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Determination of Efficiency of Weapon Systems Maintenance as Condition for DM Distribution

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Abstract:

The article discusses the mathematical model of technical condition-based maintenance of weapon systems. The model his developed based on a semi-Markov stochastic process. The diffusion-monotonic (DM) distribution law, which is specific for airfield technical condition-based maintenance of aircraft, is has been used as a failure model, and type I errors are considered. For standard operating conditions, graphs of the dependence of the coefficient of technical use and specific costs per hour of operation in good condition from the basic parameters are shown. The optimal maintenances interval ensuring maximum maintenance coefficient value has been proved. The principal results have been achieved by using multiple calculation method.

Keywords:

condition-based maintenance, diffusion-monotonic distribution law, maintenance and repair system, technical and economic model

1 Introduction

In modern warfare, it is possible to achieve success only by joint efforts of all forces and means of armed warfare, their comprehensive use in all geographical spheres. Thus, one of the most important actions forms within the structure of the modern armed operations is the form that is designed for repelling the adversary's air attack and ensuring coverage of troops and facilities, installations from air attacks. Those who are responsible for weapon systems (WS) maintenance are interested in the level of their technical effectiveness. This raises the question of how it is possible to influence the maintenance process to achieve maximum effectiveness values.

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Generally accepted weapon systems maintenance effectiveness values are the readiness coefficient and technical use coefficient.

The development of the appropriate model of the technical maintenance process is essential for the calculation of effectiveness values. Methodological errors resulted from the choice of inadequate theoretical model can be quite large.

Probable distribution laws include exponential, Weibull, lognormal distributions recommended by the Ukrainian standard for practical use depending on facility type and character of current tasks. Probable physical distributions have a certain advantage over just probable distributions because their parameters can be defined based on both statistical failure characteristics and analysis of failures of physical process. Now the diffusion laws of distribution are the most topical such as diffusion-monotonic and diffusion-nonmonotonic distributions. The mentioned facts are of considerable practical value specifying the study applicability.

Certain failure models are used while solving the problems of servicing technical facilities, which significantly affect the accuracy of the estimates obtained. Methodological errors due to the choice of a theoretical failure model can be quite significant.

As a model of failures in the article, we will use the diffusion-monotonic (DM) distribution, which is recommended by Ukrainian National Standards for mechanical type products, for example, airfield fueling tankers (AFT), airport electric power supply units (AEPSU), unified gas charging stations (UGCS), ground multipurpose air-conditioning units (GMACU), and air refuellers with a distribution function:

$$F\left(t,\mu,\nu\right)=\Phi\left(\frac{t-\mu}{\nu\sqrt{\mu t}}\right)$$

where $\Phi(x)$ is the Gauss (normal) distribution function of the form:

$$\Phi\left(x\right)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}\exp\left(-\frac{u^2}{2}\right)\mathrm{d}u$$

where μ – location parameter, ν – scale parameter, t – current time.

Such a distribution is far-reaching enough and relatively rarely used in scientific research.

2 Problem Formulation

The [1, 2] represent mathematical condition-based maintenance models with the use of diffusion distributions of their failures. There are calculated analytical dependencies of values of the mentioned equipment maintenance effectiveness on characteristics of their reliability and maintenance parameters.

Moreover, passing into the service of advanced aviation equipment and air weapons demands increased requirements to vitality and mobility of air defense systems (ADS). Thus, while developing prospective (modernizing the existing) ADS, the procedures of evaluation of the defined value of effectiveness retention coefficient are executed during justification of proposals on the use of mobility means of the appropriate type. In addition, while planning the combat use and logistical support of air defense systems, the key personnel accomplish procedures of evaluation of the effectiveness of combat operations. Performance of this procedure requires consideration of the defined number of the output data consisting of reliability values of the appropriate ADS. These problems were resolved in many works [3-10], where unfortunately the issues of reliability of ground combat facilities or their radio-electronic devices provided that the ADS mobility facilities do not fail or correspond to the provided probability of successful complex march at a defined distance.

In [3], the use of the semi-Markov process is presented, based on three operating states: operation, ready-to-be-used and repair, to study a transport system consisting of special vehicles. On the example of a sample consisting of police patrol cars, experimental studies of the intensity of fleet utilization, time of failure-free operation of vehicles were carried out, and it was demonstrated that the examined transport system is characterized by a satisfactory, stationary readiness coefficient.

In [4], effectiveness indicators play a significant role in the rationalization of functioning of maintenance services. The paper investigates four effectiveness indicators employed by the maintenance services of the company in question, i.e. mean time to failure, mean time between failures, mean time to repair and overall equipment effectiveness. In addition, the study has shown whether the use of correctly determined indicators and results interpretation could lead to a higher effectiveness of the actions taken by the maintenance services department.

In [5], the author optimized a dynamic condition-based maintenance policy for a slowly degrading system subject to soft failure and condition monitoring at equidistant, discrete time epochs. A random-coefficient autoregressive model with time effect has been developed to describe the system degradation. Stochastic behavior for both the age-dependent and the state dependent term are considered, and a Bayesian approach for periodically updating the estimates of the stochastic coefficients has been developed to combine information from a degradation database with real-time condition-monitoring information.

