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Improvement of Barrel Weapon Internal Ballistic Model Using Dynamic Vivacity Function

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Abstract:

The paper discusses a possibility of application of dynamic vivacity function in interior ballistic process mathematical model. At present time, various technologies of powder grain surfacing are applied, mainly in case of powder grains used in assemblies of small-calibre cartridges. The grain surfacing can desirably influence the interior ballistic action. It can further change basic powder thermodynamic properties, and particularly, it can change the character of the powder grain burning, which cannot be described by geometric burning concept. According to the established standards, a necessity of the dynamic vivacity function L evaluation is determined for artillery powders only, but the way of its evaluation can be applied for arbitrary powder grains. The application of the dynamic vivacity function then allows to get results of solution of the interior ballistic tasks with higher accuracy, as well as the selection of suitable grain surfacing technology.

Keywords:

interior ballistics, dynamic vivacity, geometric burning, powder grain, surfaced powder grain, shot process

1 Introduction

Interior ballistics of barrel weapons solves a whole range of theoretical and practical tasks which serve for interior ballistic system primary design, which means for the evaluation of various experiments and proper firings. The results of these tasks solution allow the determination of the required propulsive powder charge properties and provide basis for the evaluation of the weapon systems properties and technical state.

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One of the benefits of the essential internal ballistic theoretical task solution is the design of the propulsive powder charge mass and properties for an achievement of the required weapon parameters. Primarily the required muzzle velocity of the projectile is achieved with given mass under condition that the maximum pressure in the barrel is lower than the determined maximum permissible pressure. The result of the solution lies in the determination of the basic interior ballistic parameters courses in time, primarily the time courses of the powder gases pressure and projectile velocity in the barrel or the projectile trajectory in barrel.

The theoretical solution of the interior ballistic process uses various mathematic models based usually on the common assumptions of the geometrical concept of the powder charge burning and powder linear burning law. Geometrical concept of the powder charge burning as the one of grain burning is included into the classical thermodynamic models in the form of the powder gases development equation which is introduced according to [1] in the following form:

$$Z = \kappa f \left(1 + \lambda f + \mu f^2\right) \tag{1}$$

where Z is the powder relative burnt mass, f is the powder grain relative burnt web thickness, κ; λ and μ are the powder grain shape characteristics. The linear burning law is applied by the equation [1]:

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{P}{I_\mathrm{t}} \tag{2}$$

where p is the instant pressure in the barrel bore, I_t is the total gas pressure impulse usually determined in a closed vessel and t is the time.

Producers' assertions about the powder grains average dimensions and properties seem strictly speaking incorrect as it can be seen in Fig. 1.

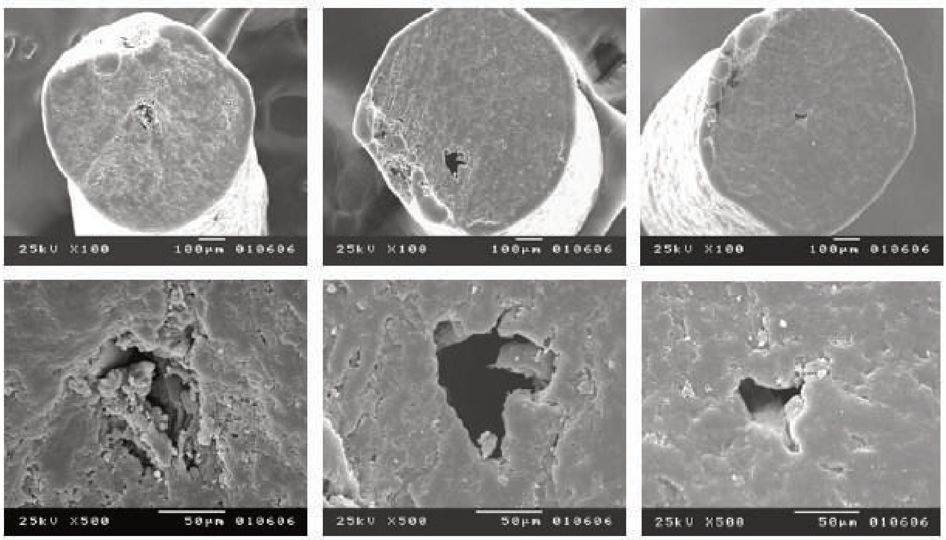

Fig. 1 Real possible shapes of tubular powder grain [2]

