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Target Programming with Multicriterial Restrictions Application to the Defense Budget Optimization

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Abstract:

The analysis of the main factors of decision-making on media planning, methods of automation the advertising targeting and existing models and algorithms discovered, the contradictions between the possibilities of traditional methods and classical models of advertising budget distribution. The requirements to automate decision-making support in managing the advertising process became important when organizing and conducting public Purchases for the needs of the Armed Forces of Ukraine. In order to increase the efficiency of defense resources management for rational advertising budget distribution between different types of advertising platforms, an advanced mathematical model for making decisions on the application of targeting informational technology to advertising has been developed.

Keywords:

method of leading priorities, target function, utility function, targeting informational technology, weighting factors

1. Introduction

At the current stage of information technologies (IT) implementation to all sectors of public life, the simulation of economic processes allows to predict the development of these processes and to ensure their adequate management. Taking this into account and based on the ERP system of the company SAP AG, the creation and implementation of the Unified System for the Management of Administrative and Economic Processes of the Armed Forces of Ukraine was started, in which defense resources management

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was established on a modern information and analytical system (IAS) with the functions of finding optimal solutions, forecasting modeling of the development of control objects, finding weak points of structures in order to eliminate them, identifying patterns, etc. [1-4]. At the same time, making decisions about choosing a strategy in case of goals and interests of competing companies antagonism remains a complicated process, for which uncertainty is a typical feature. This is the uncertainty of choice and achievement of goals in multi-criteria decision-making tasks, the uncertainty of the influence of uncontrolled factors, as well as information uncertainty of conflicts when countering competitors.

The task of finding a rational compromise allows to construct the Pareto set and to obtain additional information that enables the decision maker (DMP) to evaluate the impact of the change of one target function on the other. The importance of different target functions may depend on a number of objective and subjective factors. Additionally, choosing a goal priority may specify additional conditions or restrictions. The introduction of weight coefficients enables us to reduce the multi-purpose problem to ordinary one-critical tasks. According to this, the improvement of the mathematical model of a competitive game and the hierarchical distribution of IAS parameters with the help of ballpoint scoring tables should be considered in order to make a decision on the information technology targeting application, which makes it possible to narrow the search area of the optimal solution and ensure a rational allocation of the budget while managing the defense resources.

2. Formulation of Problem

Quantitative modeling was used to solve the budget distribution task, which is classified as heuristic modeling tasks, or the task of multi-criteria decision-making [1]. Some studies were conducted in terms of customer relationships, the effects of information placement, and the allocation of resources, using analytical hierarchical processes and targeted programming simultaneously in order to provide a more systematic solution in the real situation [5, 6]. Multi-criteria decision-making can simultaneously consider different aspects of the task of choosing information tools and scheduling information to get a satisfactory solution to the problem. In [5-8] it is argued that the mathematical constraints of the model for choice compel to create an artificial structuring of the media criteria [9]. In [10], a model was proposed for determining the amount of information that provides a significant improvement in the solution of this problem in comparison with the previous model of linear programming [11]. Criteria are considered which include the percentage estimation of the reach and frequency of occurrence on different information carriers, aimed at different segments of the market at different time periods, while taking into account the costs of targeting placement. The study [12] discussed the best possible combination of quantity and quality of information not only to describe the existing properties and features of the investigated means, but also from the point of view of the formation and achievement of the objectives of ODA based on its vision of the necessary properties and features of alternatives, as well as ways and means of implementing information technologies depending on budget constraints. Technical achievements and information capabilities of telecommunication networks and information computer technologies created fundamentally new conditions of entrepreneurial activity under the influence of competition. The paper [13] presents case studies that take into account two operations: investing in information technology targeting and the use of traditional media.

In order to solve the problem of strategic decision-making in the duopoly mode, a mixed target programming model was developed [14, 15].

But many features and disadvantages are due to the differences in the strategies of opposing companies, resulting in one side gaining more than the other, or losing out. Therefore, there are reasons to believe that insufficient account of the influence of factors of uncertainty, risk, and competition causes practical interest in further research and development of the mathematical apparatus for formalizing the tasks of uncovering and optimizing disclosure in the face of counteraction by the parties, for example, during the organization and conduct of public procurement for the needs of the Armed Forces of Ukraine, with the purpose of further use in the Unified system of management of administrative and economic processes of the Armed Forces of Ukraine.

The purpose of this study is to develop a scheme of equilibrium of the network for modeling and analyzing the behavior of competing firms, which, at the request of the Ministry of Defense of Ukraine, fulfil the tasks of providing the necessary information on public procurement procedures in mass media and the Internet through the placement of Internet advertising on many websites. The model should allow both the budget equilibrium for online advertising and the cost of advertising on different websites.

To achieve this goal, the following tasks were solved:

- to increase the value of the function of their own goals on the basis of rational choice,
- to minimize the risk of possible damage due to the uncertainty of the actions of competitors based on the choice of a rational strategy of action based on the pre-formed set of Pareto.

