



## Critical Flight Conditions at High Angles of Attack, Related to Loss of Control in Lateral Motion

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### Abstract:

*The paper deals with developing algorithms for stall diagnostics and airplane protection from inadvertently encountering post stall gyration and entering spin modes. It summarizes the results of the linear theory of stall developing as theoretical foundation of nonlinear approach for diagnostics of flight conditions relating to the loss of aircraft control which is understood as a moment when the angular rates exceed some critical (threshold) values, which in general depend on the angle of attack and airspeed and they correspond to control surfaces deflection.*

### Keywords:

*Airplane dynamics, high angles of attack, airplane loss-of-control, critical flight modes, stall and spin immunity, warning system, airplane control automation*

### 1. Introduction

As statistics analysis shows, more than 70 % of flight incidents result in fatal crashes because pilots incorrectly or too late assess the situation and wrongly identify the implications of development of a special situation, when they inadvertently get an airplane into critical conditions or dangerous situations [1 – 21]. Delayed pilot actions for airplane recovery from hazardous flight conditions result in significant loss of altitude that can threaten the dangerous situation to grow into an emergency. The error in identifying the possible consequences of a dangerous situation may result in such flight conditions when an emergency cannot be prevented from by any following crew actions, and time reserve or altitude margin do not enable an emergency escape.

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Human being is inclined to make errors by nature. Thus, along with higher aircraft reliability and safety, automation of airplane control systems and improved flight envelope protection systems, human factor consideration is an important aspect in improving flight safety and ensuring crew rescue. Simultaneously with perfecting pilot skill and flight personnel training level, it is necessary to equip the airplanes with airborne systems, which could timely and correctly inform the pilot about encountering critical or hazardous flight conditions and forecast the outcome of subsequent development of the situation.

As statistics analysis shows, most flight accidents happen due to either various technical failures (25 – 30 %) or to pilot's errors (over 40 %) [12 – 14]. It is difficult to develop an automated system intended for identification of a great number of possible types of dangerous situations. However, using the analysis of flight accidents statistics it is possible to pick out, from among the whole multitude, the set of the most frequent standard situations characterised by certain common signs by which they can be identified.

High angle of attack flight modes are of great interest from the point of view of analysis of dangerous situations, especially in low-altitude flights.

Modern airplanes are characterised by a great variety of behaviour patterns during stall and instability of stall and spin features. Stall can be abrupt and it can strongly disorientate the pilot unaccustomed to these modes. This results in pilot recognizing the mode too late and interfering in control to prevent airplane stall and spin with a big delay. In this connection, the development of algorithms for diagnosing critical flight conditions related to the loss of control becomes extremely relevant, which allows to identify the control loss moment, the angular rotation direction timely and correctly, and warn the pilot accordingly.

At present, the limitation of operational range of the angles of attack is used for ensuring flight safety and stall protection. For this purpose, various automatic devices, such as angle-of-attack limiting system, restriction signalling system, permissible flight envelope limiter, which warn the pilot about approaching maximum permissible angles of attack and prevent further AOA increase, are widely used in modern airplanes [8 – 11].

Often, during an aggressive manoeuvre, the pilot can exceed the permissible angle of attack for a short time. It should be noted that exceeding permissible angle of attack does not always result in stall. So, for example, competently flown manoeuvrable airplanes can reach very high angles of attack (far beyond  $\alpha_{stall}$  limits), remain at these angles of attack during a certain relatively short period of time and then return back to small operational angles of attack ( $\alpha < \alpha_{stall}$ ) avoiding stall and spin. Mathematical and scaled-down simulation results [17 – 19], free-flying models study results [10], full-scale flight test results and demonstration flights of modern airplanes at international airshows convincingly prove this fact. However, as analysis of flight accidents statistics shows [12 – 14], when combat pilots reach high angles of attack exceeding permissible, they, unlike test pilots, cannot always manage the situation and often recognise and identify it incorrectly or too late, which results in emergency or disastrous consequences.

Thus, the problem related to the development of algorithms for diagnostics of critical flight conditions is quite acute. Integration of such algorithms with control automation algorithms would allow to significantly improve flight safety of modern and next-generation airplanes.

In an attempt to solve the stated task, the authors of the paper in many aspects relied upon the results of studies carried out by national and foreign researchers in the field of airplane dynamics in critical flight conditions at high angle-of-attack during the last three decades.

The starting point for the work were the papers by Behel J. M., McNamara W. G. [5], Glenn Larson [3], Berko V. S, Berko G. S, Zhivov Y.G., Poedinok A.M., Syrovatsky V.A., Falko S.V., Vlasova A.E., Shibaev V. M, Kisilev Y.F., Novikov A.V., Kolchin A.A., Naumov V.A, Shenfinkel Y.I. [9 – 11], Berestov L.M. and Favorova G.N. [16] and other authors who were developing algorithms for stall diagnostics and airplane protection from inadvertently encountering these modes.

The works by Kelviste [19], G.S. Bushgens and R.V. Studnev [12], which summarized some results of the linear theory of stall and have already become classics and the papers of one of the authors under the supervision of M.G. Goman [25 – 27] became a theoretical foundation of the present work.

## 2. Diagnostics of Flight Conditions Relating to Loss of Control

Normal Loss of control and stall of an airplane occur due to the degradation of stability and damping characteristics, degraded efficiency of controls and increased values of "harmful (parasitic)" inertia and aerodynamic moments emerging at high angles of attack. According to the definition which has become conventional in American aviation scientific and technical terminology, the flight mode in which the airplane "ceases to obey" the pilot and doesn't respond to pilot's actions by adequate changes in angular motion parameters is understood as loss of control. Stall is understood as the flight mode, in which the pattern of airplane motion is inadequate to the deflection of control surfaces and it drastically changes as compared to the expected motion [1].

According to Russian terminology [23, 24], stall is understood as self-induced divergent aperiodic or oscillatory airplane motion occurring at high angles of attack, which does not cease without decreasing the angle of attack, or as converging oscillations of significant amplitude, which increase with increased angle of attack. At that, stall modes can be versatile.

Thus, it is logical to understand controllability as adequate response of an airplane to the deflection of control surfaces. In this case, the loss of control is a distortion of relationship between positions of levers or control surfaces and dynamics of airplane angular motion conventional for the pilot.

If during the loss of control and stall, the angular rates exceed the values which can be realized at maximum deflections of control surfaces during controlled motion, then, for identifying stall modes, it suffices to compare current angular rates with some critical (threshold) values which in general depend on the angle of attack and flight speed. Namely, such approach to the development of algorithms for diagnosing stall and spin modes has been implemented on some airplanes developed in the 70-ies and 80-ies [17, 18]. This approach was justified by the fact that the stall of most of manoeuvrable airplanes of those years had quite a definite character: most often the loss of directional (yaw) dynamic stability resulting in yaw divergence ("nose slice" mode) or fast oscillatory loss of stability in lateral motion with dramatic increase of roll  $p$  and yaw  $r$  rates. The loss of control and stall warning was made in the event of excessive angular rates  $|p| > p^*$  and  $|r| > r^*$  at angles of attack exceeding admissible

value  $\alpha^* > \alpha_{stall}$ . The above mentioned algorithms were flight tested and implemented on F-14, F-15, F-16, F-18 airplanes [3 – 5].

For last generation airplanes notable for diversity of stall modes, the discussed diagnostic algorithms turn to be not quite efficient. On the one hand, as flight tests have shown, new generation airplanes are capable of performing short-term controlled manoeuvres at big angular rates and very high beyond-stall angles of attack. On the other hand, the loss of control and stall of these airplanes can occur with angular rates of absolute insignificant values compared to the values in controlled flight. Thus, in the present paper, it is proposed to take analysis of correspondence of airplane behaviour in angular motion to current controls displacement as the basis of the new approach for developing algorithms for diagnostics of critical flight conditions related to the loss of control, unlike comparing current angular rates with their threshold values.

