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Modelling Missile Flight Characteristics by Estimating Mass and Ballistic Parameters

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Abstract:

This paper deals with one of the possible methods of a qualified estimation of tactical and technical parameters of surface-to-air or air-to-air guided missiles. This method is designed to estimate missile flight parameters if they are unavailable from public sources. Although the method does not provide exact values, it is sufficient for modelling and subsequent simulation. This method is based on the use of so-called "characteristic numbers" of the missile and on the assumption of validity of the hypothesis about the similarity of characteristic missile numbers of equal or, respectively, similar technological levels. To validate the obtained parameters, a method of mathematical modelling of the missile guidance process is used. The method can also be used for a qualified estimation of missile parameters in the case of an acquisition process.

Keywords:

mass parameters, ballistic parameter, characteristic number, mathematical modelling

1. Introduction

In many decision-making processes related e.g. to the weaponry purchases, upgrading and use of equipment or modelling purposes, it is necessary to have an objective evaluation of tactical-technical level and combat capabilities. For such an objective assessment of the complex technical systems it is necessary to have adequate information available. However, the proper information is usually not available.

In connection with the upcoming necessity to replace the obsolete surface-to-air missile (SAM) system SA-6 in the Czech Armed Forces, the issue of sufficient information about the market on the new missile systems is also of current importance. The problem, however, is that most of the tactical and technical parameters and military-technical characteristics of contemporary and newly introduced SAM systems are so-called sensitive and are therefore not freely available. They can be acquired only in the

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case of official contact with the supplier. This fact significantly limits the possibility to accurately determine the required parameters and characteristics.

To address the above mentioned issues, this paper presents one of possible methods of quantitative estimation of tactical and technical parameters of SAM weapon systems, which are usually unavailable from public sources. The basic idea of this method is to use so-called guided missile characteristic numbers to evaluate the missile flight characteristics by assuming that most of the contemporary guided missiles have similar characteristics and technological level. For parameters validation, we have used a method of mathematical modelling of the surface-to-air missile guiding process.

The results can then be used, for example, in education process, in air defence simulations (activities connected with the preparation of operations centres personnel) [1-3], or finally for objective assessment of SAM systems properties within the acquisition processes.

2. Guided Missiles Characteristic Numbers

Characteristic missile numbers are defined as mass ratios of various parts of missile assembly, including the mass of the propellant. Characteristic numbers also take into account the number of missile flight engine stages, respectively, working modes of the missile engine (start and flight mode). First, we define three basic characteristics:

- The "*transport number p*" is defined as the ratio of the total starting mass of the missile to the mass of the load.
- The "*construction number s*" is defined as the ratio of the total starting mass of the missile reduced by the mass of the load (which is usually represented by the combat charge), to the mass of the missile frame (body).
- The "*speed number c*" is defined as the ratio of the total starting (initial) mass of the missile to its initial mass, minus the mass of the propellant consumed.

For two-stage missiles (two engines) or for multistage single-engine missiles, the characteristic numbers shall be determined.

Such defined "characteristic numbers" then characterize the technological level of the missile - its ability to deliver a payload at its given initial mass, at the desired speed, to desired point in space (position of a target). Alternatively, it is possible to estimate several missile parameters, such as the amount of Solid Propellant for Missile Engine (SPME) needed to achieve the desired speed and to transport a payload of a given mass. The characteristic numbers of the single-stage missiles, but with a two-mode engine, and the relationships for their calculation are given in Tab. 1.

In addition to the basic definitional relationships for characteristic missile numbers, relationships between individual characteristic numbers can also be calculated. These additional numbers can then be used to calculate numbers that cannot be obtained directly from basic definitions. For example:

$$p_{\mathrm{M}} = c_{\mathrm{M}} \frac{s_{\mathrm{M}} - 1}{s_{\mathrm{M}} - c_{\mathrm{M}}}, \tag{1}$$

$$s_{\mathrm{M}} = c_{\mathrm{M}} \frac{p_{\mathrm{M}} - 1}{p_{\mathrm{M}} - c_{\mathrm{M}}}, \tag{2}$$

$$c_{\mathrm{M}} = p_{\mathrm{M}} \frac{s_{\mathrm{M}}}{p_{\mathrm{M}} + s_{\mathrm{M}} - 1}, \tag{3}$$