3. Description of Method and Basic Mathematical Equations

A scheme of the network equilibrium for modeling and analysis of the competing firms behavior was developed. It provides necessary information on public procurement procedures in mass media and the Internet through the placement of Internet advertising on many websites at the request of the Ministry of Defense of Ukraine. The strategic goals of companies are antagonistic. Parties deliberately misinform, both in terms of goals and certain parameters. Situations that arise depend on both the external environment and the strategy of the parties. The actions of the parties determine the need to change the parameters and goals in the development of the situation. The model allows to determine both the budget equilibrium for online advertising and the cost of advertising on various websites. To implement the model, the methods of targeted programming are used, which is based on the idea of organizing goals for importance. The main purpose of the methods is the transformation of the linear programming problem with several target functions into one task, or the sequence of tasks with one target function. With the help of weight coefficient methods, a single target function is formed as a weighted sum of initial partial target functions. Using the priority method, priorities are set for the importance of goals; the initial task is solved by sequentially solving linear programming problems, each of which contains only one target function in such a way that solving a problem with a lower priority does not spoil the optimal solution from higher priority. In this case, with the introduction of additional variables (deviation variables), a more general model is constructed, in which goals are included as a constraint.

The model was developed to determine the number of advertising on procurement procedures which should be placed on different media and the optimal allocation of the budget allocated to these media, is presented. The main task of this model is to maximize the availability of information placed on different media to the desired group of people (or companies, firms, etc.) within the maximum allowable budget without violating the maximum and minimum number of advertising goals. As carriers, various popular portals and a banner network were considered. The model has been formulated in such a way that the advertising reaches exactly the target group, and not the group that is not a potential consumer. The random-purpose target programming model was developed after the parameter corresponding to the reach of different carriers was considered as a random variable. Random values in this case were considered as values with known mathematical expectations and mean square deviation [16].

Since the parameters of the task constraints and the parameters of the target function are random variables and contain random components, the optimization problem is a probabilistic problem and is solved using stochastic programming methods. The probabilistic nature of planning tasks is often due to incomplete information about their conditions [17]. It happens, however, and for the exact solution of a complex deterministic problem, it requires too much computation, then it is expedient to reduce it to probabilistic, although all information is known. The volume of computations was thus significantly reduced. Figuratively speaking, the model is considered in this way: parts disappear, but the overall structure of the task becomes more perceptible [18].

Quite often, two-stage linear models of stochastic programming are used. ATS in the first stage performs certain actions (a certain optimal plan is established, the task is deterministic and the result is a vector with deterministic components) [19], after which there are random events that influence the outcome of the first stage decision. At the second stage, a corrective solution can be adopted that compensates for any undesirable consequences of the first stage decision in accordance with the actual conditions [20]. An optimal solution of such a model is a single decision of the first stage and a set of corrective decisions defining the action to be performed at the second stage in response to each random result.

The target programming model (GP) is one of the well-known models of multi-criteria mathematical programming. This model allows one to take into account simultaneously several criteria in the task of choosing the most acceptable alternative in the set of permissible decisions. More precisely, the GP is designed to find a solution that minimizes the deviation between the level of achievement of the criteria and the goals set for them. In case of exceeding, the criterion of deviation is considered positive (positive); in case of failure to achieve the goal, the criterion of the deviation is considered negative (negative). Developed in [5, 6] and used in [19, 21], the GP model has become popular and its use has spread to various areas, such as water tank management, solid management, accounting and financial aspects of fund management, marketing, quality control, human resources, production, transportation and location, space research, telecommunications, agriculture and forestry and aircraft.

The general form of GP model is as follows: minimize $\bar{a}=\{g_1(\bar{n},\bar{p}),g_2(\bar{n},\bar{p}),\dots,g_k(\bar{n},\bar{p})\}$, provided $f_i(\bar{x})+n_i-p_i=b_i,\bar{x},\bar{n},\bar{p}\geq 0$, where $g_k(\bar{n},\bar{p}),$ is a linear function of deviation variables. The dimension of $\bar{a}$ is the number of k levels of the preceding priority, b_i represents the level of desirability associated with the criterion $f_i(\bar{x})$. Variables n_i, p_i indicate respectively the negative and positive deviations of the $f_i(\bar{x})$ achievement level from the expected level. Target or expected levels intended for different criteria may be probabilistic, where the decision is finally unknown.

The problem of selecting the optimal amount of advertising on various Internet resources is considered. It is needed to find the amount of promotional messages you want to place on different sites within the budget limit, in order to maximize the expected reach of your target audience. Here as a platform were considered various Internet resources and various formats of advertising. The price aspects for various formats of advertisements were determined by monitoring various Internet sites and analyzing the statistical data obtained from the analytical services of automated systems for managing Internet advertising. The task was formulated as a task-oriented programming with multi-criteria random constraints. Advertising should reach those who are potential consumers of the product and it should not reach those who are not. The task is modeled as a task-oriented program with random limitations, since the reachability parameters for the target group are considered as random variables. It is assumed that random variables that correspond to the reach are values with known mathematical expectations and mean square deviation. The parameter that corresponds to the reach can be determined by finding the ideal solution and the law on which the parameter values change. The method of solving the problem of multi-criteria programming with random constraints will be given in this section.

First, criteria are selected for determining the target audience. Since the aim is to promote innovative technologies in the field of Internet services to ensure the government procurement of material resources for the needs of the Armed Forces of Ukraine, then the main criteria of consumers should be: monthly profit – more than 4000 USD, status – qualified specialist, office worker, top management, region – Kiev and region, age – 25-54, frequency – every day and place of use of the Internet – in the office and at home. The next step is to establish the relative importance of the selected criteria.

