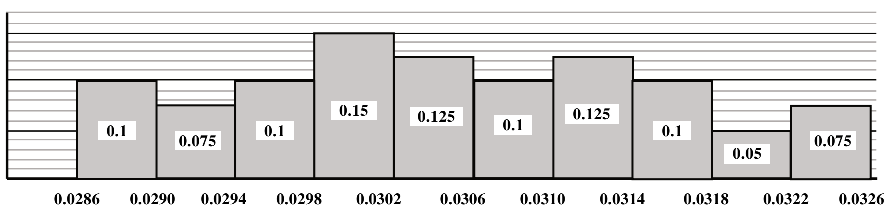

Fig. 3b Histogram of statistical series of parameter d of the system (1)

The appearance of histograms of statistical series shows that the obtained values of parameter b are grouped around the statistical average, while the values of parameter d have a pronounced nature of small fluctuations in the range of their possible values. Provided that other identical parameters of system (1) will be of a similar nature, such a model can be represented as a system of stochastic differential equations (SDE) with multiplicative Wiener noise:

$$\left.\begin{aligned} dx &= -0.0293\,y\,dt + 0.0302\left[x(0)-x\right]dw_t \\ dy &= -0.0215\,x\,dt + 0.0264\left[y(0)-y\right]dw_t \end{aligned}\right\} \quad (3)$$

The use of specialized methods of numerical integration of SDEs in order to solve system (3) [8, 9] allows to obtain results in the form of both approximate strong and approximate weak solutions. Strong solutions are used when it is necessary to calculate the most accurate approximate solutions based on a separate implementation of the SDE for given conditions. In other cases, there is a need to find the probability distributions of the required values, and individual implementations are not crucial. Weak solutions are used to meet this need, and in the context of the problem of adapting the parameters of AM (based on simulation calculations), the obtained probability distributions for weak solutions are of greater interest. It should be added that, by some analogy with simulation runs, weak solutions of the SDE imply the need to obtain a set of independent trajectories of the process. In the future, this will allow the use of a wide arsenal of data analysis methods to compare simulation distributions and distributions as a result of solving the SDE (in addition, weak methods are easier to implement). The results of the numerical solution of system (3) by the Milstein method (in the weak sense) are presented in Fig. 4.

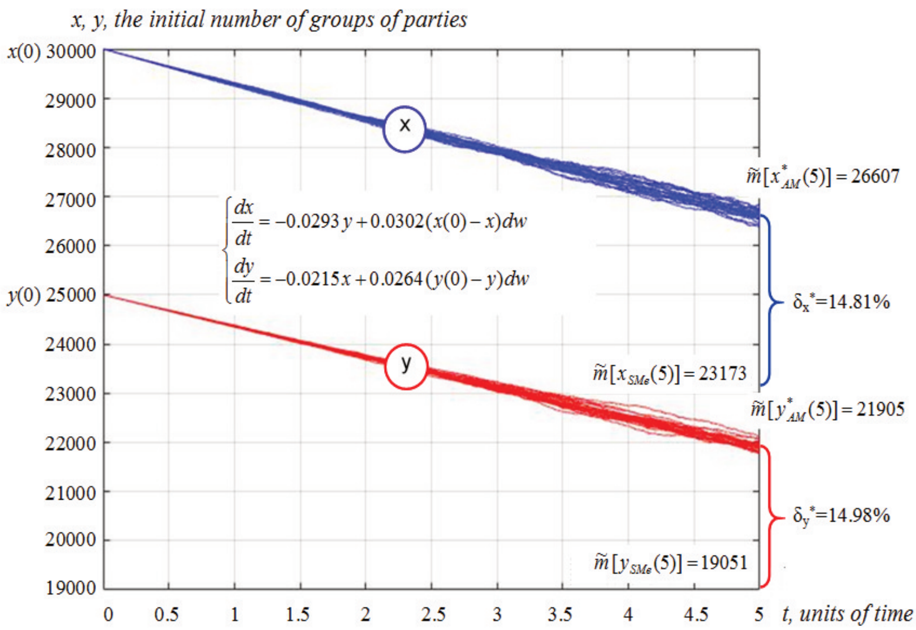

Fig. 4 Results of solving system (3) *for certain initial conditions by Milstein method (fragment for* 20 *trajectories)*

Estimation of solution stability, as in the case of ordinary differential equations, is an important qualitative characteristic of model (3), but in the case of using SDE the concept of stability has a somewhat broader interpretation, including interpretation related to methods of solving such equations. Taking into account the confirmed stability of the deterministic basic system (Fig. 2), certain computational features of the solution of the model (3) deserve more attention. Based on practical considerations and the nature of the problem, the following features should be considered: coordination of the integration step (Δt) and the number of implementations of the model (N), as well as the procedure for estimating the mutual shift of simulation distributions and distributions of weak SDE solutions. With regard to the first feature, it is advisable to use an approximate ratio of the species $\Delta t \sim N^{-1/4}$.

