



## Fuel Cell 3-D Modelling Using a Logarithmic Approximation in MATLAB<sup>®</sup> & Simulink<sup>®</sup>

K. Zaplatílek\* and J. Leuchter

*Department of Electrical Engineering, University of Defence, Brno, Czech Republic*

The manuscript was received on 11 January 2016 and was accepted after revision for publication on 16 May 2016.

### **Abstract:**

*The topic of this paper is a building process of a real fuel cell mathematical model, based on the mixed logarithmic and polynomial approximation, focused on the 3-D load characteristics. The model's input is the values of load resistance and temperature (independent variables), whereas the output is the value of voltage. The coordinates of the operating point are determined analytically, using the double approximation (both transverse and longitudinal). The resulting 3-D model is then implemented in the MATLAB (text source code) and Simulink (block model) system environments. The core of the model is a system of optimal logarithmic approximation functions. The mathematical principle of the method and a numerical example are then presented. The main idea of the paper is to offer an accurate fuel cell model for more extensive computer experiments and to verify the presented original approximation methods.*

### **Keywords:**

*Fuel cell, logarithmic and polynomial approximation, MATLAB<sup>®</sup> & Simulink<sup>®</sup>*

### **1. Introduction**

Fuel cells belong to the main alternative sources of electrical energy. Although they are still in the stage of material research their practical applications occur with an increasing rate [1 – 3]. The number of published models is relatively high [4 – 10]. The subject of this article is the Horizon H-30 fuel cell which is at the Department of Electrical Engineering one of the types used for the purposes of research and education [11, 12]. Fig. 1 shows the measured 3-D load characteristic  $V = f(R, T)$ , where  $V$  is the voltage,  $R$  is the load resistance and  $T$  is the temperature.

The measured network of characteristics was created in a special laboratory using an automatic measuring system with the Vötsch VC<sup>3</sup> 7034 thermal chamber and VEE

---

\* Corresponding author: Department of Electrical Engineering, University of Defence, Kounicova 65, CZ-662 10 Brno, Czech Republic. Phone: +420 973 442 709, Fax: +420 973 443 773, E-mail: karel.zaplatilek@unob.cz

Pro 9.3 program [12], see Fig. 3.

An independent variable can be either the electric current to the load or the value of the load resistance which is more practical for the given measurements.