Monthly earnings criterion will have weight W_1^1; the weight of status criterion is W_2^1; region criterion will have weight W_3^1; the weight of age criterion is W_4^1; similarly, the weight of the frequency and place of the Internet use criteria are of weight W_5^1 and W_6^1 respectively. These relative importance values are based on preliminary data of managerial decisions and expert assessments. Values of criteria were obtained using methods of hierarchy analysis. To make a selection of the target network and budget, you need to perform the following sequence of actions (Fig. 1) [22].

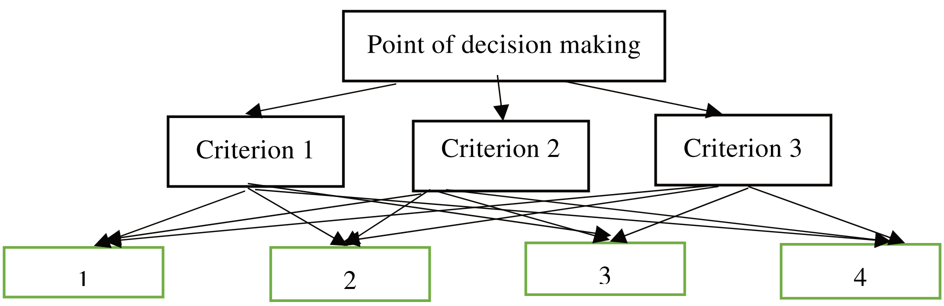

Fig. 1 A hierarchical approach to the problem of advertising budget distribution

Fix an ad budget. It is necessary to distribute the budget optimally. Let the total budget allocated for advertising on the network is equal M_1. Advertising information should be placed on various important sites and the decision is made according to the following points: the number of visitors to the site, the number of available types of targeting, the ability to see the results and to adjust the progress of the advertising campaign.

Based on these criteria, 4 banner networks and 4 separate sites were selected for the purpose of placing advertisements (hereinafter: Website 1, Website 2, Website 3, Website 4 and Network 1, Network 2, Network 3, Network 4).

Let $c_{r1}^1, c_{r2}^1, \ldots, c_{r4}^1$ is the price of one advertisement in each of the networks, r means the number corresponding to the ad format (graphic banner, flash banner, top plan, text ad, backgammon, respectively). That is, r takes values from 1 to 5. Let $K_1^1, K_2^1, \ldots, K_4^1$ be the number of demonstrations per day.

The profile of the visitors matrix based on a random sample of 1 000 in the four selected networks is presented in the Tab. 1. The sample size was determined based on the expense of data collection and in order to decrease variances of sample estimates. Selection of a small sample may lead to wide confidence intervals or risks of low quality of the resulting estimates.

Let x_{rk}^1 for r = 1, 2, …,5, k = 1, 2, 3, 4 are solution variables that match the number of advertisements in different networks. If $a_i^1, i = 1, \ldots, 4,$ mean the reach to the target audience of the ad on the respective networks, then

$$\begin{aligned} a_1^1 &= (W_1^1 p_{11}^1 + W_2^1 p_{21}^1 + W_3^1 p_{51}^1 + W_4^1 p_{41}^1 + W_5^1 p_{51}^1 + W_6^1 p_{61}^1) k_1^1, \\ a_2^1 &= (W_1^1 p_{12}^1 + W_2^1 p_{22}^1 + W_3^1 p_{32}^1 + W_4^1 p_{42}^1 + W_5^1 p_{52}^1 + W_6^1 p_{62}^1) k_2^1, \\ a_3^1 &= (W_1^1 p_{13}^1 + W_2^1 p_{23}^1 + W_3^1 p_{33}^1 + W_4^1 p_{43}^1 + W_5^1 p_{53}^1 + W_6^1 p_{63}^1) k_3^1, \\ a_4^1 &= (W_1^1 p_{14}^1 + W_2^1 p_{24}^1 + W_3^1 p_{34}^1 + W_4^1 p_{44}^1 + W_5^1 p_{54}^1 + W_6^1 p_{64}^1) k_4^1, \end{aligned} \tag{1}$$

where p_{1j}^1 is the percentage of visitors to the network j that satisfy the i-th criterion.

Tab.1 Visitor profile matrix

Criterion	**Network 1**	**Network 2**	**Network 3**	**Network 4**
Income	p_{11}^1	p_{12}^1	p_{13}^1	p_{14}^1
Status	p_{21}^1	p_{22}^1	p_{23}^1	p_{24}^1
Region	p_{31}^1	p_{32}^1	p_{33}^1	p_{34}^1
Age	p_{41}^1	p_{42}^1	p_{43}^1	p_{44}^1
The frequency of access to the network	p_{51}^1	p_{52}^1	p_{53}^1	p_{54}^1
The place of access to the network	p_{61}^1	p_{62}^1	p_{63}^1	p_{64}^1

Thus, the target function in this case can be represented as follows:

maximize $$Z=\sum_{r=1}^{5}(a_1^1x_{r1}^1+a_2^1x_{r2}^1+a_3^1x_{r3}^1+a_4^1x_{r4}^1),$$

provided:

$$\begin{aligned}&\sum_{r=1}^{5}(c_{r1}^1x_{r1}^1+c_{r2}^1x_{r2}^1+c_{r3}^1x_{r3}^1+c_{r4}^1x_{r4}^1)\le M_1\\&x_{r1}^1\ge t_{r1}^1, x_{r1}^1\le u_{r1}^1, x_{r2}^1\ge t_{r2}^1, x_{r2}^1\le u_{r2}^1, x_{r3}^1\ge t_{r3}^1, x_{r3}^1\le u_{r3}^1, x_{r4}^1\ge t_{r4}^1, x_{r4}^1\le u_{r4}^1\\&x_{rk}^1\ge 0, r=1,2,...,5, k=1,2,3,4.\end{aligned}\tag{2}$$

Here t_{rk}^1 is the minimum number of advertisements of different formats in each of the four networks. Similarly, u_{rk}^1 is the maximum number of advertisements of different formats in the corresponding network. Let Site 1, Site 2, Site 3, and Site 4 are selected professional sites and search engines. Selection criteria were the popularity and number of target audience.