Tab. 1 Characteristic numbers of missiles and their definition

Characteristic numbers	**Relationships for calculation**
Transport number for 1st (Start) phase of missile flight	$p_1 = \frac{m_0}{m_0 - m_{P1}}$
Transport number for 2nd (Start) phase of missile flight	$p_2 = \frac{m_0 - m_{P1}}{m_L}$
Construction number for 1st (Start) phase of missile flight	$s_1 = \frac{m_{MF} + m_{P0}}{m_{MF}}$
Construction number for 2nd (Start) phase of missile flight	$s_2 = \frac{m_{MF} + m_{P2}}{m_{MF}}$
Speed number for 1st (Start) phase of missile flight	$c_1 = \frac{m_0}{m_0 - m_{P1}}$
Speed number for 2nd (Start) phase of missile flight	$c_2 = \frac{m_0 - m_{P1}}{m_0 - m_{P0}}$
The total missile transport number	$p_M = \frac{m_0}{m_L} = p_1 p_2$
The total missile construction number	$s_M = \frac{m_{MF} + m_{P0}}{m_{MF}} = s_1$
The total missile speed number	$c_M = \frac{m_0}{m_0 - m_{P0}} = c_1 c_2$
The construction number of engine section	$s_E = \frac{m_{MS}}{m_{P0}}$

Variables in the relations for each characteristic missile number mean:

m_0 – the total initial missile mass,
m_L – the mass of load (e.g. warhead),
m_{P0} – the initial solid propellant (SPME) mass,
m_{MF} – the mass of missile frame,
m_{P1} – the mass of SPME for the first flight stage,
m_{P2} – the mass of SPME for the second flight stage,
m_{MS} – the mass of the whole engine section.

Another useful relationship for estimating the basic mass and ballistic parameters of the missile, namely the theoretical velocity or the required mass of SPME, is Tsiolkovsky's equation. This equation defines the relationship between the total initial and final mass of the missile, respectively. The mass of SPME consumed, and SPME efficiency on the one hand, and the theoretical speed v_{TS} of the missile on the other. Then:

$$v_{TS} = I_{SP} \ln c_M = I_{SP} \ln\left(\frac{m_0}{m_0 - m_{F0}}\right), \quad (4)$$

or also

$$m_{F0} = m_0 \left(1 - e^{-\frac{v_{TS}}{I_{SP}}} \right), \tag{5}$$

where the I_{SP} is the specific impulse of the missile SPME [N·s·kg^{-1}].

However, Tsiolkovsky's equation is valid in the absence of external forces. For this, it is necessary to respond appropriately to the parameters such as surface-to-air missiles moving in the gravitational field and the Earth's atmosphere. This can be achieved, for example, by an appropriate increase in the required theoretical speed of the missile.

3. Using Characteristic Numbers to Estimate Missile Parameters

To use the described method of estimating mass and ballistic parameters of missiles, it is firstly necessary to determine the parameters and characteristic numbers of the reference missile. To do this, we will use information about modern surface-to-air or air-to-air missiles available in various sources, such as [4, 5]. Because of sensitivity of relevant information, we demonstrate a procedure using a hypothetical missile, marked with the RM symbol (Reference Missile), whose parameters are similar to known real missiles.

In the next steps, we can calculate (estimate) the originally not available mass and ballistic parameters of the missiles according to ratios given in Tab. 1 and assuming the hypothesis of the similarity of identical missiles of the same design and similar technological levels. Examples of obtained results are given in Tabs 2-5.

The described method has also some shortcomings, such as for example the fact that not all missile parameters which we use as reference will always be clearly defined.

Most commonly, this refers to the total mass of the missile, the mass of the SPME, and the time of the engine operation, which can be given differently in various sources. Deviations are usually not significant, but they can have some influence on flight characteristics.

An important parameter of the maximum missile speed is also problematic, because there are many differences in open sources.

Tabs 2-5 contain numbers marked bold in the last column. These numbers were taken from open sources. Other values were calculated from characteristic numbers or from Reference Missile (RM) recalculation.

For the missiles analysed in Tabs 2-4, only the geometric dimensions (in particular calibre), the total starting mass and the mass of the warhead are usually available.

Furthermore, there is again relatively little credibility with the maximum speed. This allows for a quite large variance of important mass and time parameters, such as SPME mass and operating time in each engine mode, which then affect flight parameters.