Fig. 1 shows the following: the missing grain channel (on the left), the channel located out of the grain cross-section centre (in the middle), and the partly not passable grain channel (on the right). In details, the channel's imperfections are shown down the figure. These powder grain imperfections (differences from the geometric concept) lead to big differences between calculated and measured fire courses.

Both the geometrical burning concept of the powder grains and the linear burning law of the powder are based only on theoretical assumptions. Both can be replaced by other dependences to approach the results of the interior ballistic model solutions to the experimental results. The first possible dependence is a Charbonnier's burning function $\sigma(Z)$ expressed as a ratio of the grain instantaneous and initial burning surfaces evaluated from measured pressure course [3], which can substitute Eq. (1).

The second possible dependence is derived from the emission function Γ [1]. According to [4-6], the function Γ is defined by the same relation as the dynamic vivacity function L:

$$L = \Gamma = \frac{1}{P}\frac{\mathrm{d}Z}{\mathrm{d}t} \tag{3}$$

In the paper, the possibility of the interior ballistic process mathematical model solution with the higher accuracy is expressed by application of the dynamic vivacity function L evaluated from experiments. Such approach has not been published yet. The clause in the standards [4] and [5] about the necessity of this characteristic evaluation is valid for artillery powders only, but it can be applied for all powders, and consequently it can be used for a specification of the interior ballistic process model as a necessary condition, primarily for the calculation using the surfaced powder grains. The powder grains surfacing leads not only to changes of the basic powder thermodynamic characteristics determined by the measurement in the closed vessel, but primarily to the change of the powder grain burning behaviour. In this case, the unit-burning rate will not be a constant value anymore.

2 Determination of Dynamic Vivacity Function

According to standards [4] or [5], the dynamic vivacity function L is determined using the gas pressure values acquired during the measurement of the given powder sample burning in the closed vessel. For the determination of function L, the single base nitrocellulose tubular powder grain marked S060-01 has been chosen with surfacing by 1.96 % Centralite I. The experiments in the closed vessel with the volume of $c_0 = 5.9\ \mathrm{cm}^3$ were carried out in the testing laboratory of Explosia, Plc. Pardubice. For the pressure measurement, the piezoelectric sensor KISTLER 6215 was used and the gas pressure course was analysed by the ballistic analyser BA04 produced by R.M.I. Similar conditions of the tests and the process of their evaluation are introduced in [2]. The measured and smoothed time course of pressure at the loading density $\Delta = 200\ \mathrm{kg\,m^{-3}}$ is introduced in Fig. 2. The maximum pressure value is $P_\mathrm{m} = 266.7$ MPa.

The dynamic vivacity is defined in [4-7] by relation:

$$L = \frac{\mathrm{d}P}{\mathrm{d}t}\frac{1}{P \cdot P_\mathrm{m}} \tag{4}$$

where the pressure values P are evaluated from the measured record for corresponding time instants t. Pressure derivation with respect to time (see Fig. 3) is evaluated according to [4, 5] from the measured and smoothed time course of the pressure by mathematical analysis (for example by polynomic regression method). Pressure values from the measured record are not adjusted to a heat loss. Using the Eq. (4), the relation $L = L(P)$ is obtained.

However, the function L is better expressed for the application in the mathematical model solution of the interior ballistic process as the function of the burnt powder mass fraction Z.