This approach also allows to embrace the cases of the loss of control determined by high airplane asymmetric aerodynamic and weight loads induced by the failure of one of the engines or partial destruction of the structure (for example, breakdown of one outer wing panel or tail plane). If such disturbances exceed the control moments from deflection of control surfaces, the airplane loses control and does not obey the pilot.

Therefore, for the diagnostics of critical flight conditions related to the loss of control, it is necessary to develop the algorithms which would use current information measured on-board and would in real time identify the moment of airplane going out of pilot's control and its further behaviour in angular motion becoming unpredictable. In this connection, it is necessary to formalize the relationship between airplane behaviour and deflection of control surfaces. Distortion of the correspondence determined by formal signs can be considered the loss of control of an airplane.

### ***2.1. Main Principles for Development of Diagnostic Algorithms***

The issue of formal description of the relationship between airplane behaviour and control inputs of control surfaces conventional for the pilot can be settled through a selection of the reference model with certain accuracy describing dynamics of an airplane depending on current control parameters. The reference model can be built in different ways. In particular, both logical description and description by means of a transfer function is possible:

$$\hat{X} = W(\bar{X}, p)\bar{\delta}, \quad (2.1)$$

where  $\hat{X}$  – evaluation of airplane response to control input  $\bar{\delta}$ .

$W(\bar{X}, p)$  – transfer function generally depending on parameters of motion  $\bar{X}$ , aerodynamic and mass-inertia characteristics of an airplane.

The error  $\delta\bar{X}$  representing the modulus of difference between motion parameter values  $\hat{X}$  obtained by simulation and the values of parameters  $\bar{X}$  in real processes depends on the following two factors. On the one hand, it is determined by the degree of accuracy of modelling (in a specific case, it is a transfer function  $W(p)$ ), and on the other hand, it depends on the level of external disturbances, for example, atmospheric turbulence.

Relations of airplane characteristics as part of the transfer function  $W(\bar{X}, p)$  can depend on parameter values in real processes; they can be known quite well in one domain and can be determined only with a big error in another domain. Besides, if the

reference model is linear in relation to control, then at large control disturbances  $\bar{\delta}, \dot{\bar{\delta}}$  its adequacy can also be upset.

Therefore, the accuracy of description of the controlled flight by means of a transfer function  $W(\bar{X}, p)$  can depend on both the current status of airplane dynamics and the level of control inputs. Thus, the error value of the reference model  $\delta\bar{X}$  can change as a function of  $\bar{X}$  and  $\bar{\delta}$ .

The reference model should possess a number of properties, with the main following:

a) The model should be simple. Its description should depend whenever possible on the minimum number of parameters and characteristics of the controlled object.

б) At the same time, the reference model should describe the behaviour of the controlled object sufficiently and reliably, so that in the operational region (area) the difference between the response of the object  $\bar{X}$  and its evaluation  $(\bar{\delta}, \bar{X})$  to the same disturbance  $\bar{\delta}$  at all possible modes (in the whole expected domain of  $\delta\bar{X}$  variation) should be minimum, i.e.:

$$\delta\bar{X} = \left| \bar{X} - \hat{\bar{X}} \right| \rightarrow \min. \quad (2.2)$$

Both discussed requirements relating to the reference model are contradictory. Simplification of the model is inevitably followed by the deterioration of relationship between the variation of parameters  $\hat{\bar{X}}$  obtained during simulation of dynamics of the object and variations of its parameters in real processes  $\bar{X}$ . In other words, simplification of the model results in degradation of its accuracy.

The difference  $\delta\bar{X}$  between  $\bar{X}$  and  $\hat{\bar{X}}$  can be limited from top (above) by  $\Delta\bar{X}$  value. By its physical sense,  $\Delta\bar{X}$  value is determined by both internal and external factors relative to the object, which were not taken into account.  $\Delta\bar{X}$  parameter determines an admissible level of nonconformity between the reference (ideal) behaviour of the adjustable object (airplane) and its real behaviour.

The algorithms for diagnostics of critical flight conditions related to the loss of control are expected to be developed on the basis of the following principle:

A simple reference model is built, which at small angles of attack, with no failures (as will be shown below on concrete examples) can describe the dynamics of the controlled motion of an airplane accurately enough. This fact is corroborated by the results of mathematical simulation with the use of data obtained in full-scale flight tests.

If harmful cross inertial, kinematic and aerodynamic moments, as a rule arising with increasing angles of attack or as a result of failures of one of the engines, or partial destruction of the structure, are small, the pilot virtually does not notice them. Control is facile for the pilot: the airplane quickly and precisely responds to control inputs; and the airplane can be easily controlled. In other words, the airplane adequately responds to controls deflections and its behaviour is close to the reference behaviour.

With the increased values of „harmful“ moments hindering control, it becomes more and more difficult for the pilot to overcome them. The behaviour of an airplane „deteriorates“ and increasingly differs from the reference behaviour. If the mismatch between the response of an airplane  $\bar{X}$  and the evaluation of this response  $\hat{\bar{X}}$  (according to the reference model) exceeds the value bigger than  $\Delta\bar{X}$ , the level of mismatch

between the real and reference behaviour of an airplane can be considered the loss of control.

Therefore, we shall understand normal (i.e. controlled) airplane behaviour as:

$$\widehat{X} - \Delta\bar{X} < \bar{X} < \widehat{X} + \Delta\bar{X} \quad (2.3)$$

It is necessary to note that the most reliable functioning of such an algorithm for identifying the onset of the loss of control can be achieved by an appropriate selection of the value of maximum admissible mismatch of airplane dynamics to the reference sample  $\Delta\bar{X}$ , which can generally depend on both  $\widehat{X}$  and  $\bar{\delta}$   $\Delta\bar{X} = \Delta\bar{X}(\bar{X}, \bar{\delta})$ . Here, the reliable functioning of an algorithm should be understood as the absence of malfunctioning.

The selection of  $\Delta\bar{X}$  value can be based upon the results of theoretical analysis and practical experience and it follows from:

- Generalization and understanding of flight test results, including expert assessments of flight personnel;
- Analysis of peculiarities of airplane dynamics in flight modes accompanied by the loss of control according to both flight test results and mathematical and bench simulation results.

Thus, algorithms for diagnostics of flight modes related to the loss of control can be based upon the following three principles:

- 1) The selected reference model representing airplane response to control  $\bar{\delta}$  input familiar to the pilot is included into algorithms of on-board automatics. Controls deflection and/or some motion parameters (if necessary) are supplied to the model input. Airplane response  $\widehat{X}$  is assessed at the output.
- 2) The comparison of airplane actual behaviour  $\bar{X}$  with its reference behaviour  $\widehat{X}$  is carried out continuously. The onset of the loss of control is understood as the moment when the difference between  $\widehat{X}$  and  $\bar{X}$  begins exceeding, by modulus, some critical value  $\Delta\bar{X}$  which can generally depend on current aircraft behaviour  $\bar{X}$  and current control  $\bar{\delta}$ .
- 3) Selection of the value  $\Delta\bar{X}(\bar{X}, \bar{\delta})$  should be based on analysis of mathematical and scaled-down simulation, flight tests and expert evaluations of pilots.

In the present paper, the main attention is paid to the development of algorithms for diagnostics of stall modes related to the loss of control of an airplane in lateral motion, roll and yaw.

## ***2.2. Algorithms of Warning about Control Degradation and Control Loss in Lateral Motion (Taking into Account Roll/Yaw Interaction Coupling)***

In order to develop the algorithms for diagnostics of critical modes related to the loss of control, it is suggested to take into account the interaction of movements in roll and yaw channels (coupling of roll and yaw motion) in reference model development.

### ***2.2.1. Principles for Development of Diagnostic Algorithms***

It is necessary to note that the selection of the reference model representing airplane dynamics significantly influences reliable operation of algorithms of warning about the loss of control. As it has already been noted above, the reference model is the formal description of the relationship between the airplane behaviour and the deflection of controls familiar to the pilot [3 – 5].