In [6], the method of evaluation of availability and readiness of technical objects (vehicles) in an executive subsystem. The semi-Markov model was used to determine the probabilistic characteristics are presented. The analysis of probabilistic characteristics of objects readiness, as well as the time spent in the distinguished operating states, enables the search for optimal algorithms of using and operating vehicles.

In [7], a comparison of the results of the examination of the process of non-parametric distribution with an analysis in which its exponential form was assumed is presented. The aim of the research/ study was to draw attention to the inconsistencies obtained and to the importance of a preliminary assessment of the data collected for examination. The diagnostics of the machine readiness operating in the studied production company was additionally performed. This allowed to evaluate its operational potential, especially in the context of solving process optimization problems.

In [8], the authors presented the complex measure of surface-to-air missile system effectiveness the coefficient of effectiveness sustainment of surface-to-air missile system combat (technical) mobility means. The coefficient used here is considered to be the function of mean distance between failures of mobility means and their number.

In [9], the author presented the way of estimating semi-Markov model parameters on the basis of actual observations obtained from the transport company, which allowed for diagnostics and evaluation of the company in terms of its readiness.

In [10], the case where the Markov renewal model is derived by lumping in a continuous-time finite Markov process with exponential holding times is given special attention, and the study includes an analysis of the effect of processing rates that differs with state or time.

However, these works could be used neither for the calculation of the effectiveness of weapon systems effectiveness values, nor for the airfield technical maintenance (ATM) of condition-based maintained aircraft.

The article is aimed at developing the adequate mathematical model of WS maintenance ATM of condition-based maintained aircraft.

At present, the current operational strategy is considered relevant. We will evaluate the effectiveness of maintenance using the total availability factor (K_{tu}), from which it is easy to go to the availability factor. Determination was linked with the construction of the model maintenance of WS.

3 Description of the Method and Basic Mathematical Equations

The mathematical model of the maintenance of WS is described in sufficient detail in [1].

We commence the solution of the task on determining K_{tu} with the development of a mathematical model on using automobile, electric and gas equipment. The graphic description of this model is depicted in Fig. 1.

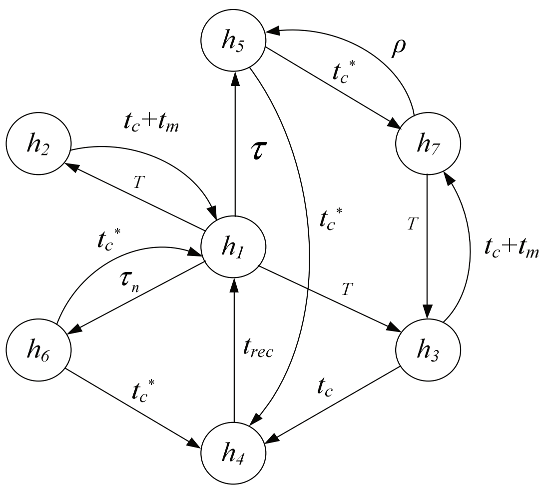

Fig. 1 The graphic of states for the model of using samples of automobile and electric and gas equipment

Such a model includes the following possible states of the WS that are to be further defined as the object of control (OC):

- h_1 – OC is functioning well,
- h_2 – the outer control system (OCS) is used to carry out the control of the technical state in the course of conducting maintenance, whilst the object of control has no denial,
- h_3 – the OCS is used to conduct the control of the technical state in the course of maintenance activity, whilst within the object of control in an accidental time-frame between 0 and T the denial appears, that has not been pointed out by the integrated control system,

- h_4 – the used system conducts a full recovery of the object of control that has been denied,
- h_5 – at an accidental moment within the time-frame between 0 and T the object of control carried out a denial that has been pointed out by the OCS,
- h_6 – at an accidental moment within the time-frame between 0 and T the OCS has pointed out a signal of false alarm,
- h_7 – the object of control is used in case of an undefined denial.

In Fig. 1 the arrows demonstrate the directions of transmission. The time of presence of the point of control in a particular state is above the arrows.

Fig. 1 uses the following identifications:

- T – the period for conducting maintenance activities,
- t_{m} – the continuity of conducting maintenance activities,
- t_{rec} – the continuity for full recovery of the point of control,
- t_{c} – the continuity of checking the point of control with the aid of the OCS,
- t_{c}^{*} – the continuity of checking the point of control with the aid of an integrated control system,
- τ – the accidental moment of receiving a signal from the OCS on the denial of the point of control,
- τ_n – the accidental moment of receiving from the OCS a false signal on the denial of the point of control,
- ρ – the accidental moment for the appearance of denial, that has been omitted whilst checking the point of control by the OCS.