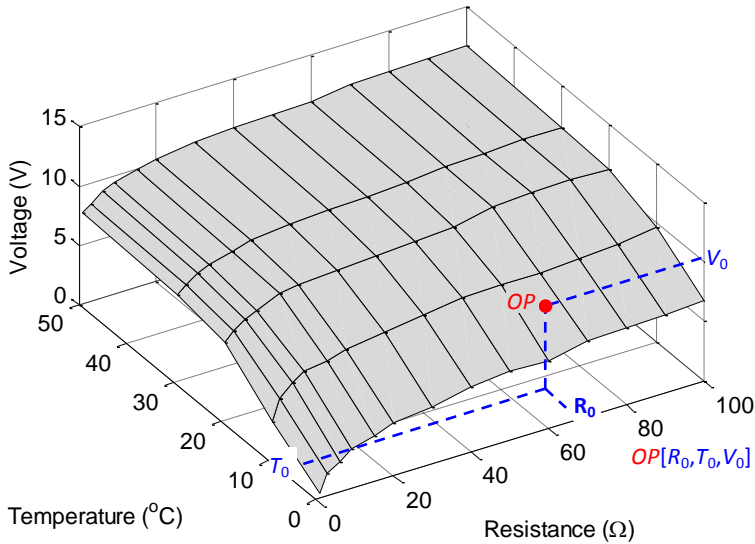


Fig. 1 Temperature profile of the load characteristic of the H-30 fuel cell

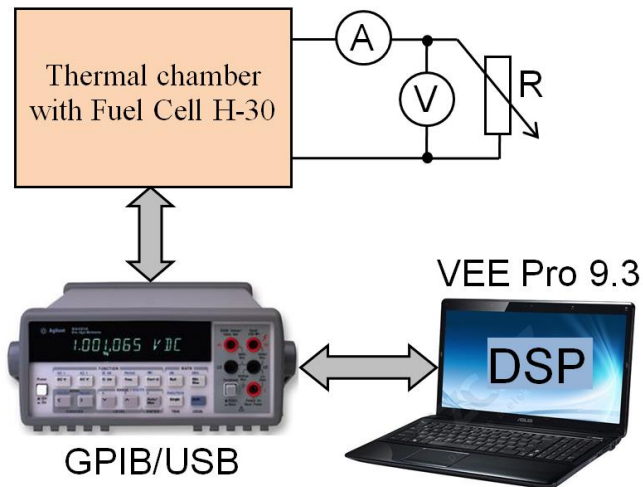


Fig. 2 Block scheme of the automated measuring station

Fig. 2 shows a simplified block scheme of the automated measuring station. Fig. 3 shows the thermal chamber and a detail of the Horizon H-30 fuel cell measured, with the accessories (output power 30 W, operating voltage (7.0 ÷ 12.6) V, operating current 4 A). The whole measuring station is unique and capable of exact temperature measuring of various devices of the size given. The actual measuring is quite

complicated in the sense of technology, mainly because of the chemical equipment (hydrogen) and the operations with temperature.

The coordinates of the operating point in Fig. 1 are given by the expression  $OP [R_0, T_0, V_0]$  where  $R_0$ ,  $T_0$  and  $V_0$  are particular values of the load resistance, temperature and voltage. The main objective of this article is to describe and implement an algorithm that enables us to do an analytical calculation of the operating point coordinates in any place of the given intervals.

## 2. Basic Principle of Model Building

Fig. 4 shows a general block scheme of the algorithm for model building in 3-D. Except the measured data, the input includes also the values of load resistance and temperature (scalar quantities). The output is the value of the voltage on the fuel cell output.



*Fig. 3 Thermal chamber of the measuring station with the H-30 fuel cell*

Fig. 5 shows more complex block scheme of the 3-D model building process. At its core, there are two processes of optimal approximations marked as VR-approximation and VT-approximation.

The output of the VR-approximation process is one optimal logarithmic function that very well approximates the course of  $V = f(R), T = const.$ , see Fig. 6. The optimal logarithm  $\ln$  expression emphasizes the actual process of finding the constants  $K_1$  and  $K_2$  of modified approximation natural logarithms in (1).

The modified optimal logarithmic function of the output can be described by the following equation:

$$V(R) = K_2 [\ln(1 + R) + K_1]. \quad (1)$$

The calculation of the  $K_1$  and  $K_2$  can be proven by the following equations (2):

$$\begin{aligned} K_1 &= V(1) - \ln[1 + R(1)], \\ K_2 &= V(end) / [K_1 + \ln(1 + R(end))], \end{aligned} \quad (2)$$

where  $V$  and  $R$  are the vectors of the measured voltage and resistance of the characteristic  $V = f(R), T = const.$