Let's analyze the equations describing the isolated lateral motion of the airplane:

$$\left\{ \begin{array}{l} \dot{p} = \frac{P_d S l}{J_x} c_{l\Sigma}; \\ \dot{r} = \frac{P_d S l}{J_z} c_{n\Sigma}; \\ \dot{\beta} = p \sin \alpha - r \csc \alpha + C_S^\beta \beta; \\ \dot{\varphi}_V = p \cos \alpha + r \sin \alpha, \end{array} \right. \quad (2.4)$$

where

$$c_{l\Sigma} = c_l^{\bar{p}} \bar{p} - c_l^{\bar{r}} \bar{r} + c_l^\beta \beta + c_l^{\Delta\delta_h} \Delta\delta_h + c_l^{\delta_a} \delta_a + c_l^{\delta_r} \delta_r$$

$$c_{n\Sigma} = c_n^{\bar{p}} \bar{p} - c_n^{\bar{r}} \bar{r} + c_n^\beta \beta + c_n^{\Delta\delta_h} \Delta\delta_h + c_n^{\delta_a} \delta_a + c_n^{\delta_r} \delta_r.$$

Analysis of the given equations (2.4) shows that the loss of control over roll and yaw rates  $p$  and  $r$  should result in an uncontrollable change of roll angle  $\gamma$  and side slip angle.

The following should be taken as the basis for the development of a reference model:

- $\dot{\varphi}$  angle in this case is not a phase variable;
- the third equation of the system (2.4) describing variation of a side slip angle  $\beta$  is used as an auxiliary equation in the solution of the given system in the operator kind ( $p = d/dt$ ) in order to avoid the use of the side slip angle  $\beta$  as a parameter along with control inputs;
- the choice of roll and yaw rates as the controlled parameters meets the requirements for sufficiency and nonredundancy.

Record the chosen equations in the operator form:

$$\left\{ \begin{array}{l} \left[ -\frac{J_x}{P_d S l} p + c_l^{\bar{p}} \frac{1}{2V} \right] p - c_l^{\bar{r}} \frac{1}{2V} r + c_l^\beta \beta = -c_l^{\delta_a} \delta_a - c_l^{\delta_r} \delta_r - c_l^{\Delta\delta_h} \Delta\delta_h; \\ \left[ -\frac{J_z}{P_d S l} p - c_n^{\bar{r}} \frac{1}{2V} \right] r + c_n^{\bar{p}} \frac{1}{2V} p + c_n^\beta \beta = -c_n^{\delta_a} \delta_a - c_n^{\delta_r} \delta_r - c_n^{\Delta\delta_h} \Delta\delta_h; \\ p \sin \alpha - r \cos \alpha - p\beta = 0. \end{array} \right. \quad (2.5)$$

and having resolved the given system relative to  $p$ ,  $r$ ,  $\beta$  variables, we get the expressions for roll and yaw rates  $\hat{p}$  and  $\hat{r}$  in the following form:

$$\hat{p} = \sum_{i=1}^3 \frac{T_{x_l}^{\delta_i} p^2 + T_x^{\delta_i} p + A_x^{\delta_i}}{T_1 p^3 + T_1 p^2 + T_0 + A_0} \delta_i,$$

$$\hat{r} = \sum_{i=1}^3 \frac{T_{z_l}^{\delta_i} p^2 + T_z^{\delta_i} p + A_z^{\delta_i}}{T_1 p^3 + T_1 p^2 + T_0 + A_0} \delta_i, \quad (2.6)$$

where  $\delta_1 = \Delta\delta_h$ ,  $\delta_2 = \delta_a$ ,  $\delta_3 = \delta_r$  – the control surface deflections, and  $l$  – the wing span,  $V$  – the airspeed,  $P_d$  – the dynamic pressure.

$T_2, T_1, T_0, T_{x_l}^{\delta_i}, T_{y_l}^{\delta_i}, T_x^{\delta_i}, T_y^{\delta_i}, A_{x_l}^{\delta_i}, A_{y_l}^{\delta_i}, A_o$  coefficients are computed during system (2.5) solution and are generally the functions of the angle of attack  $\alpha$ , the airspeed ( $V$ ), Mach number ( $M$ ) and the air density at current altitude  $\rho$  ( $H$ ).

It is necessary to note that for the reference model development, it is possible to confine oneself to aperiodic form of describing dynamics of the airplane controlled motion that is basically a justified procedure; in addition, it considerably simplifies the reference model structure. In fact, as the results of mathematical modelling have demonstrated (see Fig. 1), it is possible to neglect the members of the order of magnitude higher than the first. Then the equation (2.6) becomes:

$$\begin{aligned} \hat{p} &= \sum_{i=1}^3 \frac{T_x^{\delta_i} p + A_x^{\delta_i}}{T_0 p + A_0} \delta_i; \\ \hat{r} &= \sum_{i=1}^3 \frac{T_z^{\delta_i} p + A_z^{\delta_i}}{T_0 p + A_0} \delta_i \end{aligned} \quad (2.7)$$

where  $\delta_1 = \Delta\delta_h$ ,  $\delta_2 = \delta_a$ ,  $\delta_3 = \delta_r$   
or in more detail:

$$\begin{aligned} T_0 &= \frac{l^2}{4V^2} (c_l^{\bar{r}} c_n^{\bar{p}} - c_l^{\bar{p}} c_n^{\bar{r}}) - \frac{J_x}{P_d S l} c_n^{\beta} \cos \alpha - \frac{J_x}{P_d S l} c_l^{\beta} \sin \alpha; \\ A_0 &= \frac{l}{4V} (c_n^{\beta} c_l^{\bar{r}} \sin \alpha - c_n^{\beta} c_n^{\bar{p}} \cos \alpha + c_l^{\beta} c_n^{\bar{p}} \cos \alpha - c_l^{\beta} c_l^{\bar{r}} \sin \alpha); \\ A_x^{\delta_i} &= (c_l^{\delta_i} c_n^{\beta} - c_n^{\delta_i} c_l^{\beta}) \cos \alpha; \\ T_x^{\delta_i} &= (c_l^{\delta_i} c_n^{\bar{r}} - c_n^{\delta_i} c_l^{\bar{r}}) \frac{l}{2V}; \\ A_x^{\delta_i} &= (c_n^{\delta_i} c_l^{\beta} - c_l^{\delta_i} c_n^{\beta}) \sin \alpha; \\ T_z^{\delta_i} &= (c_n^{\delta_i} c_l^{\bar{p}} - c_l^{\delta_i} c_n^{\bar{p}}) \frac{l}{2V}. \end{aligned}$$

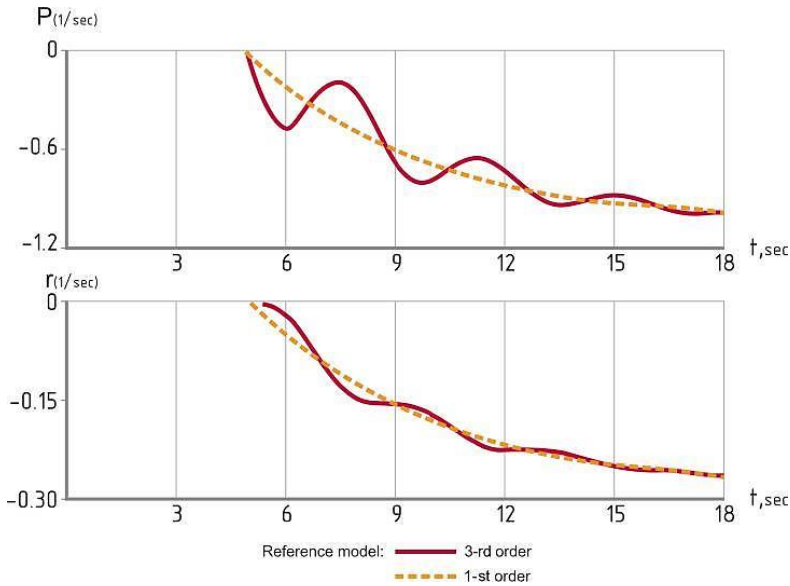


Fig. 1 Results of mathematical modelling to choose reference model structures (airplane roll response to step aileron input– 3).