There is an assumption within the model, that the values T, t_{m}, t_{rec}, t_{c}, t_{c}^{*} are not accidental. This is done with the aim to simplify the further calculations. The assumption made does not allow using the mark for modelling of accidental processes, for the function of distributing the transformation continuity from the state h_i (i = 1, 2, 3, 4, 5, 6, 7) to the h_j (j = 1, 2, 3, 4, 5, 6, 7) and for these processes it is to be strictly exponential.

This model involves scheduled preventive maintenance of weapon systems, as well as emergency repairs. This model is characterized by the presence of errors of the first and second type.

The total availability factor K_{tu} can be calculated as follows [1]:

$$K_{tu} = \frac{\sum_{i=1}^{7} \pi_i(T) \times \omega_i(T)}{\sum_{i=1}^{7} \pi_i(T) \times \eta_i(T)} \tag{1}$$

where $\pi_i(T)$ is the average frequency of the return of the Markov chain to the state h_i, $\omega_i(T)$ – the average residence time of WS in good condition, $\eta_i(T)$ – the average residence time of WS in i-state.

We assume that in each WS product there are built-in monitoring tools that control generalized technical parameters which can be used to evaluate the performance of a particular product. In addition to the built-in monitoring tools, there are control tools used during regular maintenance, as well as preventative maintenance. Such controls will be called external.

Let the operating parameters of the aircraft ATM means be such as those indicated in Tab. 1.

Tab. 1 Operational parameters of the studied means of airfield technical maintenance of aircraft

N°	ATM product / Operation parameters, unit of measurement	airport electric power supply unit	ground multipurpose air-conditioning unit	unified gas charging station	airfield fueling tanker
1	Scale parameter μ [–]	230	179	759	170
2	Form parameter ν [–]	0.5	0.5	0.5	0.5
3	False alarm rate λ, [1/hour]	10^{-3}	10^{-3}	10^{-3}	10^{-3}
4	The intensity of the manifestations of failures displayed in WS λ_m, [1/hour]	10^{-2}	10^{-2}	10^{-2}	10^{-2}
5	The duration of WS control by external means of control t_m, [hour]	1	1	1	1
6	The duration of regular maintenance T, [hour]	100	100	100	100
7	The duration of control with integrated controls t_c^*, [hour]	0.5	0.5	0.5	0.5
8	The duration of preventive maintenance t_{pm}, [hour]	5	4	2	4
9	The duration of WS restoration t_{rec}, [hour]	10	8	4	8
10	The credibility of WS control by external means of control d_{em}	0.8	0.8	0.8	0.8
11	The reliability of determining a fault condition by built-in monitoring tools d_{em}^*	0.55	0.55	0.55	0.55
12	The reliability of the correct determination of the working condition by built-in monitoring tools d_{cd}	0.7	0.7	0.7	0.7
13	The probability of a signal about the failure of the WS product from the built-in control system c	0.7	0.7	0.7	0.7

For our model, the average uptime $\omega_i(T) = M\,[\min(\tau, \tau_n)]$.

Rest $\omega_i(T)$, where $i = 2, 3, 4, 5, 6, 7$ is equal to zero since the WS for these conditions cannot be used for its intended purpose

$$\eta_j(T) = \sum_{i=1}^{7} P_{ij}(T) \times \int_0^\infty t \times \mathrm{d}F_{ij}(t) \tag{2}$$

where $F_{ij}(t)$ – the law of distribution of the duration of the transition of a random process from state i to state j, $\boldsymbol{P}(T)$ – the matrix of probabilities of transitions from state i to state j.

$$\begin{cases} \vec{\pi}(T) \leftarrow \vec{\pi}(T) \times P_{ij}(T) \\ \sum_{i=1}^{7} \pi_i(T) = 1 \end{cases} \tag{3}$$

The transition probability matrix for our model has the form Eq. (4),

where $P_{12} = [1 - F(T)] \times \mathrm{e}^{-\lambda t}$, $P_{13} = (1-\rho) \times \int_0^T \mathrm{e}^{-\lambda t} \mathrm{d}F(t)$, $P_{15} = \rho \times \int_0^T \mathrm{e}^{-\lambda t} \mathrm{d}F(t)$, $P_{16} = \lambda \times \int_0^T \mathrm{e}^{-\lambda t} [1 - F(t)] \mathrm{d}t$, $F(t)$ – the diffusion-monotonic distribution function.

$$\boldsymbol{P}(T)=\begin{Vmatrix} 0 & P_{12} & P_{13} & 0 & P_{15} & P_{16} & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_{\mathrm{em}} & 0 & 0 & 1-d_{\mathrm{em}} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_{\mathrm{em}}^{*} & 0 & 0 & 1-d_{\mathrm{em}}^{*} \\ d_{\mathrm{cd}} & 0 & 0 & 1-d_{\mathrm{cd}} & 0 & 0 & 0 \\ 0 & 0 & \mathrm{e}^{-\lambda_{\mathrm{m}}T} & 0 & 1-\mathrm{e}^{-\lambda_{\mathrm{m}}T} & 0 & 0 \end{Vmatrix} \tag{4}$$