The obtained mass and ballistic parameters of the missiles under investigation will be used in the mathematical model of missile movement when guided to an aerial target. As a result, the missile and target trajectories are calculated, together with missile velocity curve. These parameters will be used to verify the missile mass and ballistic parameters estimations and the maximum missile range estimation. For example, if the speed drops to a set limit, most often at 200 m·s^{-1}, missile manoeuvring capabilities and the flight time of the missile stop at a calculated distance. In Figs 1 and 2, for example, one of the results of the analysed missiles motion is presented as the velocity depending on the range of the flight.

The graphs in Fig. 3 then show the flight time of a certain type of missile at a given distance, for example at the internal or external boundary of the effective missile area.

Tab. 2 Example of the estimation of the Derby SR missile

Parameter	**Symbol**	**Unit**	**RM**	**DERBY SR**
Total initial mass of the missile	m_0	[kg]	**165.00**	**118.00**
Warhead mass	m_{BH}	[kg]	**20.00**	**23.00**
SPME total mass	m_{PH}	[kg]	**57.65**	43.25
SPME specific impulse	I_{SP}	[N·s·kg^{-1}]	**2 460.00**	2 460.00
1st (start) phase acceleration	a_{x1}	[m·s^{-2}]	**190.00**	190.00
1st (start) phase engine thrust	P_1	[N]	**31 350.00**	22 500.00
1st ph. SPME consumption	m_{s1}	[kg·s^{-1}]	**12.00**	9.15
1st phase time	Δt_1	[s]	**2.70**	2.74
Tot. 1st ph. SPME consumed	m_{PH1}	[kg]	**32.40**	25.06
2nd phase acceleration	a_{x2}	[m·s^{-2}]	**121.00**	127.00
2nd phase engine thrust	P_2	[N]	**16 000.00**	11 800.00
2nd phase SPME consumption	m_{s2}	[kg·s^{-1}]	**6.53**	4.80
2nd phase time	Δt_2	[s]	**3.15**	3.79
Total 2nd phase SPME consumed	m_{PH2}	[kg]	**20.57**	18.19
Total engine run time	t	[s]	**5.86**	6.53
Final missile mass	m_k	[kg]	**112.00**	74.75
Open sources max. missile speed	v_{max}	[M]	**3.00**	**3.00**
Theoretical missile speed	v_{TS}	[m·s^{-1}]	**1 057.00**	1 123.00
Modelled max. missile speed	$v_{M\ max}$	[m·s^{-1}]	**900.00**	925.00
Total missile transport num.	p_R	[–]	8.250	5.130
Total missile constr. num.	c_R	[–]	1.537	1.579
Total missile speed number	s_R	[–]	1.660	1.836
Speed number of 1st stage	s_1	[–]	1.244	1.270
Constr. number of 1st stage	c_1	[–]	1.660	1.836
Speed number of 2nd stage	s_2	[–]	1.235	1.243
Constr. number of 2nd stage	c_2	[–]	1.289	1.352

Tab. 3 Example of the estimation of the Russian SA-17 – BUK 9M38M1 missile

Parameter	Symbol	Unit	RM	9M38M1
Total initial mass of the missile	m_0	[kg]	**165.00**	690.00
Warhead mass	m_{BH}	[kg]	**20.00**	70.00
SPME total mass	m_{PH}	[kg]	**57.65**	340.00
SPME specific impulse	I_{SP}	[$N \cdot s \cdot kg^{-1}$]	**2 460.00**	2 460.00
1st (start) phase acceleration	a_{x1}	[$m \cdot s^{-2}$]	**190.00**	190.00
1st (start) phase engine thrust	P_1	[N]	**3 1350.00**	69 000.00
1st ph. SPME consumption	m_{s1}	[$kg \cdot s^{-1}$]	**12.00**	28.05
1st phase time	Δt_1	[s]	**2.70**	4.00
Tot. 1st ph. SPME consumed	m_{PH1}	[kg]	**32.40**	112.20
2nd phase acceleration	a_{x2}	[$m \cdot s^{-2}$]	**121.00**	121.00
2nd phase engine thrust	P_2	[N]	**16 000.00**	35 025.00
2nd phase SPME consumption	m_{s2}	[$kg \cdot s^{-1}$]	**6.53**	14.24
2nd phase time	Δt_2	[s]	**3.15**	16.00
Total 2nd phase SPME consumed	m_{PH2}	[kg]	**20.57**	227.80
Total engine run time	t	[s]	**5.86**	56.91
Final missile mass	m_k	[kg]	**112.00**	350.00
Open sources max. missile speed	v_{max}	[M]	**3.00**	3.00
Theoretical missile speed	v_{TS}	[$m \cdot s^{-1}$]	**1 057.00**	1 650.00
Modelled max. missile speed	$v_{M\ max}$	[$m \cdot s^{-1}$]	**900.00**	1 075.00
Total missile transport num.	p_R	[–]	8.250	9.857
Total missile constr. num.	c_R	[–]	1.537	1.971
Total missile speed number	s_R	[–]	1.660	2.214
Speed number of 1st stage	s_1	[–]	1.244	1.194
Constr. number of 1st stage	c_1	[–]	1.660	2.214
Speed number of 2nd stage	s_2	[–]	1.235	1.651
Constr. number of 2nd stage	c_2	[–]	1.289	1.814