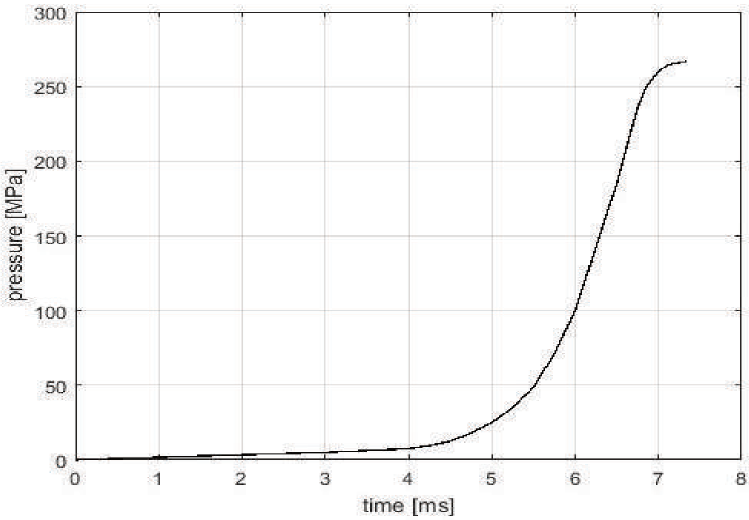

Fig. 2 Gas pressure-time course measured in closed vessel at the loading density Δ = 200 kg m^{-3}

The expression of this function is derived from the equation of state. Under the assumption that the igniter burns out instantly, the equation of state for the constant volume (closed vessel volume) is as follows:

$$P\left[V_0 - \frac{C}{\rho}(1-Z) - bCZ - b_\mathrm{I}C_\mathrm{I}\right] = FCZ + F_\mathrm{I}C_\mathrm{I} \tag{5}$$

where C – the powder charge mass, ρ – the powder density, F – the powder specific energy, b – the powder gas covolume. The quantities with index I mean the igniter parameters.

The pressure in the closed vessel caused by the igniter burnup is determined when substituting $Z = 0$ in Eq. (4):

$$P_I = \frac{\dfrac{F_\mathrm{I}C_\mathrm{I}}{C}}{\dfrac{1}{\Delta} - \dfrac{1}{\rho} - b_\mathrm{I}\dfrac{C_\mathrm{I}}{C}} \tag{6}$$

Furthermore, after substituting a cartridge density $\Delta = C/V_0$into Eq. (5) we can express the powder relative burnt mass Z [1]:

$$Z = \frac{\dfrac{1}{\Delta} - \dfrac{1}{\rho} - b_\mathrm{I}\dfrac{C_\mathrm{I}}{C} - \dfrac{1}{P}\dfrac{F_\mathrm{I}C_\mathrm{I}}{C}}{\dfrac{F}{P} + b - \dfrac{1}{\rho}} \tag{7}$$

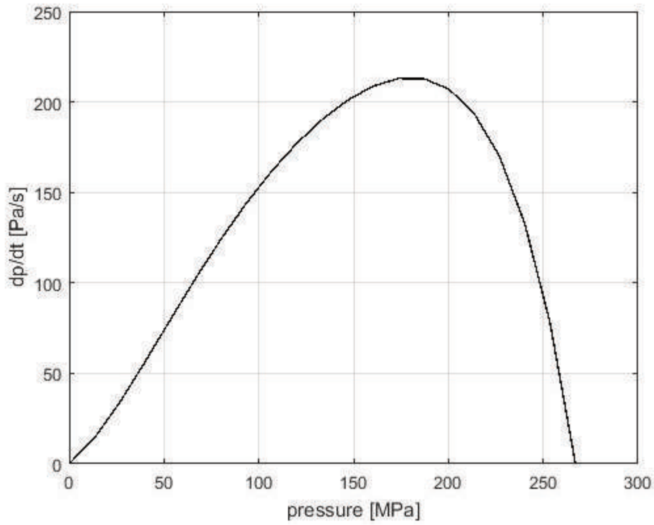

Fig. 3 Calculated course of the pressure derivation with respect to time in dependence on pressure