After substituting matrix (4) into Eq. (3), we obtain the system of equations

$$\left.\begin{array}{l} p_i(T)=p_2(T)+p_4(T)+d_{cd}\times p_6(T) \\ p_2(T)=a_1\times p(T) \\ p_3(T)=a_2\times p_1(T)+a_3\times p_7(T) \\ p_4(T)=d_{\mathrm{em}}\times p_3(T)+d_{\mathrm{em}}^{*}\times p_5(T)+\left(1\text{-}d_{cd}\right)\times p_6(T) \\ p_5(T)=a_4\times p_1(T)+a_5\times p_7(T) \\ p_6(T)=a_6\times p_1(T) \\ p_7(T)=\left(1-d_{\mathrm{em}}\right)\times p_3(T)+\left(1-d_{\mathrm{em}}^{*}\right)\times p_5(T) \\ \sum\limits_{i=1}^{7} p_i(T)=1 \end{array}\right\} \tag{5}$$

In the system of Eq. (5) $a_1=\left[1-F(T)\right]\times\mathrm{e}^{-\lambda T}$, $a_2=(1-\rho)\times\int_0^T \mathrm{e}^{-\lambda t}\times\mathrm{d}F(t)$, $a_3=\mathrm{e}^{-\lambda_{\mathrm{m}}T}$, $a_4=\rho\times\int_0^T \mathrm{e}^{-\lambda t}\times\mathrm{d}F(t)$, $a_5=1-\mathrm{e}^{-\lambda_{\mathrm{m}}T}$, $a_6=\lambda\times\int_o^T \mathrm{e}^{-\lambda t}\times\left[1-F(t)\right]\times\mathrm{d}t$.

The solution of the system of Eq. (5) will be

$$\begin{cases} \pi_1(T)=\dfrac{M(t)}{C(t)} \\ \pi_2(T)=a_1\times\dfrac{M(t)}{C(t)} \\ \pi_3(T)=a_2\times\dfrac{M(t)}{C(t)}+a_3\times\dfrac{a_2\left(1-d_{\mathrm{em}}\right)+a_4\left(1-d_{\mathrm{em}}^{*}\right)}{C(t)} \\ \pi_4(T)=\dfrac{M(t)\times\left[d_{\mathrm{em}}a_2+d_{\mathrm{em}}^{*}a_4+(1-d_{\mathrm{cd}}^{*})\right]\times a_6}{C(t)}+\dfrac{\left[d_{\mathrm{em}}a_3+d_{\mathrm{em}}^{*}a_5\right]\times\left[a_2\left(1-d_{\mathrm{em}}\right)+a_4\left(1-d_{\mathrm{em}}^{*}\right)\right]}{C(t)} \\ \pi_5(T)=a_4\times\dfrac{M(t)}{C(t)}+a_5\times\dfrac{a_2-d_{\mathrm{em}}a_2+a_4-d_{\mathrm{em}}^{*}a_4}{C(t)} \\ \pi_6(T)=a_6\times\dfrac{M(t)}{C(t)} \\ \pi_7(T)=\dfrac{a_2\left(1-d_{\mathrm{em}}\right)+a_4\left(1-d_{\mathrm{em}}^{*}\right)}{C(t)} \end{cases} \tag{6}$$

In the system of Eq. (6)

$$M(t)=1-\left(1-d_{\mathrm{em}}\right)\times a_3-\left(1-d_{\mathrm{em}}^{*}\right)\times a_5$$

$$C(t)=\left(1+a_1+a_2+a_4+2a_6+d_{\mathrm{em}}\times a_2+d^*_{\mathrm{em}}\times a_4-d_{\mathrm{cd}}\times a_6\right)\times$$
$$\times\left[1-a_3\times\left(1-d_{\mathrm{em}}\right)-a_5\times\left(1-d^*_{\mathrm{em}}\right)\right]+\left[a_1\times\left(1+d_{\mathrm{em}}\right)+a_5\times\left(1-d^*_{\mathrm{em}}\right)+1\right]\times$$
$$\times\left[a_2\times\left(1-d_{\mathrm{em}}\right)+a_4\times\left(1-d^*_{\mathrm{em}}\right)\right]$$

For the initial conditions (Tab. 1) for the airfield power source, the frequency vector of the Markov chain falls into the state i, where i = 1, 2, 3, 4, 5, 6, 7, for the Diffusion-monotonic distribution law is:
$p_1(T) = 0.47656$, $p_2(T) = 0.41257$, $p_3(T) = 0.00975$, $p_4(T) = 0.03245$, $p_5(T) = 0.02025$, $p_6(T) = 0.04506$, $p_7(T) = 0.01106$.

It can be verified that the sum of the components of the vector $\vec{\pi}(T)$ is equal to unity with sufficiently high accuracy. This indicates the correctness of the calculations of the system of Eq. (5) by the numerical method.

To determine K_{tu}, it is necessary to know the vector $\eta(t)$ of average durations of the semi-Markov process in the state i, where i = 1, 2, 3, 4, 5, 6, 7.