Tab. 4 Example of the estimation of the MICA missile

Parameter	**Symbol**	**Unit**	**RM**	**MICA**
Total initial mass of the missile	m_0	[kg]	**165.00**	**112.00**
Warhead mass	m_{BH}	[kg]	**20.00**	**12.00**
SPME total mass	m_{PH}	[kg]	**57.65**	41.90
SPME specific impulse	I_{SP}	[N·s·kg⁻¹]	**2 460.00**	2 460.00
1st (start) phase acceleration	a_{x1}	[m·s⁻²]	**190.00**	190.00
1st (start) phase engine thrust	P_1	[N]	**31 350.00**	21 200.00
1st ph. SPME consumption	m_{s1}	[kg·s⁻¹]	**12.00**	8.62
1st phase time	Δt_1	[s]	**2.70**	2.75
Tot. 1st ph. SPME consumed	m_{PH1}	[kg]	**32.40**	23.70
2nd phase acceleration	a_{x2}	[m·s⁻²]	**121.00**	127.00
2nd phase engine thrust	P_2	[N]	**16 000.00**	11 200.00
2nd phase SPME consumption	m_{s2}	[kg·s⁻¹]	**6.53**	4.55
2nd phase time	Δt_2	[s]	**3.15**	3.99
Total 2nd phase SPME consumed	m_{PH2}	[kg]	**20.57**	18.18
Total engine run time	t	[s]	**5.86**	6.74
Final missile mass	m_k	[kg]	**112.00**	70.12
Open sources max. missile speed	v_{max}	[M]	**3.00**	**3.00**
Theoretical missile speed	v_{TS}	[m·s⁻¹]	**1 057.00**	1 152.00
Modelled max. missile speed	$v_{M\ max}$	[m·s⁻¹]	**900.00**	926.00
Total missile transport num.	p_R	[–]	8.250	5.130
Total missile constr. num.	c_R	[–]	1.537	1.579
Total missile speed number	s_R	[–]	1.660	1.836
Speed number of 1st stage	s_1	[–]	1.244	1.270
Constr. number of 1st stage	c_1	[–]	1.660	1.836
Speed number of 2nd stage	s_2	[–]	1.235	1.243
Constr. number of 2nd stage	c_2	[–]	1.289	1.352

Tab. 5 Example of the estimation of the IRIS-T missile

Parameter	**Symbol**	**Unit**	**RM**	**IRIS-T**
Total initial mass of the missile	m_0	[kg]	**165.00**	**87.50**
Warhead mass	m_{BH}	[kg]	**20.00**	**11.40**
SPME total mass	m_{PH}	[kg]	**57.65**	31.20
SPME specific impulse	I_{SP}	[N·s·kg^{-1}]	**2 460.00**	2460
1st (start) phase acceleration	a_{x1}	[m·s^{-2}]	**190.00**	190
1st (start) phase engine thrust	P_1	[N]	**3 1350.00**	16 500
1st ph. SPME consumption	m_{s1}	[kg·s^{-1}]	**12.00**	6.71
1st phase time	Δt_1	[s]	**2.70**	2.70
Tot. 1st ph. SPME consumed	m_{PH1}	[kg]	**32.40**	18.11
2nd phase acceleration	a_{x2}	[m·s^{-2}]	**121.00**	127
2nd phase engine thrust	P_2	[N]	**16 000.00**	8 800
2nd phase SPME consumption	m_{s2}	[kg·s^{-1}]	**6.53**	3.58
2nd phase time	Δt_2	[s]	**3.15**	3.66
Total 2nd phase SPME consumed	m_{PH2}	[kg]	**20.57**	13.09
Total engine run time	t	[s]	**5.86**	6.36
Final missile mass	m_k	[kg]	**112.00**	56.30
Open sources max. missile speed	v_{max}	[M]	**3.00**	**3**
Theoretical missile speed	v_{TS}	[m·s^{-1}]	**1 057.00**	1085
Modelled max. missile speed	$v_{M\ max}$	[m·s^{-1}]	**900.00**	920
Total missile transport num.	p_R	[–]	8.250	5.130
Total missile constr. num.	c_R	[–]	1.537	1.579
Total missile speed number	s_R	[–]	1.660	1.836
Speed number of 1st stage	s_1	[–]	1.244	1.270
Constr. number of 1st stage	c_1	[–]	1.660	1.836
Speed number of 2nd stage	s_2	[–]	1.235	1.243
Constr. number of 2nd stage	c_2	[–]	1.289	1.352