By expressing transfer function coefficients through controllability and stability criteria of the linear stall theory [22], we obtain:

$$\begin{aligned}
 T_o &= \frac{l^2}{4V^4} \sigma_\omega(\alpha) + \frac{J_x}{P_{dSI}} \sigma_\beta(\alpha); \\
 A_o &= \frac{l}{2V} \tilde{\lambda}; \\
 A_x^{\delta_i} &= \sigma_{\delta_i} \cos \alpha; \\
 A_z^{\delta_i} &= -\sigma_{\delta_i} \sin \alpha; \\
 T_x^{\delta_i} &= (c_l^{\delta_i} c_n^{\bar{r}} - c_n^{\delta_i} c_l^{\bar{r}}) \frac{l}{2V}; \\
 T_z^{\delta_i} &= (c_n^{\delta_i} c_l^{\bar{p}} - c_l^{\delta_i} c_n^{\bar{p}}) \frac{l}{2V},
 \end{aligned}
 \tag{2.8}$$

where:

$\sigma_\beta$  – the nose slice parameter showing dynamic directional stability;

$\bar{\omega}$  – the criterion of dead-beat stability in lateral motion, introduced in the paper [20];

$\sigma_{\delta_i}$  – the parameters of lateral motion controllability.

$\tilde{\lambda}$  – the Numerator in the expression of the roll radical.

The expressions (2.8) allow to get the additional evidence of the appropriateness of the chosen approach used for selecting the reference model. The linearized system (2.4) in its right parts has saved the criteria used for stability and controllability analysis within the limits of the linear theory of stall [22] ( $\sigma_\beta, \sigma_\omega, \sigma_{\delta_n}, \sigma_{\delta_y}, \sigma_{\delta_{\Delta\varphi}}, \tilde{\lambda}$ ).

The analysis of the dependence of coefficients of system transfer members (2.7) on the angle of attack for some particular airplanes demonstrated some instability regions; this is unacceptable from the point of view of using the system (2.7) as a reference model, supposed to serve for formal description of an airplane as a regulated object. To avoid this, it is possible to impose some limitations upon  $T_o, A_o, A_x^{\delta_i}, A_y^{\delta_i}, T_x^{\delta_i}, T_y^{\delta_i}$  coefficients. The essence (or the physics) of such limitations can be reduced to meeting stability and controllability conditions according to criteria of stall linear theory (see Fig. 2), i.e.:

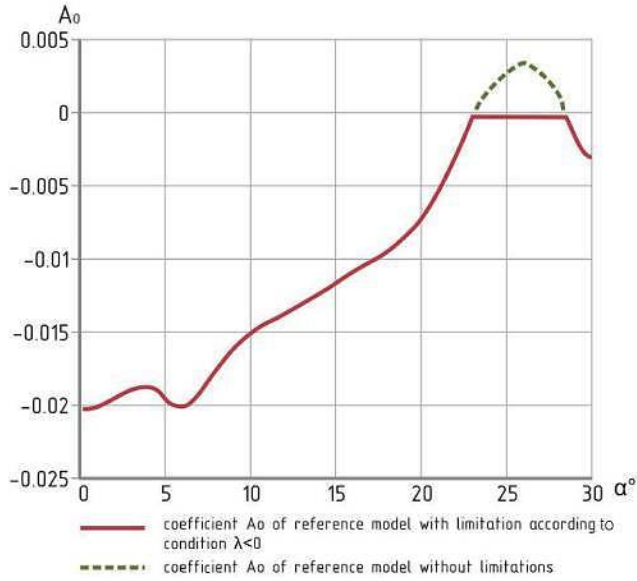


Fig. 2 Example of reference model coefficient limitation based on meeting criteria of stall linear theory

$$\begin{aligned} \sigma_{\beta} < 0; \sigma_{\omega} < 0; \tilde{\lambda} < 0; \\ \sigma_{\delta_H} < 0; \sigma_{\delta_3} < 0; \sigma_{\delta_{\Delta\varphi}} < 0. \end{aligned} \quad (2.9)$$

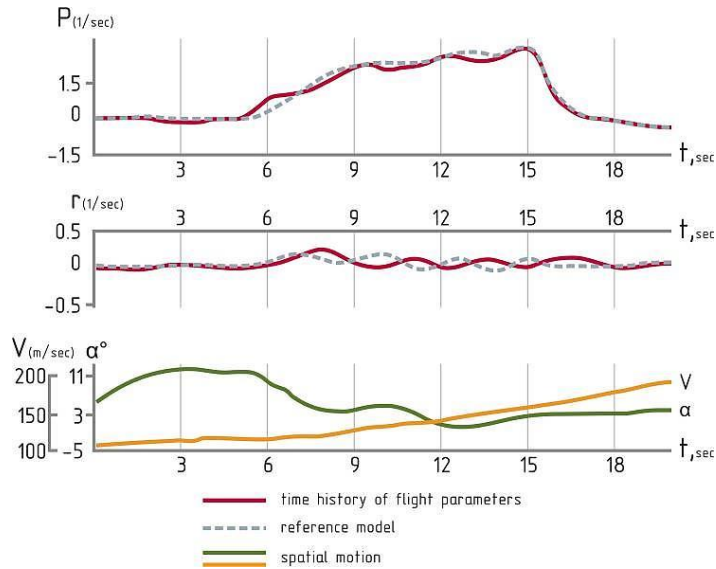
As the system stability analysis demonstrates (2.7), there are no instability regions provided the conditions (2.9) are satisfied.

Thus, the adequate behaviour of a particular airplane can be formally described as follows:

$$\begin{aligned} \hat{p}, \hat{r} &= \sum_{i=1}^3 \frac{T_{x,y}^{\delta_i} p + A_{x,y}^{\delta_i}}{T_0 p + A_0} \delta_i; \\ \tilde{T}_0 &= T_0(V, \sigma_{\beta}(\alpha), \sigma_{\omega}(\alpha)); \\ \tilde{A}_0 &= A_0(V, \tilde{\lambda}(\alpha)); \\ \tilde{T}_{x,z}^{\delta_i} &= T_{x,y}^{\delta_i}(V, \alpha); \\ \tilde{A}_{x,z}^{\delta_i} &= A_{x,z}^{\delta_i}(\sigma_{\delta_i}(\alpha)); \\ \sigma_{\beta} &> 0; \\ \sigma_{\omega} &< 0; \\ \tilde{\lambda} &< 0; \\ \sigma_{\delta_i} &> 0; i = 1, 2; \sigma_{\delta_3} < 0. \end{aligned} \quad (2.10)$$

In order to assess how accurately the reference model (2.10) represents the airplane dynamics during controlled flight at low angles of attack, the mathematical

modelling was performed. The analysis of its results has shown that with the same control inputs the difference between the reference behaviour and the airplane behaviour, which was modelled by means of complete equations of spatial motion, significantly depends on intensive longitudinal manoeuvring. Thus, during simulation of airplane rolling rotation with pitch control stick in fixed position, the difference in roll and yaw rates in the reference and the airplane model is small and does not exceed 15 % (see Fig. 3).



*Fig. 3 Mathematical modelling – assessment of airplane response with use of reference model, which does not take into account inertia coupling (without manoeuvring in longitudinal motion)*

During the simulation of roll rotation, when the pilot increases angle of attack by pulling the control stick, the standard assessments of roll and yaw rates already differ 40 % from roll and yaw rate values, gained as a result of complete equation integration (see Fig. 4). Such difference is big enough and does not allow to assert that the reference model (2.10) represents the airplane response to stick forces well enough. As was mentioned above, the bigger difference between roll and yaw rates  $p$  and  $r$  and their standard assessments  $\hat{p}$  and  $\hat{r}$  is directly related to the longitudinal motion.