Five different promotional packages have been selected depending on the importance and number of visits. The task remains the same, namely, to decide what volume of advertisements, in which format, on which sites and in which packages should be placed, in order to reach the desired amount of target audience within the limits of the available advertising budget and restrictions on the minimum and maximum number of advertisements on each of the sites and in each package. The following packages are considered: morning 5-11, day 11-15, day 15-19, evening 19-23, night 23-4. It is assumed that the price of an advertisement depends on the package. It is also assumed that a certain minimum and maximum number of advertisements must be placed on different sites at different time intervals. Limiting the number of ads at different times on different sites should be considered as different goals. The minimum and maximum number of ads on different sites in different packages is determined by managers based on their experience of previous advertising companies.

The profile of visitors for different sites and different time packages s, where $s = 1, 2, \ldots, 5$ is shown in Tab. 2.

Tab. 2 Compliance of selected sites with criteria

Criterion	**Site1**	**Site 2**	**Site 3**	**Site 4**
Income	q_{11}^{2s}	q_{12}^{2s}	q_{13}^{2s}	q_{14}^{2s}
Status	q_{21}^{2s}	q_{22}^{2s}	q_{23}^{2s}	q_{24}^{2s}
Region	q_{31}^{2s}	q_{32}^{2s}	q_{33}^{2s}	q_{34}^{2s}
Age	q_{41}^{2s}	q_{42}^{2s}	q_{43}^{2s}	q_{44}^{2s}
The frequency of access to the network	q_{51}^{2s}	q_{52}^{2s}	q_{53}^{2s}	q_{54}^{2s}
The place of access to the network	q_{61}^{2s}	q_{62}^{2s}	q_{63}^{2s}	q_{64}^{2s}

In Tab. 2 q_{ij}^{2s} is the percentage of visitors to the site j satisfying the i-th criterion.

Let k_{si}^2 be the average number of site visitors in the s-th time slot.

Let $c_{s1}^2, c_{s2}^2, \ldots, c_{s4}^2$ be the cost of one ad on each site, s-denotes the number corresponding to the time packet. Let the budget, for advertising on individual sites and search engines is equal M_2.

If a_{si}^2, $i = 1, 2, 3, 4$ mean the reach of an advertisement on the relevant sites the target audience, and they are calculated by the Eq. (1), and x_{sk}^2 for $s = 1, 2, \ldots, 5$, $k = 1, 2, 3, 4$ are decision variables that correspond to the number of advertisements on each of the sites in each package, then the mathematical formulation of the problem will look like this:

maximize $$Z_2 = \sum_{s=1}^{5}\left(a_{s1}^2 x_{s1}^2 + a_{s2}^2 x_{s2}^2 + a_{s3}^2 x_{s3}^2 + a_{s4}^2 x_{s4}^2\right),$$

provided:

$$\begin{gathered} \sum_{s=1}^{5}\left(c_{s1}^2 x_{s1}^2 + c_{s2}^2 x_{s2}^2 + c_{s3}^2 x_{s3}^2 + c_{s4}^2 x_{s4}^2\right) \le M_2 \\ x_{s1}^2 \ge t_{s1}^2, x_{s1}^2 \le u_{21}^2, x_{s2}^2 \ge t_{s2}^2, x_{s2}^2 \le u_{s2}^2, x_{s3}^2 \ge t_{s3}^2, x_{s3}^2 \le u_{s3}^2, x_{s4}^2 \ge t_{s4}^2, x_{s4}^2 \le u_{s4}^2 \\ x_{sk}^2 \ge 0, s = 1, 2, ..., 5, k = 1, 2, 3, 4. \end{gathered} \tag{3}$$

Here t_{sk}^2 is the minimum number of advertisements in different time packs on each of the four sites. Similarly, u_{sk}^2 is the maximum number of advertisements in different time packs at different sites.

Thus conditions (2) and (3) are the boundary conditions for the acceptable range of use of the proposed model.

So, the multi-purpose deterministic problem, which unites both of the above-stated problems, can be formulated as follows:

maximize $$Z_1 = \sum_{r=1}^{5}\left(a_1^1 x_{r1}^1 + a_2^1 x_{r2}^1 + a_3^1 x_{r3}^1 + a_4^1 x_{r4}^1\right),$$

maximize $$Z_2 = \sum_{s=1}^{5}\left(a_{s1}^2 x_{s1}^2 + a_{s2}^2 x_{s2}^2 + a_{s3}^2 x_{s3}^2 + a_{s4}^2 x_{s4}^2\right),$$

provided:

$$\begin{gathered} \sum_{r=1}^{5}\left(c_{r1}^1 x_{r1}^1 + c_{r2}^1 x_{r2}^1 + c_{r3}^1 x_{r3}^1 + c_{r4}^1 x_{r4}^1\right) \le M_1, \\ \sum_{s=1}^{5}\left(c_{s1}^2 x_{s1}^2 + c_{s2}^2 x_{s2}^2 + c_{s3}^2 x_{s3}^2 + c_{s4}^2 x_{s4}^2\right) \le M_2. \end{gathered} \tag{4}$$

To solve the multi-purpose deterministic problem (4), a targeted programming model with random constraints was developed based on the structure of the foreground priorities.