Substituting value $Z = 1$ into Eq. (5), we can express the maximum pressure value in the closed vessel:

$$P_m = \frac{F + \frac{F_I C_I}{C}}{\frac{1}{\Delta} - b - b_I \frac{C_I}{C}} \tag{8}$$

We exclude expression $F_I C_I / C$ from Eq. (5) and substitute it into Eqs (6) and (7). Further, we exclude quantity F from converted Eq. (8) and we substitute it into Eq. (7). After consequent modifications described for example in [8], we get an equation in the form:

$$Z = \frac{f(P)}{k + f(P)(1-k)} \tag{9}$$

where we have introduced the function of pressure [3]

$$f(P) = \frac{P - P_I}{P_m - P_I}$$

and constant [8]

$$k = \frac{\frac{1}{\Delta} - b - b_I \frac{C_I}{C}}{\frac{1}{\Delta} - \frac{1}{\rho} - b_I \frac{C_I}{C}}$$

If the igniter effect is omitted, we get a simpler form of the relative burnt mass Z expression. If we express the pressure in the closed vessel from the equation of state, we will get the dependence [8]:

$$P = \frac{F\Delta Z}{1 - \frac{\Delta}{\rho}(1 - Z) - b\Delta Z}$$

After we express value Z from that equation, we can use the substitutions:

$$k_1 = \frac{1}{F}\left(\frac{1}{\Delta} - \frac{1}{\rho}\right) \text{ and } k_2 = b - \frac{1}{\rho}$$

The relative burnt mass is then:

$$Z = \frac{k_1 P}{1 + k_2 P} \tag{10}$$

This process gives values Z corresponding to the measured values of pressure. Thus, we can replace function $L = L(P)$ by function $L = L(Z)$. Its course for given powder grain is illustrated in Fig. 4. Further, we can express the derivation from Eq. (3) to the form [1]:

$$\frac{\mathrm{d}Z}{\mathrm{d}t} = \frac{\mathrm{d}P}{\mathrm{d}t} \cdot \frac{\mathrm{d}Z}{\mathrm{d}P}$$

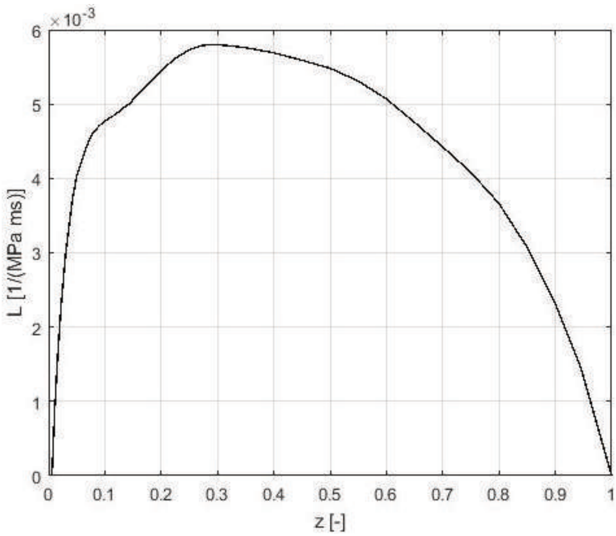

Fig. 4 Calculated course of function L in dependence on relative burst mass

In the equation above, we know the derivative of pressure to time (see Fig. 3) and we can derive an equation for the expression of the derivative of relative burnt mass to pressure by the derivative of equation of state to pressure without the igniter consideration [3]. We get the form:

$$\frac{\mathrm{d}Z}{\mathrm{d}P} = \frac{1 + \left(b - \frac{1}{\rho}\right)\frac{P_\mathrm{m}}{F}}{P_\mathrm{m}\left[1 + \left(b - \frac{1}{\rho}\right)\frac{P}{F}\right]^2}$$