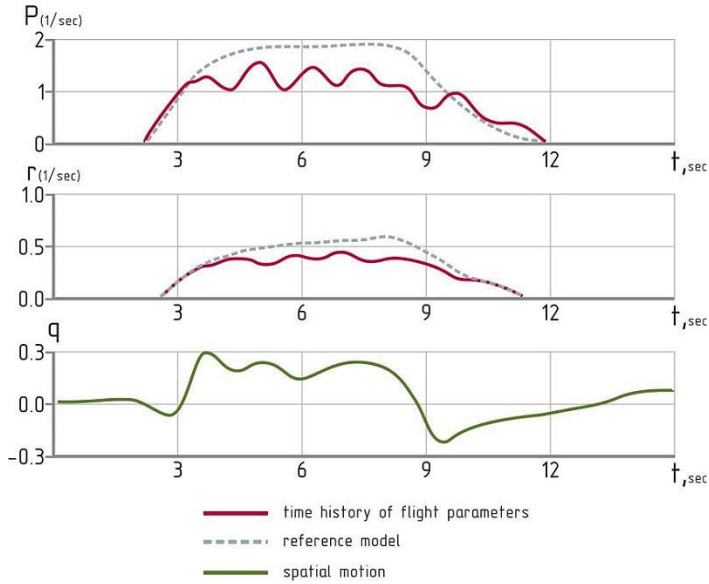


Fig. 4 Mathematical modelling – assessment of airplane roll response using reference model taking into account and not taking into account inertia coupling (with manoeuvring in longitudinal motion)

Pitch rate value  $q$ , implemented in the course of angle of attack increase (see Fig. 4), determines the value of inertia moments in roll and yaw, thereby significantly influencing lateral motion parameters of an airplane. Thus, the conclusion is made about the need to take account of inertia coupling in the development of a reference model.

Let's try to improve the reference model. For this purpose, we use the equations (2.4) and add the terms which take into account inertia coupling of longitudinal and lateral motion [3 – 5]:

$$\left. \begin{cases} \dot{p} = \frac{P_d S l}{J_x} c_{l\Sigma} - \frac{J_z - J_y}{J_x} q r; \\ \dot{r} = \frac{P_d S l}{J_z} c_{n\Sigma} - \frac{J_y - J_x}{J_z} q p; \\ \dot{\beta} = p \sin \alpha - r \cos \alpha; \\ \dot{\phi} = p, \end{cases} \right\} (2.11)$$

$$\text{where } \begin{aligned} c_{l\Sigma} &= c_l^{\bar{p}} \bar{p} - c_l^{\bar{r}} \bar{r} + c_l^{\beta} \beta + c_l^{\Delta\delta_h} \Delta\delta_h + c_l^{\delta_a} \delta_a + c_l^{\delta_r} \delta_r; \\ c_{n\Sigma} &= c_n^{\bar{p}} \bar{p} - c_n^{\bar{r}} \bar{r} + c_n^{\beta} \beta + c_n^{\Delta\delta_h} \Delta\delta_h + c_n^{\delta_a} \delta_a + c_n^{\delta_r} \delta_r; \end{aligned}$$

Having replicated the considerations presented during development of the reference model (2.10) underlain by the set of equations (2.4), we obtain the formulas for parameters of the reference model which takes into account inertia coupling:

$$\begin{aligned}
 \hat{p} &= \sum_{i=1}^3 \frac{T_x^{\delta_i} p + A_x^{\delta_i}}{T_0 p + A_0} \delta_i; \\
 \hat{r} &= \sum_{i=1}^3 \frac{T_z^{\delta_i} p + A_z^{\delta_i}}{T_0 p + A_0} \delta_i,
 \end{aligned}$$

where  $\delta_1 = \Delta\delta_h$ ,  $\delta_2 = \delta_a$ ,  $\delta_3 = \delta_r$ .

$$\begin{aligned}
 \bar{T}_0 &= \tilde{T}_0 + \frac{(J_z - J_y)(J_y - J_x)}{(P_d S l)^2} q^2 + \frac{q}{2P_d S l} \left( (J_y - J_x) c_l^{\bar{r}} + (J_z - J_y) \right) c_n^{\bar{p}}; \\
 \bar{A}_0 &= \tilde{A}_0 + \frac{q}{P_d S l} (J_z - J_y) c_n^{\beta} \sin \alpha + (J_y - J_x) c_l^{\beta} \cos \alpha; \\
 \bar{A}_{x,z}^{\delta_i} &= \tilde{A}_{x,z}^{\delta_i}; \\
 \bar{T}_x^{\delta_i} &= \tilde{T}_x^{\delta_i} - c_n^{\delta_i} \frac{(J_z - J_y)}{P_d S l} q; \\
 \bar{T}_z^{\delta_i} &= \tilde{T}_z^{\delta_i} - c_l^{\delta_i} \frac{(J_y - J_x)}{P_d S l} q.
 \end{aligned}
 \tag{2.12}$$

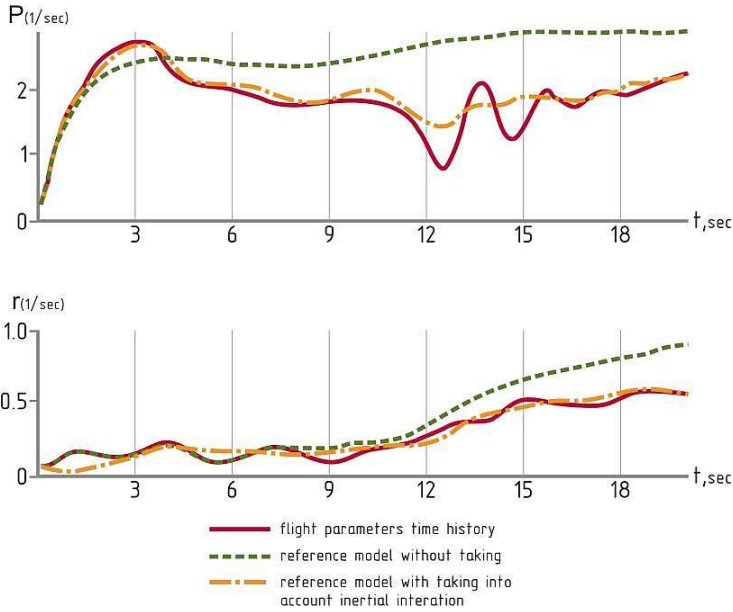
Let's impose restrictions upon  $\bar{T}_0$ ,  $\bar{A}_0$ ,  $\bar{A}_{x,y}^{\delta_i}$ ,  $\bar{T}_{x,y}^{\delta_i}$  coefficients, which would ensure stability of this system:

$$\begin{aligned}
 \frac{\bar{T}_0}{\bar{A}_0} &> 0; \\
 \left| \frac{\bar{T}_0}{\bar{A}_0} \right| &> \left| \frac{\bar{T}_{x,z}^{\delta_i}}{\bar{A}_0} \right|;
 \end{aligned}
 \tag{2.13}$$

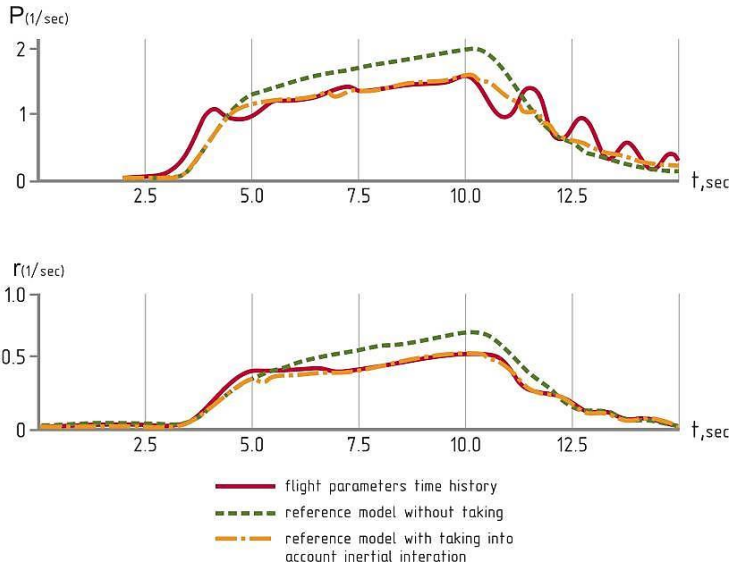
or  $\bar{T}_0 < 0$ ,  $\bar{A}_0 < 0$ ,  $|\bar{T}_0| > \left| \bar{T}_{x,z}^{\delta_i} \right|$ , and requirements of control adequacy:

$$\bar{A}_x^{\delta_a} > 0; \bar{A}_z^{\delta_r} > 0; \bar{T}_x^{\delta_h} > 0.
 \tag{2.14}$$

Thus, as mathematical modelling results have shown, provided the requirements (2.13) and (2.14) are met, the reference model (2.12) chosen anew allows to achieve a very good relationship between roll and yaw rates ( $p$  and  $r$ ) and their reference (ideal/standard) assessments ( $\hat{p}$  and  $\hat{r}$ ) in a broad range of flight conditions in different manoeuvres (see Figs. 5 and 6).