4. Conclusion

The flight characteristics of the missiles obtained by estimating the mass and ballistic parameters described in this paper may be used to extend or possibly verify information on the missiles tactical and technical parameters not otherwise routinely available. According to the method results, there could also be a correction of views or opinions on the weapon system's capabilities and the way in which the SAM system is used.

Mathematical modelling results also show some differences in flight characteristics between new missiles and missiles that were (e.g. V-601P, 5V27D, V-755, 5Ja-23) or still are (e.g. 3M9M3E) in armament of the Czech Armed Forces. Older missiles were equipped with engines with a longer operating time (about tens of seconds), which gave them a smaller speed (usually M2.5 to M3), but retained that velocity for most of the declared range (see the graph in Fig. 2).

The new missiles which we have analysed, on the other hand, have powerful engines for solid SPMEs, which gives them a high speed (M3 to M4), but with a relatively short operating time (less than 10 seconds). At the end of the engine run, the missiles fly on inertia and their speed falls within declared range quite rapidly (see the graphs in Figs 1 and 2).

These missiles fly to the interpolation point close to the internal boundary of the missile effective area earlier than the older missiles, but they lose some of this advantage when they fly to the outer boundary of the missile effective zone (see the graph in Fig. 3). Everything, of course, also depends on the target flight parameters. Targets in the higher levels of the atmosphere can usually be destroyed at greater distances, since the missile at these altitudes does not overcome enormous air resistance.