After introducing variations of variance into budget target constraints (4), the target programming model will look like this:

$$\sum_{r=1}^{5}\left(c_{r1}^{1}x_{r1}^{1}+c_{r2}^{1}x_{r2}^{1}+c_{r3}^{1}x_{r3}^{1}+c_{r4}^{1}x_{r4}^{1}\right)+n_3-p_3=M_1,$$
$$\sum_{s=1}^{5}\left(c_{s1}^{2}x_{s1}^{2}+c_{s2}^{2}x_{s2}^{2}+c_{s3}^{2}x_{s3}^{2}+c_{s4}^{2}x_{s4}^{2}\right)+n_4-p_4=M_2. \tag{5}$$

Here n_3, n_4, p_3, p_4 are the corresponding negative and positive variables of the deviation for both purposes. Minimizing the excess of budget goals ensures that these goals are met. Objectives of the budget constraint must be fully met and, as a result, they should have a higher priority. The formulation in terms of targeted programming of the criteria for minimum and maximum quantities after the introduction of the variables of the deviation in (2) and (4) is as follows:

$$x_{r1}^{1}+n_{r3}-p_{r3}=t_{r1}^{1}, x_{r1}^{1}+n'_{r3}-p'_{r3}=u_{r1}^{1}, x_{r2}^{1}+n_{r4}-p_{r4}=t_{r2}^{1}, x_{r2}^{1}+n'_{r4}-p'_{r4}=u_{r2}^{1},$$
$$x_{r3}^{1}+n_{r5}-p_{r5}=t_{r3}^{1}, x_{r3}^{1}+n'_{r5}-p'_{r5}=u_{r3}^{1}, x_{r4}^{1}+n_{r6}-p_{r6}=t_{r4}^{1}, x_{r4}^{1}+n'_{r6}-p'_{r6}=u_{r4}^{1},$$
$$x_{s1}^{2}+n_{s7}-p_{s7}=t_{s1}^{2}, x_{s1}^{21}+n'_{s7}-p'_{s7}=u_{s1}^{2}, x_{s2}^{2}+n_{s8}-p_{s8}=t_{s2}^{2}, x_{s2}^{2}+n'_{s8}-p'_{s8}=u_{s2}^{2},$$
$$x_{s3}^{2}+n_{s9}-p_{s9}=t_{s3}^{2}, x_{s3}^{2}+n'_{s9}-p'_{s9}=u_{s3}^{2}, x_{s4}^{2}+n_{s10}-p_{s10}=t_{s4}^{2}, x_{s4}^{2}+n'_{s10}-p'_{s10}=u_{s4}^{2},$$

Here $n_{ri}, n'_{ri}, p_{ri}, p'_{ri}$ are negative and positive variables of deviation.

To find the equivalent for the reach criterion, it is necessary to do the following. Let A_1 and A_2 are desired levels of reach for the first and second target functions. These desirable levels can be obtained by finding the ideal solutions for the first and second criteria. Maximization of the reach criterion can be written as an objective programming task with random constraints, in which the probability that the estimated value of reach to the desired group of people will be greater than the ideal solution (estimated target value for reach) is greater than or equal to α or β (some acceptable range of probability values).

$$P\left(\sum_{r=1}^{5}\sum_{k=1}^{4}a_{rk}^{1}x_{rk}^{1}\geq A_1\right)\geq\alpha,$$
$$P\left(\sum_{s=1}^{5}\sum_{k-1}^{4}a_{sk}^{2}x_{sk}^{2}\geq A_2\right)\geq\beta,$$

where A_1 are A_2 are the estimates of reach.

$$P\left[\frac{\sum_{r=1}^{5}\sum_{k=1}^{4}a_{rk}^{1}x_{rk1}^{1}-M(A_1)}{\sqrt{D(A_1)}}\geq\frac{A_1-M(A_1)}{\sqrt{D(A_1)}}\right]=P\left[\frac{A_1-M(A_1)}{\sqrt{D(A_1)}}\leq\frac{\sum_{r=1}^{5}\sum_{k=1}^{4}a_{rk}^{1}x_{rk1}^{1}-M(A_1)}{\sqrt{D(A_1)}}\right]\geq\alpha.$$

Here $\frac{A_1-M(A_1)}{\sqrt{D(A_1)}}$ is the normal standard deviation with the mean value of 0 and the dispersion equal to 1. Let e be the value of the standard normal variable for which $\varphi(e)=\alpha$.

$$\varphi\left[\frac{\sum_{r=1}^{5}\sum_{k=1}^{4}a_{rk}^{1}x_{rk1}^{1}-M(A_1)}{\sqrt{D(A_1)}}\right]\geq\varphi(e)\Rightarrow\frac{\sum_{r=1}^{5}\sum_{k=1}^{4}a_{rk}^{1}x_{rk1}^{1}-M(A_1)}{\sqrt{D(A_1)}}\geq e\Rightarrow$$

$$\sum_{r=1}^{5}\sum_{k=1}^{4}a_{rk}^{1}x_{rk1}^{1}-M(A_1)-e\sqrt{D(A_1)}\geq 0.$$