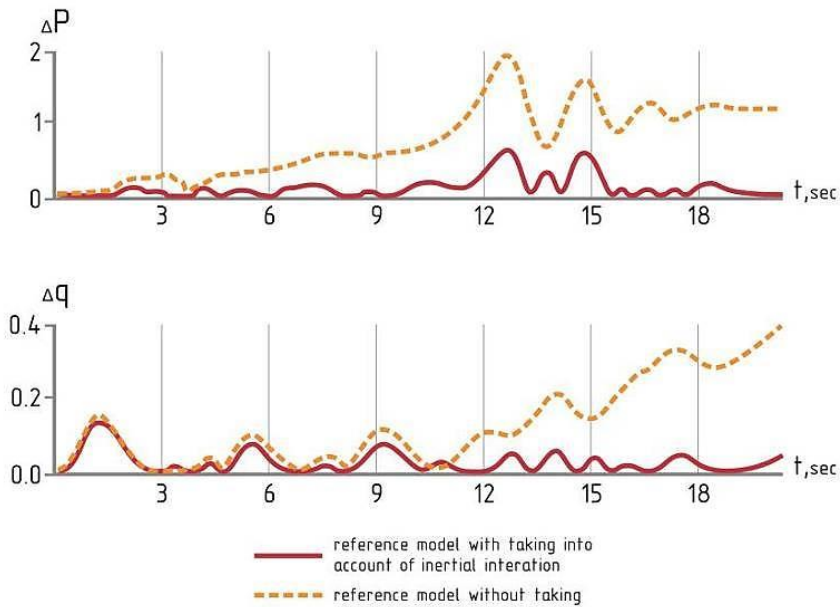
*Fig. 5 Mathematical modelling – comparison of parameters fidelity of various reference models (with and without account of inertia coupling) to airplane spatial motion parameters modelled according to complete equations (barrel-roll)*



*Fig. 6 Mathematical modelling – comparison of fidelity of parameters of various reference models (with and without account of inertia coupling) to airplane spatial motion parameters modelled according to complete equations (step input with control stick returned into neutral roll position)*

It is rather difficult to present, within the context of this paper, all simulated flight modes discussed to compare the adequacy of suggested reference models (2.10) and (2.12). The body of information is too large to be an obvious and accurate evidence of the above conclusions. Therefore, it is necessary to develop the criteria-based approach to the assessment and comparison of reference models.

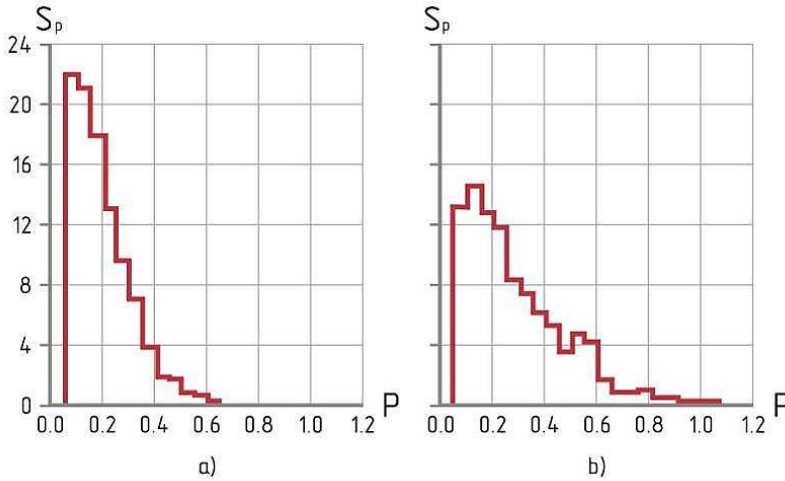
It should be mentioned that the mismatch value between the actual response of airplane model to controls deflections and its assessment using reference models (2.10) and (2.12) depends on flight parameters, positions of controls and time in a complicated way (see Fig. 7). If  $\omega_0$  value is set in advance, it is possible to choose time intervals  $[t_k, t_{k+1}]$ , during which the mismatch is in any of intervals. Thus, the obtained information can be presented as a diagram which shows the portion of the modelled flight (time) with mismatch on the interval present  $[n\omega_0, (n + 1)\omega_0]$ .



*Fig. 7 Mathematical modelling of roll rotation – difference of reference model parameters (with and without account of inertia coupling)*

Such representation of obtained information allows to demonstrate the difference and advantages of the discussed reference models on the examples of both of the modelled flight modes and the whole range of modes.

In Figs. 8a, b and 9a, b, the graphs plotted according to the above procedure are presented. The values 0.05 rad/sec and 0.01 rad/sec, respectively, are chosen as mismatch intervals between roll and yaw rates of the airplane mathematical model and their assessment obtained using reference models (2.10) and (2.12). The graphs are based on a large number of mathematical modelling results of different flight modes at high angles of attack (50 modes) with manoeuvring during lateral motion (such as coordinated left and right rolls, „S-turn“ type modes, straight flights with slide) at various initial conditions.



*Fig. 8 Bar graph of mismatch allocation between current roll rate and its standard assessment in time, based on the results of processing of mathematical modelling of modes with manoeuvring in roll motion at high angles of attack (sample size - 50 modes, mismatch intervals - 0,05 rad/sec)*

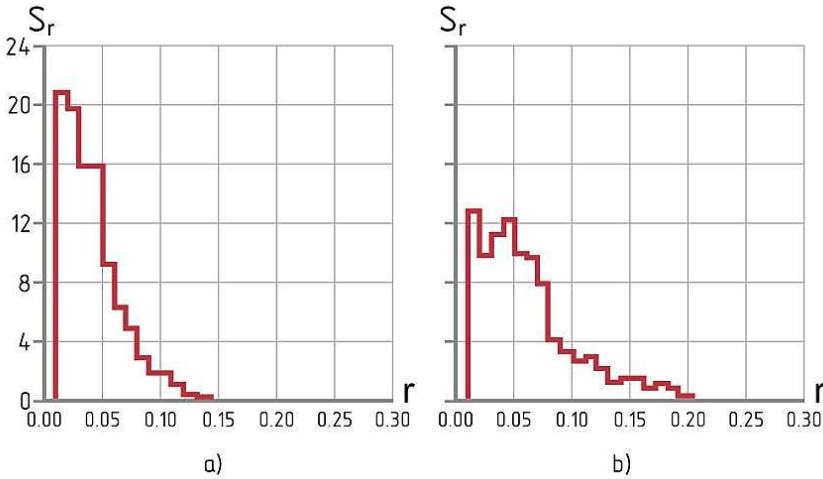
Simultaneously, the assessment of airplane response to control deflections using reference models which have accounted (2.12) and have not accounted (2.10) inertia coupling was calculated. From Fig. 8a it is clear that the mismatch between roll rate and its assessment according to the reference model (2.12) which takes into account inertia coupling, does not exceed the value of 0.4 rad/sec during the main part of time (96 %), and only 6 % of time of all mismatch modes falls into the interval between 0.4 and 0.6 rad/second. The mismatch between roll rate and its assessment according to the reference model (2.10) which does not take into account inertia coupling (see Fig. 8b) and for the main part of time (20 %) lies in the range between 0.4 and 0.6 rad/sec and, in addition, about 6 % of mismatch time falls into the interval from 0.6 to 1.0 rad/sec, which is unacceptable for the concept of assessment of airplane response adequacy to control inputs, since the mismatch of 0.6 – 1.0 rad/sec is comparable with roll rates attained in the course of vigorous manoeuvring in lateral motion.