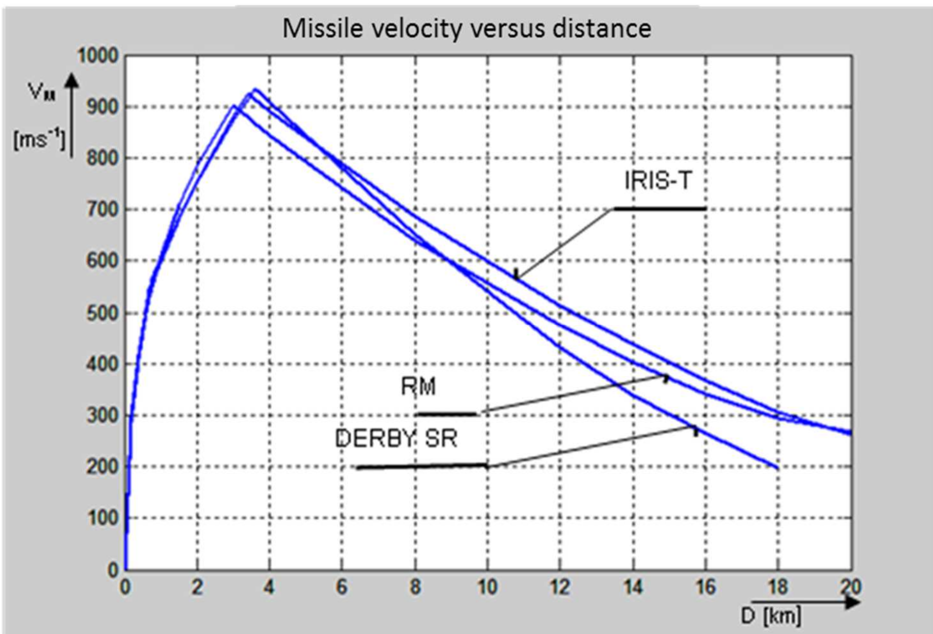

Fig. 1 Dependence of missile velocity on distance

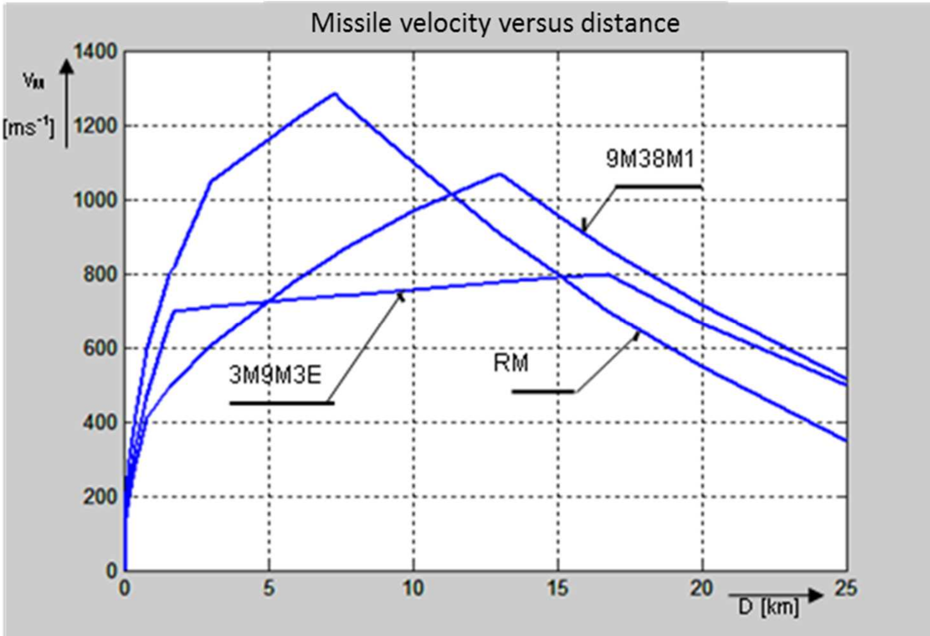

Fig. 2 Graph of dependence of missile velocity on distance (range)

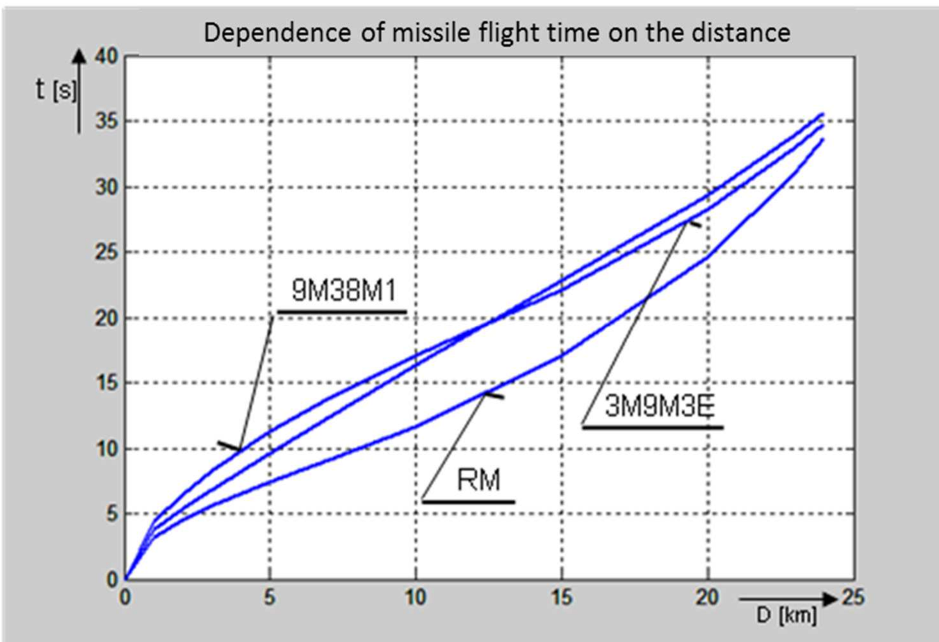

Fig. 3 Dependence of missile flight time on distance (range)

Subject Matter Experts (SME) estimates about current and future anti-aircraft missiles and missile guidance mathematical modelling represent an important part of the theoretical basis for the creation of a ground-based tactical air defence simulator [1, 2]. Such simulators could be applicable both to the training of personnel in air defence operations centres and practical exercises at the University of Defence.

The missile estimation method and the mathematical modelling can play significant role in missile guidance process simulation and in supporting decision-making in case of acquisition processes.

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