Eq. (3) can be modified in the form:

$$L = \frac{\mathrm{d}P}{\mathrm{d}Z}\frac{\mathrm{d}Z}{\mathrm{d}t}\frac{1}{P \cdot P_{\mathrm{m}}}$$

from which we get the equation:

$$\frac{\mathrm{d}Z}{\mathrm{d}t} = L \cdot P \cdot P_{\mathrm{m}} \frac{\mathrm{d}Z}{\mathrm{d}P} = L(Z)P \tag{11}$$

in which function $L(Z)$ is found as the best approximation of the evaluated data from experiments shown in Fig. 4. Under the assumption that the evaluated dependence of function $L = L(Z)$ from the measured pressure course in closed vessel is known for given powder, Eq. (11) can be used in the interior ballistic process solution of other ballistic systems.

However, the measurement in the closed vessel also serves for the determination of the powder specific energy F, the powder gases covolume b and the total powder gases pressure impulse I_{t}. Evaluating the measurements of the given powder at other charging densities, the average value of the total pressure impulse has been determined: $I_{\mathrm{t}} = 0.3642$ MPa s. Further, using Noble-Abel equation [1, 8], the average values of the specific energy $\mathrm{F} = 1.021\ \mathrm{MJ\,kg^{-1}}$ and powder gas covolume $b = 0.00117\ \mathrm{m^3\,kg^{-1}}$ have been determined.

3 Geometric burning replacement by real one

Eqs (1) and (2) are the basic equations describing the geometric rules of powder burning with the use of the linear powder burning law. During the solution of the standard thermodynamic mathematic model of the interior ballistic process, these equations can be replaced by Eq. (11) only, which expresses the process of the real powder grain burning. The other equations derived in [3] then remain without changes.

For the comparison of results of both internal ballistic process mathematical models' solutions, the pressure measurement has been carried out formerly on the ballistic measuring instrument of calibre .308 Winchester whose structural parameters and cartridge characteristics are introduced in Tab. 1.

Five shots were fired during the experiment and measured pressure courses were evaluated using the ballistic analyser. The average pressure course is drawn by the dashed line in Fig. 5. The measured maximum pressure P_{m} was recalculated to ballistic pressure. The evaluated results are introduced in Tab. 2. The measured pressure course is arranged in such a way that the starting point of the projectile movement corresponds to the initial pressure $P_0 = 50$ MPa.

The standard thermodynamic model of the interior ballistics was built in MATLAB and it includes Eqs (1) and (2). Its solution is based on the values evaluated from the measurement of phlegmatized powder charge in the closed vessel, where especially the powder specific energy F is markedly lower than in case of powder without phlegmatization. The obtained time-dependent theoretical pressure course is drawn by the dot-and-dash code in Fig. 5.

Further, in the converted mathematic model, Eqs (1) and (2) were replaced by Eq. (11) and the function $L(Z)$ was expressed by the 10-degree polynomial function. The calculated gas pressure course with respect to introduced conversion is drawn by the solid line in Fig. 5.

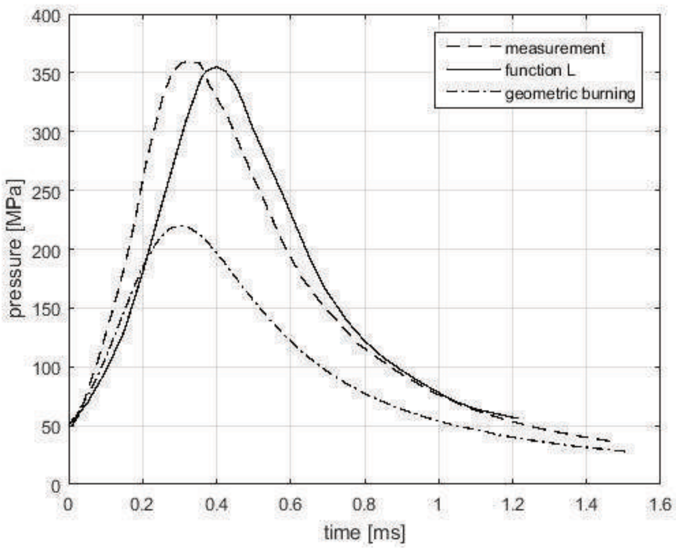

Fig. 5 Comparison of the measured and calculated gas pressure courses of the cartridge .308 Winchester