Similarly, in the case of random restrictions for the second criterion of reach

$$\sum_{s=1}^{5}\sum_{k=1}^{4}a_{sk}^{2}x_{sk}^{2}-M\left(A_2\right)-f\sqrt{D\left(A_2\right)}\geq 0.$$

By converting these inequalities into equations using variables of deviation, we obtain the following target equations:

$$\sum_{r=1}^{5}\sum_{k=1}^{4}a_{rk}^{1}x_{rk1}^{1}+n_1-p_1=M\left(A_1\right)+e\sqrt{D\left(A_1\right)}, \tag{6}$$

$$\sum_{s=1}^{5}\sum_{k=1}^{4}a_{sk}^{2}x_{sk}^{2}+n_2-p_2=M\left(A_2\right)+f\sqrt{D\left(A_2\right)} \tag{7}$$

Here e and f are the values of normal standard deviations for which $\varphi(e)=\alpha$, $\varphi(f)=\alpha$.

For this model, the following structure of the foremost priorities has been developed:
priority 1: limiting the maximum number of ads on different media was placed at the first priority level, since any value reachable to the target group may only be achieved by increasing the number of advertisements. Then there should be some criteria of higher priority than the reach criterion that needs to be limited to reach any level of value,
priority 2: budget targets for both networks and for individual sites,
priority 3: in order to maximize the reach criterion, it is necessary to minimize inaccessibility as criterion (6) and (7) at the third priority level,
priority 4: minimum number of advertisements on different media. These criteria are met by minimizing the failure of all goals at the fourth priority level.

In Tab. 3, the calculation of parameters to the criteria is represented and the comparison of criteria is performed in Tab. 4.

Tab. 3 Calculation of parameters for the criteria

Parameter	**Criterion**					
	1	**2**	**3**	**4**	**5**	**6**
K	1.209	1.264	1.229	1.153	1.045	1.071
Y	0.186	0.208	0.194	0.162	0.113	0.123
>	1.186	1.208	1.194	1.162	1.113	1.123
<	0.814	0.792	0.806	0.838	0.887	0.877

Tab. 4 Comparison of criteria

Criteria	**Criteria**						**Sum**	**Mark**
Number	**1**	**2**	**3**	**4**	**5**	**6**	30.692	
1	—	0.814	0.814	1.186	1.186	1.186	5.186	0.169
2	1.208	—	1.208	1.208	1.208	1.208	6.040	0.197
3	1.194	0.806	—	1.194	1.194	1.194	5.582	0.182
4	0.838	0.838	0.838	—	1.162	1.162	4.838	0.158
5	0.887	0.887	0.887	0.887	—	0.887	4.435	0.145
6	0.872	0.872	0.872	0.872	1.123	—	4.611	0.150

From Tab. 4 it follows that in the first place the criterion is the status of the importance, in the second – the region, in the last – the place of access to the network. The comparison matrix has the form:

$$A_{ij} = \begin{pmatrix} 1 & \frac{2}{3} & \frac{4}{5} & \frac{4}{3} & 2 & 4 \\ \frac{3}{2} & 1 & \frac{6}{5} & 2 & 3 & 6 \\ \frac{5}{4} & \frac{5}{6} & 1 & \frac{5}{3} & \frac{5}{2} & 5 \\ \frac{3}{4} & \frac{1}{2} & \frac{3}{5} & 1 & \frac{3}{2} & 3 \\ \frac{1}{2} & \frac{1}{3} & \frac{2}{5} & \frac{2}{3} & 1 & 2 \\ \frac{1}{4} & \frac{1}{6} & \frac{1}{5} & \frac{1}{3} & \frac{1}{2} & 1 \end{pmatrix}$$

We calculate for it the elements of the vector $\boldsymbol{w}$:

$$w_{rm} = \sqrt[n]{a_{r1}a_{r2}a_{r3}a_{r4}a_{r5}a_{r6}}, n = 6, k = \overline{1,6}, m = 1$$

$$w_{11} = 1.336$$

$$w_{21} = 2.004$$

$$w_{31} = 1.67$$

$$w_{41} = 1.002$$

$$w_{51} = 0.668$$

$$w_{61} = 0.334$$

$$\boldsymbol{w} = \left(\frac{1.336}{7.014}; \frac{2.004}{7.014}; \frac{1.67}{7.014}; \frac{1.002}{7.014}; \frac{0.668}{7.014}; \frac{0.334}{7.014}\right)$$

$$\boldsymbol{w} = (0.191; 0.286; 0.238; 0.143; 0.098; 0.048)$$

Find the maximum value of the average consistency of scores:

$$\alpha_1^* = 1.00275 + 1.001 + 0.9996 + 1.001 + 1.029 + 1.008 = 6.04135.$$

It is necessary to check that the condition of error is fulfilled when compiling random matrices:

$$\beta_1^* = \frac{6.04135 - 6}{5 \cdot 1.24} \cdot 100\ \% = 0.67\ \% < 10\ \%.$$

Consequently, the condition of error is fulfilled. Thus, the received ballroom expert assessments, accumulated in the matrix, can be considered as adequate to realistic estimates.