The qualitative pattern for mismatch between yaw rate and its assessment according to the reference models accounting and not accounting inertia coupling, is similar (see Fig. 9). It is necessary to note that such allocation of mismatch in time is approximately invariant, for both the whole set of considered flight modes and each mode separately.

Therefore, this procedure confirms the conclusion that the reference model (2.12), which takes into account inertia coupling, is more accurate with controllability in lateral motion present.

Along with the selection of the reference model, the determining reliability factor in the system for identifying the moment of the loss of control is the selection of acceptable divergence of airplane parameters from standard (values of acceptable mismatch), i.e. the problem of boundary selection.





*Fig. 9 Bar graph of mismatch allocation between current yaw rate and its standard assessment in time, based upon the results of full-scale flight test data processing with roll manoeuvring at large angles of attack (sample size - 50 modes, mismatch intervals - 0.01 rad/sec)*

The starting point for selecting quantitative assessment of acceptable boundary value can be the above mentioned procedure of mismatch allocation according to the time period of a modelled flight mode. In this case (see Fig. 8a), at the angles of attack less than acceptable, during the main time of modelled flight, the mismatch between roll rate and its assessment according to the reference model (2.12) does not exceed the value of 0.4 rad/sec, for yaw rate this value is (see Fig. 9a) 0.15 rad/second. Thus, if the mismatch exceeds the specified values, it is possible to speak about disturbed adequacy of airplane response to control deflections, i.e. about the loss of control. In this document, the acceptable boundary values of mismatch were adjusted based on analysis of flight test results and expert assessments of test pilots with vast experience of high-angle-of-attack flights. The relation of values upon angle of attack is presented in Fig. 10a, 10b.

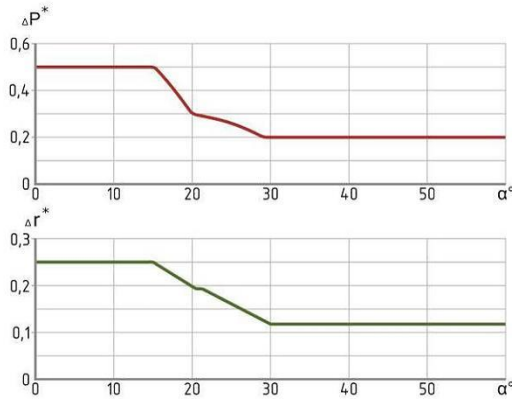


Fig. 10 Relation of acceptable boundary value of mismatch to angle of attack

**2.3. The Warning System for Loss of Lateral Control**

The warning system flow chart for the loss of control is presented in Fig. 11. The input parameters of the flow chart are pitch, roll and yaw rates, angle of attack, airspeed, rudder, ailerons and stabilizer differential deflections.

The flow chart output parameters are  $P_x$  and  $P_z$  signals, which adopt “0” or “1” values, depending on presence or lack of controllability in controlled motion channels (“0” – controllability is present, “1” – controllability is lost).

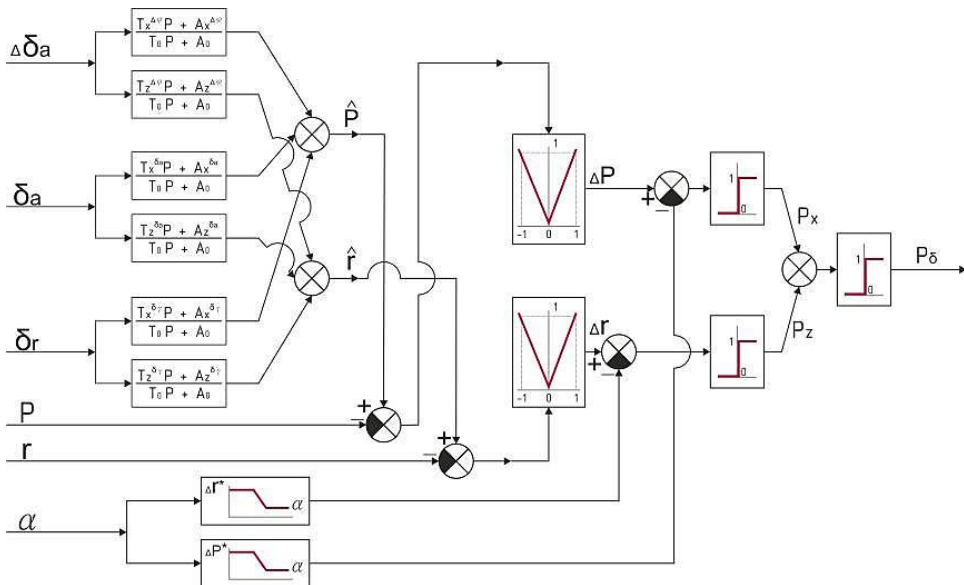


Fig. 11 Flow chart of the automatic system of airplane controllability diagnostics in lateral motion

When controllability is lost (according to the chosen procedure) at least in one of the channels, it is possible to speak about the loss of control in lateral motion, taking

into account mathematical modelling analysis, based on flight performance data.  $P_\delta$  signal characterising controllability in lateral motion takes the “1” value when at least one of  $P_x$  or  $P_z$  signals (or both simultaneously) is equal to “1”, and accordingly adopts the “0” value when both  $P_x$  and  $P_z$  signals are equal to “0”.

Thus,  $P_\delta$  parameter is the criterion of controllability in lateral motion.

### ***2.3.1. Mathematical Modelling Results for Assessment of Performance Capabilities of the Offered Warning System for Loss of Control***

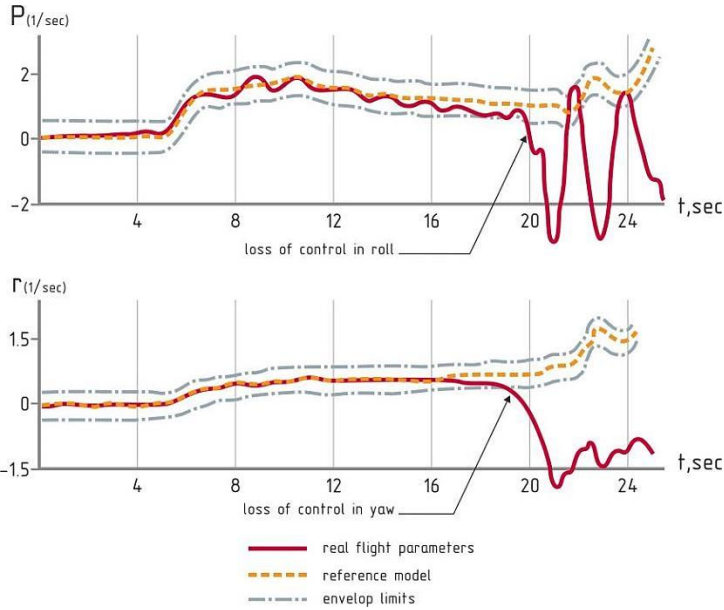
Performance capabilities and efficiency of suggested algorithms for automatic identification of the moment of the loss of control in lateral motion at high angles of attack were assessed in two ways on the computer.

In the first case, the dynamics of an agile airplane in complete equations of motion was simulated on the computer. In parallel, the algorithms of the automatic warning system were modelled. The flow chart is presented in Fig. 11. Controls deflections and the parameters of airplane motion gained as a result of dynamic equation integration, were supplied to the input of the modelled warning system.

In the second case, the airplane motion parameters and control deflections obtained during full-scale stall flight tests were supplied to the outputs of the modelled automatic warning system.