In the first state, the semi-Markov process is on average time $\eta_1(t)$, which is equal to

$$\eta_1(t)=\left[1-F(T)\right]\times \mathrm{e}^{-\lambda t}\times T+(1-\rho)\times\int_0^T \mathrm{e}^{-\lambda t}\mathrm{d}F(t)\times T+\rho\times\int_0^T \mathrm{e}^{-\lambda t}\mathrm{d}F(t)\times\int_0^T t\times \mathrm{d}F_{15}(t)+$$
$$+\lambda\times\int_0^T \mathrm{e}^{-\lambda t}\times\left[1-F(t)\right]\mathrm{d}t\times\int_0^T t\times \mathrm{d}F_{16}(t)$$

where

$$F_{16}(t)=\frac{\int_0^t \mathrm{e}^{-\lambda x}\left[1-F(x)\right]\mathrm{d}x}{\int_0^T \mathrm{e}^{-\lambda x}\left[1-F(x)\right]\mathrm{d}x}\qquad\qquad F_{15}(t)=\frac{\int_0^t \mathrm{e}^{-\lambda x}\mathrm{d}F(x)}{\int_0^T \mathrm{e}^{-\lambda x}\mathrm{d}F(x)}$$

It can be shown that for the initial conditions of the Tab. 1 for the airfield electric power supply unit $\eta_1(t) = 94.7$ h, $\eta_2(t) = 6$ h, $\eta_3(t) = 2$ h, $\eta_4(t) = 10$ h, $\eta_5(t) = 0.5$ h, $\eta_6(t) = 0.6$ h, $\eta_7(t) = 63.2$ h.

For the rest of the studied weapon systems samples, the vector components $\eta(t)$ are:
η_{GMACU} = (93.7, 5, 1.8, 8, 0.5, 0.5, 63.2),
η_{UGCS} = (95.2, 3, 1.4, 4, 0.5, 0.5, 63.2),
η_{AFT} = (93.4, 5, 1.8, 8, 0.5, 0.5, 63.2).

$\omega_i(T)$ represents the average residence time of airfield equipment in good condition. This time is the mathematical expectation of a minimum of two random variables, namely $\omega_i(T) = M\,[\min(\tau, \tau_n)]$. If we assume that false alarms are distributed according to an exponential law, that is, $\Delta(t) = 1 - \mathrm{e}^{-\lambda t}$ where λ is the intensity of false alarms, then

$$\omega(T)=M\left[\min\left(\tau,\tau_n\right)\right]=\int_0^T\left[1-F(t)\right]\times\left[1-\lambda(t)\right]\mathrm{d}t=\int_0^T\left[1-F(t)\right]\times \mathrm{e}^{-\lambda t}\mathrm{d}t=\ 94.5\text{ h}$$

To calculate the technical and economic efficiency of the operation of the airfield technical support means, we assume that the average cost of remaining the product in a condition h_i and exit from the state is equal to [11]

$$C_{\,i}(T)=C_{ii}(T)\,\eta_i(T)+\sum_{j=1}^{7}P_{ij}(T)C_{ij}(T) \tag{7}$$

where $C_{ii}(T)$ – the cost of keeping the product in a condition of the process of technical operation, UAH/h. (UAH (hryvnia) – the monetary unit of Ukraine). To calculate $C_i(T)$, it is necessary to know the transition probability matrix $P_{ij}(T)$.

For the initial conditions of Tab. 1, the matrices $P_{ij}(T)$ for the airfield electric power supply at $T = 100$ h have the following form:

$$P_{ij\,\mathrm{AEPSU}}(100) = \begin{Vmatrix} 0 & 0.8657 & 0.0119 & 0 & 0.0278 & 0.0945 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & 0 & 0 & 0.2 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.55 & 0 & 0 & 0.45 \\ 0.7 & 0 & 0 & 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0.3679 & 0 & 0.6321 & 0 & 0 \end{Vmatrix}$$

For other ATM, the dependences are similar.

The stationary probabilities of the embedded Markov chain are:

$P_{i\mathrm{AEPSU}} = (0.4766, 0.4126, 0.0097, 0.0325, 0.0202, 0.0451, 0.011)$,

$P_{i\mathrm{GMACU}} = (0.4502, 0.359, 0.0254, 0.0619, 0.0528, 0.0419, 0.0288)$,

$P_{i\mathrm{UGCS}} = (0.49296, 0.44605, 2\times10^{-7}, 0.014073, 4.1\times10^{-7}, 0.04691, 2.3\times10^{-7})$,

$P_{i\mathrm{AFT}} = (0.4429, 0.3441, 0.0298, 0.0701, 0.0618, 0.041, 0.0338)$.

To determine the average cost of staying in a state, it is necessary to know the cost of the sample. The cost (B) is determined by the formula [12]:

$$B = P_{\mathrm{acq}} \times K_{\mathrm{ind}}$$

where P_{acq} – initial cost (acquisition price) of weapon systems, K_{ind} – indexation coefficient, which was taken equal to 3.4277.