Thus, after combining the criteria, the initial multi-criterion target programming problem (5) can be written as follows:
find x_{rk}^1, x_{sk}^2, to minimize

$$P_1\left[\sum_{r=1}^{5}\left(p'_{r3} + p'_{r4} + p'_{r5} + p'_{r6} + p'_{r7}\right) + \sum_{s=1}^{5}\left(p'_{s8} + p'_{s9} + p'_{s10} + p'_{s11} + p'_{s12}\right)\right], P_2\left(p_3 + p_4\right),$$

$$P_3\left(n_1 + n_2\right), P_4\left[\sum_{r=1}^{5}\left(n'_{r3} + n'_{r4} + n'_{r5} + n'_{r6} + n'_{r7}\right) + \sum_{s=1}^{5}\left(n'_{s8} + n'_{s9} + n'_{s10} + n'_{s11} + n'_{s12}\right)\right],$$

provided

$$\sum_{r=1}^{5}\sum_{k=1}^{4} a_{rk}^1 x_{rk1}^1 + n_1 - p_1 = M\left(A_1\right) + e\sqrt{D\left(A_1\right)},$$

$$\sum_{s=1}^{5}\sum_{k=1}^{4} a_{sk}^2 x_{sk}^2 + n_2 - p_2 = M\left(A_2\right) + f\sqrt{D\left(A_2\right)},$$

with restrictions from Eq. (5) and followings; n and p with different lower indices are negative and positive variables of deviation and larger, or equal to zero.

In reality, the mutual misinformation of counteraction parties is possible: by choosing one strategy, each party asserts that another strategy is adopted [21]. It follows that the degree of risk in this case depends both on the probability of choosing an opponent of a particular strategy and on the likelihood of recognition of the decision. The level of risk determines the losses of each player.

In this version, there are ways (alternatives) to choose advertising x: with the use of targeting and even advertising for the entire market of telecommunication services, therefore, there is a need to choose one of them. In addition, you need to take into account the impact of choosing an alternative competing company. There is no ambiguity between alternatives and a plurality of options. That is, each set of alternatives matches the set of outputs $z_{ij} = g(x_{i1}, x_{j2}), i = 1, \ldots, n, j = 1, \ldots, n$, here x_{i1} is an alternative, chosen by the first company, and x_{j2} – an alternative of a competing company.

The effectiveness of alternative advertising strategies in conditions of competition and incomplete awareness is estimated using point and interval estimates. The result of decisions about a strategy application in the case of anticipated actions of the competitor is the value of the utility function on the set of outputs [23]. The company's profitability function is the total profit P_a, since it can easily be reduced to a monetary measure, therefore $E(x) = \left[P_a(z)\right]^*$. The results $z_k\left(k = 1, \ldots, m\right)$ of choosing strategy $x_{i1}(i = 1, \ldots, n)$ have utility $P\left(z_{kj}\right) = g^k{}_{ij}(x_{i1}; x_{j2}), i = 1, \ldots, n; j = 1, \ldots, n$ and lead to a guaranteed profit g_k^*. Then the economic efficiency of using each strategy can be found by the formula: $E(x) = \max_k P(z_k)$.

To uncover the uncertainty of the behavior of opposing companies, two approaches are used: targeting achievement of the guaranteed result in the worst conditions and orientation on the most likely version of the behavior of the competing company and providing the best result under these conditions. That is, the following advertising parameters are selected in order to have the maximum possible value of the target function in the worst situation. In this case, the criterion used to find guaranteed earnings for each of the competing companies is: $g_1^* = \max_{x_1} \min_{x_2} g_1(x_1, x_2)$, $g_2^* = \max_{x_2} \min_{x_1} g_2(x_1, x_2)$.

Then for the first company it is guaranteed that $x_1 = x_1^*$ for any value x_2 the condition is fulfilled: $g_1(x_1^*, x_2) \geq g_1^*$. For the second company if $x_2 = x_2^*$ for any value x_1 the condition is fulfilled: $g_1(x_1, x_2^*) \geq g_2^*$.

A tabular, graphical, or classical method, based on the study of extreme properties of functions, can be used to find guaranteed earnings.

To find the absolute minimum and maximum of functions used in the interval comparison method, another criterion is used:

$$g_1^+ = \max_{x_1} \max_{x_2} g_1(x_1, x_2),\; g_2^+ = \max_{x_2} \max_{x_1} g_1(x_1, x_2),$$
$$g_1^- = \min_{x_1} \min_{x_2} g_1(x_1, x_2),\; g_2^- = \min_{x_2} \min_{x_1} g_1(x_1, x_2).$$

The situation is estimated by the value of the target function in the numerical interval, the minimum value of which corresponds to the worst situation, and the maximum – to the best.

For the target function of each company $g_{ij}(x_1, x_2), i, j = 1, \ldots, n,$ the interval estimate is presented in the form:

$$I_1^-(x_1, x_2) = \frac{g_1(x_1, x_2) - g_1^-}{g_1^+ - g_1^-};\, I_1^+(x, y) = \frac{g_1^+ - g_1(x_1, x_2)}{g_1^+ - g_1^-};$$
$$I_2^-(x_1, x_2) = \frac{g_2(x_1, x_2) - g_2^-}{g_2^+ - g_2^-};\, I_2^+(x_1, x_2) = \frac{g_2^+ - g_2(x_1, x_2)}{g_2^+ - g_2^-}.$$

Here $I_1^-, I_2^-, I_1^+, I_2^+$ are estimates that determine the relative level of differences in utility functions from their minimum and maximum values, respectively.

$$g_1^- \leq g_1(x_1, x_2) \leq g_1^+;\; g_2^- \leq g_2(x_1, x_2) \leq g_2^+.$$

It is easy to check that $I_1^- + I_2^+ = 1; I_1^- + I_2^+ = 1.s$ That is, it's enough to get one estimate for each function and to predefine it, or to set the permissible lower limit.