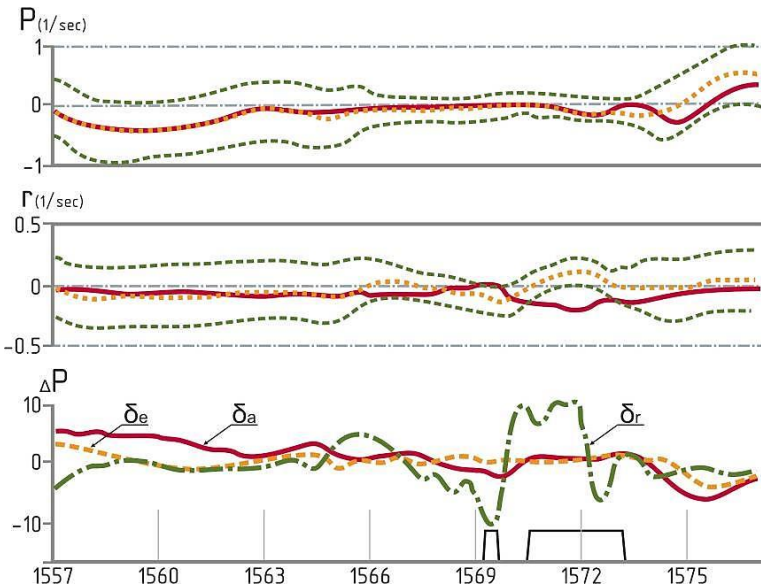
The analysis of mathematical modelling results has shown that in both cases the onset of loss of control and stall were recorded by the modelled automatic system precisely and in time.

In Fig. 12, the results of airplane dynamics modelling are presented in the mode when at the 15th second of roll rotation the stabiliser completely deflects to pitch-up. It is clear from Fig. 12 that at low angles of attack ( $\alpha < \alpha_{stall}$ ), the airplane behaviour practically does not differ from the reference one. With the increasing angle of attack, the mismatch between controlled parameters  $p$  and  $r$  and their reference assessment  $\hat{p}$  and  $\hat{r}$  grows; the moment of loss of control is fixed at the 19th second, when the difference exceeds allowable values.



*Fig. 12 Determination of the moment of loss of control during roll rotation with the increasing angle-of-attack*

In Fig. 13, the results of mathematical modelling with the use of real on-board records obtained in full-scale stall flight tests are presented.



*Fig. 13 Determination of the moment of loss of control during spatial motion*

In Fig. 14, the airplane motion parameters obtained in full-scale stall flight tests are presented. At the 1562-nd second of spatial manoeuvring, the pilot pulls the pitch

control stick and after that during the increase of the angle of attack at the 1570<sup>th</sup> second, due to occurrence of asymmetric yaw moments, the angular rate  $r$  develops; the pilot counteracts it by rudder deflection against yaw rotation, responding to  $r$ , which he sees on the electronic turn indicator.

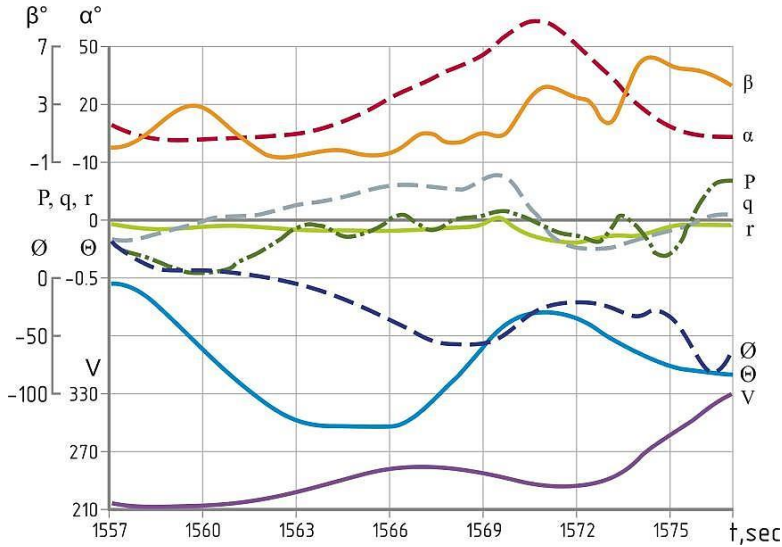


Fig. 14 Flight tests – stall mode

Addressing Fig. 13 again, we see that exactly during this moment, the difference between current yaw rate  $r$  and its reference assessment  $\hat{r}$  exceeds the acceptable value  $\Delta r$ , and the system fixes the moment of the loss of control ( $P_\delta = 1$ ).

### 3. The Use of Algorithms for Diagnostics of Loss-of-Control Modes in Airborne Systems

The developed and tested algorithms which signal about the aircraft loss of control in lateral motion during high-angle-of-attack flight, were flight-tested and they can be used for stall and spin diagnostics, and for practicing airplane control automation in manual mode in order to improve airplane immunity to stall and spin, including recovery from these modes.

The airborne system in which the suggested algorithms are used by means of director symbols and voice information reporting system, provides the pilot with the information about either the presence or the lack of control in lateral motion according to the value of control loss sign  $P_\delta = 1$ , and, depending on the direction of spontaneously developing yaw rotation of an airplane (see Fig. 15), it immediately displays to the pilot the direction to move the control stick (the pitch control stick) and pedals to prevent stall or to recover from spin.

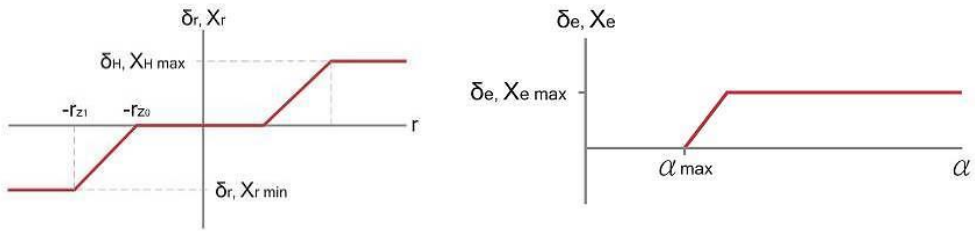


Fig. 15 The set rudder (pedals) and elevator deflections (pitch control stick)

Thus, the director information is presented to the pilot in a clear and simple way on MFD flight frame of (see Fig. 16).

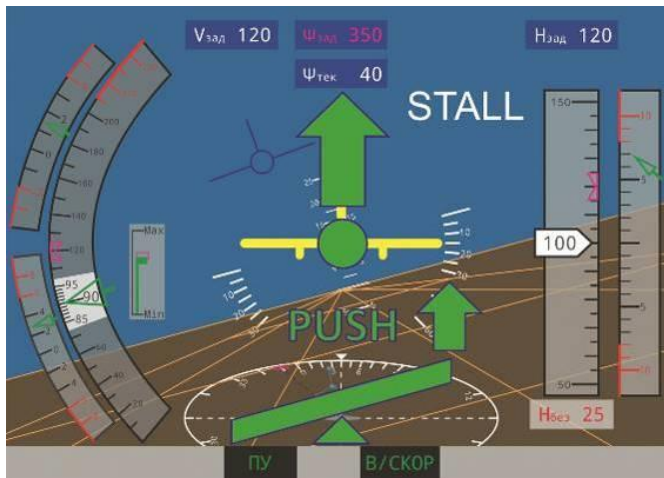


Fig. 16 The director information on MFD

#### 4. Conclusions

1. An approach to the development of algorithms for an automatic airplane loss-of-control warning system based on reference model describing airplane dynamics during controlled movement has been suggested.
2. On the basis of the suggested approach, the algorithms of an automatic loss-of-lateral-control warning system with the use of reference models which describe the airplane response to the control deflections through roll and yaw rates were developed.
3. The analysis of mathematical modelling results using real on-board records, obtained in full-scale stall flight tests, has shown that the moments of the loss of control and stall onset were fixed by modelled automation system precisely enough and in time, at all specified flight modes (over 40) at high angles of attack, accompanied by stall and spin.
4. The experimental director device intended for trying out the principles of diagnostics of loss-of-control-related critical modes in full-scale flight tests, and airplane control automation at high angles of attack in order to improve stall and spin immunity were developed.

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