$B_{\mathrm{AEPSU}} = 193\,665$ UAH,

$B_{\mathrm{GMACU}} = 3\,80817$ UAH,

$B_{\mathrm{UGCS}} = 419\,893$ UAH,

$B_{\mathrm{AFT}} = 7\,3012$ UAH.

As mentioned earlier, $C_{ii}(T)$ has a dimension – UAH/h. Thus, for WS $C_{ii}(T)$ is:

$$C_{ii\mathrm{AEPSU}} = \frac{193\,665}{8\,760\times30} = 0.7369292 \qquad C_{ii\mathrm{GMACU}} = \frac{380\,817}{8\,760\times30} = 1.4490753$$

$$C_{ii\mathrm{UGCS}} = \frac{419\,893}{8\,760\times30} = 1.5977664 \qquad C_{ii\mathrm{AFT}} = \frac{73\,012}{8\,760\times30} = 0.2778234$$

Unit costs per unit of calendar time of WS are

$$C_{uc} = \frac{\sum_{i=1}^{7} C_i(T)\times\pi_i(T)}{\sum_{i=1}^{7} \pi_i(T)\times\eta_i(T)} \tag{8}$$

where $\pi_i(T)$– the average frequency of a Markov chain falling into a state h_i, $\eta_i(T)$ – the average length of staying of the product in any condition h_i, $C_i(T)$ – the average cost of staying of the product in the state h_i.

The costs of the technical operation of airfield support equipment for flights are of the following form:

$$C_{\mathrm{AEPSU}}=\begin{pmatrix} 0.7369 & 0 & 0 & 0 & 0 & 0 & 0 \\ 197920 & 0.7369 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7369 & 197920 & 0 & 0 & 197920 \\ 65973 & 0 & 0 & 0.7369 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6597.3 & 0.7369 & 0 & 6597.3 \\ 6597.3 & 0 & 0 & 6597.3 & 0 & 0.7369 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.7369 \end{pmatrix}$$

For other ATM, the dependences are similar.

In the given matrices, the dimension of the elements located on the main diagonal is UAH/h, and the remaining elements have a dimension in hryvnias. In the first matrix, the element C_{21} = 19 792 UAH – is the cost of a scheduled maintenance, and C_{41} = 65 973 UAH – is the cost of emergency repairs, which is significantly more than the cost of scheduled repairs. Elements $C_{54} = C_{57} = C_{61} = C_{64}$ = 6 597.3 UAH – are the cost of the control operations of the power source.

The unit costs per unit of stay of WS in good condition are determined by the formula:

$$C_{1uc}=\frac{C_{uc}(T)\times\eta_{\mathrm{midl}}(T)}{P_1(T)\times\omega_1(T)}$$

where $\eta_{\mathrm{midl}}(T)=\sum_{i=0}^{7}\pi_i(T)\times\eta_i(T)=$ 48.211 h

$$\omega_1(T)=M\left[\min(\tau,\tau_n)\right]=\int_0^T\left[1-F(t)\right]\times\left[1-\pi(t)\right]\mathrm{d}t=\int_0^T\left[1-F(t)\right]\times\mathrm{e}^{-\lambda t}\mathrm{d}t=\text{ 94.7 h}$$

We calculated the costs of remaining OC and exit from the state as follows (7)

$$\begin{cases} C_1=C_{11}\eta_1+P_{12}C_{12}+P_{13}C_{13}+P_{15}C_{15}+P_{16}C_{16} \\ C_2=C_{22}\eta_2+P_{21}C_{21} \\ C_3=C_{33}\eta_3+P_{34}C_{34}+P_{37}C_{37} \\ C_4=C_{44}\eta_4+P_{41}C_{41} \\ C_5=C_{55}\eta_5+P_{54}C_{54}+P_{57}C_{57} \\ C_6=C_{66}\eta_6+P_{61}C_{61}+P_{64}C_{64} \\ C_7=C_{77}\eta_7+P_{73}C_{73}+P_{75}C_{75} \end{cases} \tag{9}$$

The cost vector for the states of the airfield technical maintenance (ATM) model will look as follows:

C_{AEPSU} = (70, 19 796, 19 793, 65 980, 6 598, 6 598, 47),
C_{GMACU} = (136, 21 324, 21 320, 71 069, 7 106, 7 106, 92),
C_{UGCS} = (152, 8 505, 8 502, 28 339, 2 834, 2 834, 101),
C_{AFT} = (26, 17 251, 17 251, 57 502, 5 750, 5 750, 18).

Next, the unit costs per unit of a calendar time of the ATM in good condition will be calculated according to Eq. (9).

For illustrative purposes, these characteristics are shown in Tab. 2.

According to Eqs (1) and (9), the total availability factor and unit costs for maintaining WS in working condition have been calculated.

The calculation results are shown in Figs 2-4, where the values K_{tu} are shown on the ordinate axis on the left, and the specific costs per unit time of the stay of WS in good condition are on the right. The values of the regular maintenance frequency are shown on the abscissa axis in Fig. 1, the reliability of monitoring the technical condition of ATM means by external means of control – in Fig. 2, the recovery time – in Fig. 3.