In this study, to ensure the procurement of material resources for the needs of the Armed Forces of Ukraine, the task of maximizing the value of its own target function and minimizing the risk of possible damage due to the uncertainty of competitors' actions was considered. It was assumed that ads would reach the maximum number of potential customers. A multi-purpose model was developed with uncertainty about the reach of the target audience. The parameter associated with the reach was considered as a random variable with known mathematical expectations and a mean square deviation, and a model of targeted programming with random constraints was developed. The desired level of goals was chosen as their ideal solution. The task was solved with the help of the structure of the foremost priorities. The notion of forward-looking priorities was taken into account in calculations using WinQSB.

The goals for the first three priorities were fully achieved and the fourth priority objective was partially met. The main goals of maximizing the reach of the target audience, both for networks and for individual sites, have been fully achieved. Budgetary constraints were fully implemented. But the goal of minimizing the number of promotional messages was achieved only partially.

An assessment of the effectiveness of an advertising strategy to ensure the government procurement of material resources for the needs of the Armed Forces of Ukraine in conditions of uncertainty of counteraction by means of a guaranteed value was based on quantitative estimates of DMP's own decisions in case of anticipated actions of the enemy. But such estimates are one-sided. They show the difference between a possible result and a certain pre-known value. Such an estimate does not give an idea of the approximation of the result simultaneously to the maximum and minimum possible values. Therefore, along with the point estimate, an interval comparison was made which allows us to estimate the values of the function of utility in a numerical interval, the boundaries of which correspond to the worst and best situations in the market of IT services.

To take into account the time between the appearances of advertisements, the model can be considered as a model of dynamic programming. In this case, it is proposed to consider the model of the relationship between the advertising effect of time-based advertising investments for various advertising strategies [24, 25]. The accuracy of evaluating the effectiveness of advertising with the proposed method depends on the quality of conducting studies related to the assessment of the relevant parameters.

With a known value of the target audience reach n (taking into account the overlap rate), as well as the profit from one purchase ε, it`s possible to find the estimated value of advertising revenue: $P = P_t(k)n\varepsilon - A - C$, where A – advertising costs; C – constant costs.

Given that only one point of space can correspond to each single point of time, a graphical interpretation of the dependence of consumers' feedback on investment in advertising over time can be represented as a three-dimensional curve, the form of which depends on the advertising strategy and set of factors at a certain point in time. The model involves the stratification of the "consumer response" axis on the levels of consumer reaction to advertising. The "time" axis should indicate the periods during which advertising companies with different budget, creative concept and involved advertising platforms were spent.

4. Discussion of Research Results

Application of automation in the decision-making process on the advertising budget distribution requires modern information and analytical system with the functions of finding optimal solutions.

However, criteria which include the percentage estimation of the reach and frequency of occurrence on different information carriers, aimed at different segments of the market at different periods of time, while taking into account the costs of targeting placement, needs of the project, financial capacity, traffic volume, availability of advertising space, target audience.

In the case of a small site, there is no acute need in the advertising management system. When considering an online resource with good traffic, you can save time by connecting to an Internet advertising management system, since 90% of manual transactions will occur automatically. Similar advantages are offered for consumers who advertise on the Internet. Conditionally, they get handy software that increases the effectiveness of advertising campaigns. Each of the existing advertising management systems has its advantages and disadvantages that have already been considered. With the help of additional functions of automatic sale of advertising you can receive additional profit. Advertising on the web allows you to reach a wider audience than advertising on a single, even large site.

This model can be used interactively by changing the desired reach level in accordance with the requirements of the decision maker.

5. Conclusions

The evaluation of the effectiveness of the decision making on media planning provided that effective targeting is based on the theory of system analysis, decision-making theory, game theory and mathematical statistics methods, confirms the feasibility of using the proposed methods of system evaluation of the degree and level of risk during the disclosure of the uncertainty of actions of the opposing parties in determining the strategy of an advertising campaign when organizing and conducting public procurement for the needs of the Armed Forces of Ukraine.

The overall effectiveness of managing defense resources in terms of allocating an advertising budget using the elements of the information and analytical system when deciding on the promotion of services in the market based on targeting modeling increases with the reach of the target group. The reliability of the results is due to the correct choice of mathematical apparatus and a large database of statistical data provided by the AdWords system.

The developed model of optimization of advertising on the condition of competition and incomplete awareness is intended to improve the management of defense resources in the unified system of management of administrative and economic processes of the Armed Forces of Ukraine. It allows using information technologies to consider a three-dimensional model of the dependence of the advertising effect on advertising investments in time for various advertising strategies. The obtained adoption allows, on the basis of statistical data or expertly, to obtain parameter estimates and to use it to predict the points in which the company faces the choice of a further strategy of advertising. On its basis, critical values of the state variable can also be found, and recommendations are developed for choosing the company's best behavior strategy at these points.

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