The comparison of the measured and calculated values of the maximum pressures P_m, the muzzle velocities v_m and the time intervals t_m of the projectile movement inside the barrel are shown in Tab. 2.

Tab. 1 Ballistic measuring instrument and .308 Winchester cartridge parameters

Calibre d [mm]	7.62	Covolume b [m^{3}kg^{-1}]	1.17×10^{-3}
Initial combustion volume V_0 [m^3]	3.25×10^{-6}	Powder density ρ [kg m^{-3}]	1 598
Projectile total trajectory l_m [m]	0.6	Temperature of explosion T_v [K]	3 010
Bore cross section s [m^2]	4.75×10^{-5}	Initial pressure P_0 [MPa]	50
Projectile mass m_q [kg]	0.00955	Resistance coefficient k_φ [–]	1.1
Powder charge mass C [kg]	0.0028		
Heat of explosion Q_v [MJ kg^{-1}]	3.758	Average grains dimensions:	
Powder specific energy F [MJ kg^{-1}]	1.021	Length $2l$ [mm]	1.2
Total pressure impulse I_t [MPa s]	0.3642	Outer diameter $2e_1$ [mm]	0.825

Tab. 2 Measured and calculated data comparison

	P_m [MPa]	v_m [m s^{-1}]	t_m [ms]
Measurement	359	830	1.525
Calculation – geometric burning	220	620	1.5
Calculation with function $L(Z)$	355	848	1.218

It is obvious from the courses in Fig. 5 that in case of the interior ballistic model solution using surfaced powder grains, it is not possible to use the geometric concept of powder burning; in addition, it is not possible to assume the powder linear burning law. The powder surfacing does not influence the powder thermodynamic properties, but it primarily does influence the process of its burning in the grain individual layers. It is not possible in any way to calculate parameters using the constant unit burning rate.

Based on the comparison of the maximum pressures and muzzle velocities values obtained from measurement evaluation and calculation using function $L(Z)$, a relatively high correspondence of theoretical and experimental results can be observed. The time shift in the maximum pressures' attainment can be explained by the fact that in real case, the initial projectile movement in the barrel starts probably under pressure which is lower than the presupposed initial pressure 50 MPa. Moreover, the standard interior ballistic model assumes constant projectile resistance during its movement through the barrel bore while it is in real variable (first it increases, then it drops).

4 Conclusion

New technologies of the powder grains production and finishing process make the mathematical models based on the powder grain burning according to the geometric concept and using the linear burning law almost impossible for the interior ballistic process solution. Requirements of producers for the production cost reduction often led to substantial shape and dimension differences of real and theoretically assumed grain shapes. Various ways of the powder grains surfacing evoked by the tendency to influence the shot process also make it impossible to use the standard mathematical models.

To obtain the desired course of the shot, the powder producer makes number of tests for finding suitable properties of the powder grains (composition of powder, shape, and eventual surfacing of powder grains). Evaluated results of such tests and their documentation can further serve as a significant source for improvement of described mathematical model of interior ballistics.

From a database of experimental results, dependences of the powder grain thermodynamic parameters on chosen surfacing technology as well as on the content of a surfacing substance in the grain surface layer can be found. Further, it could be possible to find proper fitting form of the function $L(Z)$ expression considering the chosen surfacing technology. The knowledge of these dependences will be valuable not only for the powders' producer, because it will enable to reduce the number of tests needed for the proper powder charge finding, but also for the number of theoretic tasks solutions in the interior ballistic domain.

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