Tab. 2 Cost characteristics and average lengths of stay of the semi-Markov process in model states

Nº	Mark WS / Cost characteristics and average lengths of stay of the semi-Markov process in model states.		airport electric power supply unit	ground multipurpose air-conditioning unit	unified gas charging station	airfield fueling tanker
1	Costs of stay and exit from the state C, UAH	C_1	70	136	152	26
2		C_2	19 796	21 324	8 505	17 251
3		C_3	19 793	21 320	8 502	17 251
4		C_4	65 980	71 069	28 339	57 502
5		C_5	6 598	7 106	2 834	5 750
6		C_6	6 598	7 106	2 834	5 750
7		C_7	47	92	101	18
8	Average lengths of stay in semi-Markov process states η_{midl}, h	$\eta_{\text{midl}1}$	94.7	93.7	95.2	93.4
9		$\eta_{\text{midl}2}$	6	5	3	5
10		$\eta_{\text{midl}3}$	2	1.8	1.4	1.8
11		$\eta_{\text{midl}4}$	10	8	4	8
12		$\eta_{\text{midl}5}$	0.5	0.5	0.5	0.5
13		$\eta_{\text{midl}6}$	0.5	0.5	0.5	0.5
14		$\eta_{\text{midl}7}$	63.2	63.2	63.2	63.2
15	Unit costs per unit of calendar time, UAH		225	287	91	242
16	Unit costs per unit of time in good condition C_{1uc}, UAH/h		243	318	94	270

Fig. 2 shows that for all ATM facilities there is an optimal period of maintenance work at which the maximum value is ensured. At the same time, a regularity is observed: the larger the scale parameter (for UGCS – $\mu = 759$ hours, for AEPSU – $\mu = 230$ hours, for GMACU – $\mu = 179$ hours, for AFT – $\mu = 170$ hours), the more necessary the maintenance work is. Thus, for example, for UGCS – $T_{\text{opt}} \approx 240$ hours, for AEPSU – $T_{\text{opt}} \approx 90$ hours, for GMACU – $T_{\text{opt}} \approx 72$ hours, for AFT – $T_{\text{opt}} \approx 70$ hours.

Meanwhile, it was established that regular maintenance with optimal frequency will ensure minimum economic costs. Thus, for example, carrying out regular maintenance on a power source after about 82 hours, one hour of operation of the unit will cost about 220 UAH. For an airfield air conditioner, regular maintenance after 72 hours, one hour of operation in good condition will cost approximately 310 UAH.

For a gas charging station, regular maintenance after 240 hours of operation, one hour of operation will cost about 20 UAH.

The obtained data make it possible to establish the economic costs for a certain time of operation of the airfield technical support means for flights.

Fig. 3 shows that an increase in the reliability of external control has almost no effect on the level of technical readiness for a gas charging station. Economic costs for all ATM are weakly dependent on increasing the reliability of control.

Since the recovery time t_{rec} is a special case of η_1 in Eq. (1), in Fig. 4 it can be seen that an increase in the recovery time t_{rec} leads to a decrease of K_{tu}, the low level of reliability of the products, and the unit cost per unit time of the products in good condition practically does not depend on time from the initial conditions of Tab. 1.