Regarding the second feature, in addition to estimating the deviations of the means (MEs or medians) by expression (2), it is possible to use statistical criteria to assess the reliability of such deviations (for example, criteria to classify the estimated deviations to the so-called "measurement shift"). For certain initial data of a practical example, and also results of a kind of Fig. 4, the relative deviations of the selected indicators (δ_x^* and δ_y^*), calculated for ME distributions for 10 000 implementations of system (3), are: $\delta_x^* = 14.81$ %; $\delta_y^* = 14.98$ %.

That is, the condition $\Delta Y \leq \Delta Y_D$ in this case is observed and such a simple example indicates the need for a reasonable choice of the form of application of the specified parameters, as a result of the analysis of all simulation data of the CP. Similarly, the form of parameters is specified in the case of a substantially nonstationary process, i.e. an example of AM of type (1) will reflect the process as a system of linear nonstationary equations (or combined, as a system of nonstationary SDEs). If the reasonable use of the established forms of the specified parameters does not provide fulfillment of a condition $\Delta Y \leq \Delta Y_D$, it will testify to need of adjustment already of a kind or a class of AM, instead of only its parameters. Thus, only when the results of detailed simulation and analytical modeling of unique CPs converge (within certain limits) can we talk about the possibility of generating CI based on the results of analytical calculations.

4 Conclusions

In summary, the solution of the inverse qualimetric problem in the combined mapping of unique CPs due to parametric adaptation of AM will require, first of all, the formation of PmC as part of agreed analytical and simulation models of CP, in which at least one (or only) simulation model should serve to obtain reference results. In this case, the appropriate procedure for adapting the parameters of the AM is the following sequence:

- determination of parameters of analytical models that need to be specified by simulation,
- calculation of distributions of the defined parameters on runs of the reference SM for the most likely flow options CP,
- the use of simulated parameters in analytical models in the form determined by the analysis of the results of simulation (as a set of statistical data), the class of analytical models used and the features of their software implementation,
- assessment of difference of the results of analytical and simulation modeling, adjustment (combination), if necessary, of the forms of the applied specified parameters. According to the experience of practical calculations, it is quite real to keep the

deviation of the results of analytical modeling from the reference (or accepted as such) in this way within 15 %. Otherwise, the clarification will require the very type of used analytical models, and such a question should be attributed to the scope of structural adaptation of the PmC.

In general, parametric adaptation of analytical models can be considered as effective element of solving one of the key problems of model qualimetry, namely providing their necessary qualitative properties, including the integrated application of heterogeneous models of unique complex processes. In the military spare, the practice of adapting the AM can be considered as follows: simulation modeling of the most probable options for operations at the stage of advanced planning. Clarification of parameters for the considered options; use of AM with adjusted parameters in crisis response planning (analytical modeling significantly reduces the time needed to obtain results). The prospect of further research is to issues of structural adaptation of simulation models to the given conditions of reflection of complex processes.

References

[1] MIKONI, S.V., B.V. SOKOLOV and R.M. YUSUPOV. *Qualimetry of Models and Polymodel Complexes* (in Russian). St. Petersburg: Nauka, 2018. DOI 10. 31857/S9785907036321000001.

[2] RUMYANTSEV, M.I. On the Problem of the Adequacy Estimation of Simulation Models of the Banking Business Processes. In: *Conference Proceedings SWorld.* Odessa, 2010, **15**, pp. 84-93. ISBN 978-966-555-152-2.

[3] SMITH, J.S., D.T. STURROCK and D.W. KELTON. *Simio and Simulation: Modeling, Analysis, Applications.* 4th ed. Scotts Valley: CreateSpace, 2017. ISBN 1-54-646192-0.

[4] LAW, A.M. *Simulation Modeling and Analysis.* 5th ed. New York: McGraw-Hill Education, 2015. ISBN 978-0-07-340132-4.

[5] BALCI O. Validation, Verification and Testing Techniques throughout the Life Cycle of a Simulation Study. *Annals of Operation Research*, 1994, **53**, pp. 121-173. DOI 10.1007/BF02136828.

[6] *Joint Conflict and Tactical Simulation Capabilities Brief* [online]. 2018 [viewed 2021-10-06]. Available from: https://csl.llnl.gov/sites/csl/files/JCATS-LLNL-Brochure-30May2018.pdf

[7] *JTLS-GO Executive Overview* [online]. 2021 [viewed 2021-10-10]. Available from: https://www.rolands.com/jtls/j_vdds/executive_overview.pdf

[8] ARTEMIEV, S.S. and T.A. AVERINA. *Numerical Analysis of Systems of Ordinary and Stochastic Differential Equation.* Berlin: De Gruyter, 1997. ISBN 978-90-6764-250-7.

[9] DEBRABANT, K. and A. ROESSLER. Classification of Stochastic Runge–Kutta Methods for the Weak Approximation of Stochastic Differential Equations. *Mathematics and Computers in Simulation*, 2008, **77**(4), pp. 408-420. DOI 10.1016/j.matcom.2007.04.016.