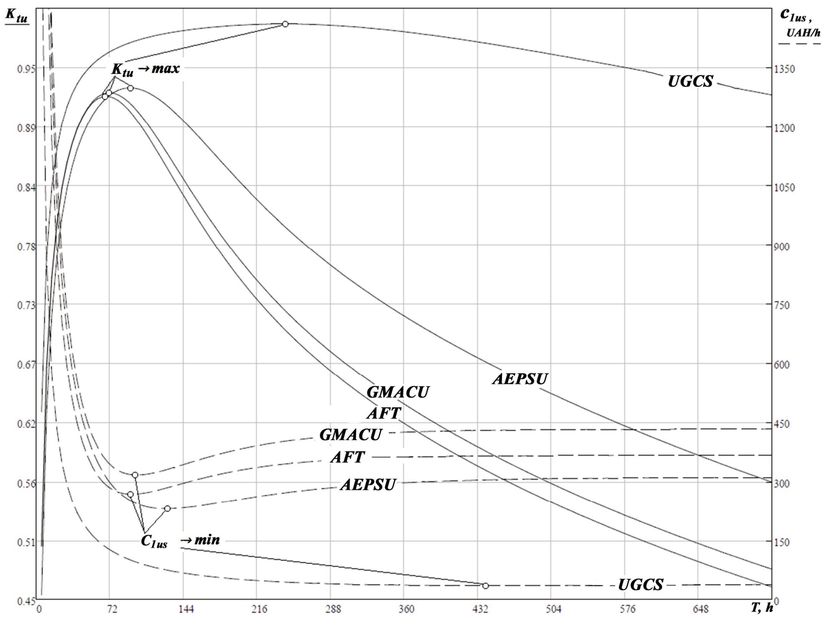

Fig. 2 Dependence K_{tu} and C_{1uc} on the frequency of regular maintenance T

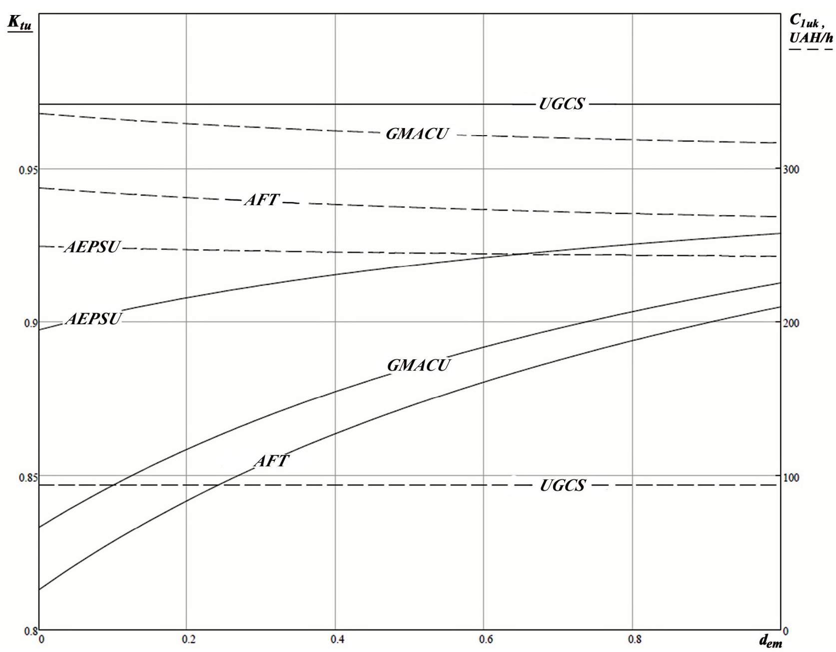

Fig. 3 Dependence K_{tu} and C_{1uc} on the reliability of external controls d_{em}

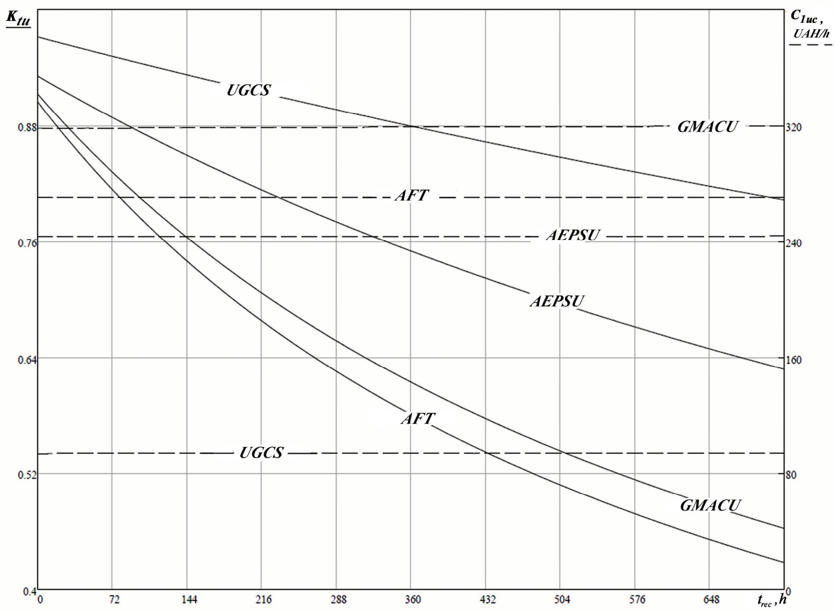

Fig. 4 Dependence K_{tu} and C_{1uc} on the duration of recovery t_{rec}

4 Conclusions

The scientific novelty of the article with a focus on a technical condition-based maintenance during the use of semi-Markov random process and diffusion-monotonic law of time distribution to failure is determined by the attempt to define the analytical dependence of maintenance coefficient on the coefficient of scale and form of diffusion-monotonic distribution law, frequency and duration of regular works, probability of information on ATM failures, control reliability and duration of preventive service and repair. The analytical dependence of specific costs per time unit of ATM good state on these parameters has been defined.

It enables the maintainers to define the level of maintenance coefficient and specific costs for real conditions of ATM facilities maintenance. The users (leaders and engineers) can use the developed correlations for similar calculations for other input values to be received.

Due to the developed models being suggested as a software product, it is possible to define the limit values of this parameter as the average duration of regular works on ATM facilities, which correspond to provided values of maintenance coefficients.

While planning the logistic support of air military unit(s) the developed models can be used by the key logisticians of operational and strategic levels for support of the defined ATM effectiveness level.

Moreover, if these facilities' service time is known, it is possible to calculate the total cost of their service.

For example, if it is specified that the power source is operated for 10 hours during a 24-hour period, the operational costs of the mentioned facility will be UAH

2 870. In the case of regular works, the coefficient of technical use $K_{tu} = 0.93$ will be ensured within 80 hours.

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