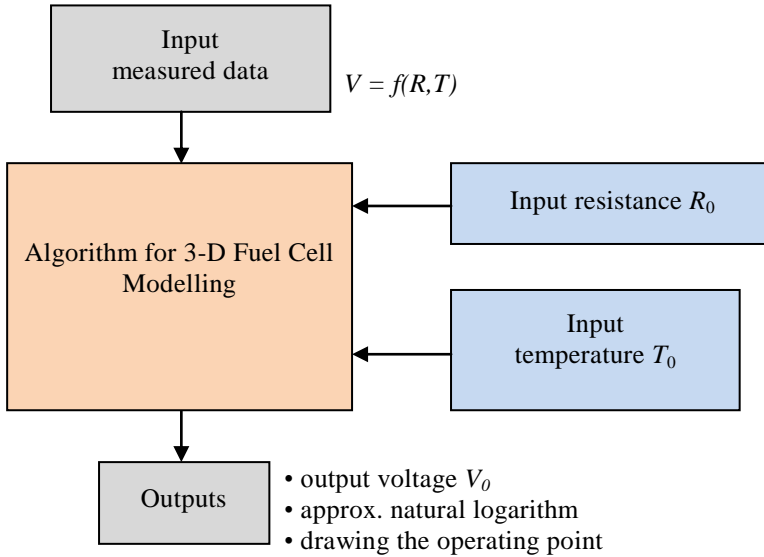


Fig. 4 Principal block scheme of the 3-D model building algorithm

The „1“ and „end“ symbols are the indexes of samples in given vectors. The equations (2) are presented without proof, which is quite simple.

The equations (2) can be used for the calculation of any measured temperature  $T_P$ . If the approximation logarithmic function (1) is known, then the coordinates of any point  $P [R_0, T_P, V_P]$  can be easily calculated thanks to (1), see Fig. 6:

$$V_P(R_0) = K_2(T_P) [\ln(1 + R_0) + K_1(T_P)]. \quad (3)$$

For the calculation of  $V_0$  coordinate of the resulting operating point, another approximation is required. This VT-approximation approximates the course of  $V = f(T), R = const.$  Its use is necessary for the continuous movement of the logarithmic

functions in the direction of the temperature axis. According to (2) it is an approximation of the  $K_1$   $K_2$  constants along the temperature.

For this type of approximation we have chosen an optimal polynomial approximation with regard to the nature of the courses. These approximations are very well done. The calculations described above have to be done for all the temperatures measured.

The result of the approximation task is then one optimal approximation polynomial for the constant  $K_1$  and another for  $K_2$  for a specific value of  $R_0$ , see Fig. 6 and (3):

$$\begin{aligned} K_1(T) &= a_n T^n + a_{n-1} T^{n-1} + \dots a_1 T + a_0, \\ K_2(T) &= b_n T^n + b_{n-1} T^{n-1} + \dots b_1 T + b_0. \end{aligned} \quad (4)$$

For a specific input temperature  $T_0$  there are new values in the fields of the constants  $K_1(T_0)$  and  $K_2(T_0)$ . These values are then along with  $R_0$  substituted into (1), so then it applies:

$$V_0(R_0, T_0) = K_2(T_0) [\ln(1 + R_0) + K_1(T_0)]. \quad (5)$$

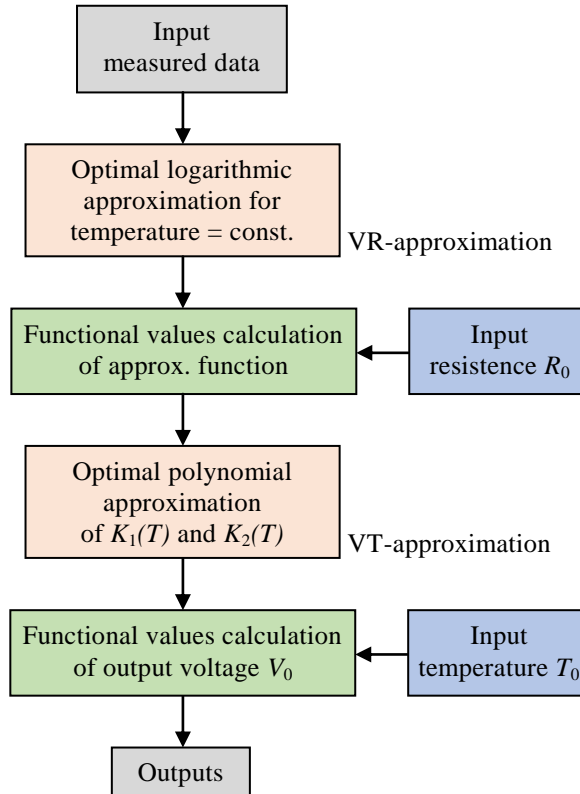


Fig. 5 Double optimal approximation algorithm in 3-D

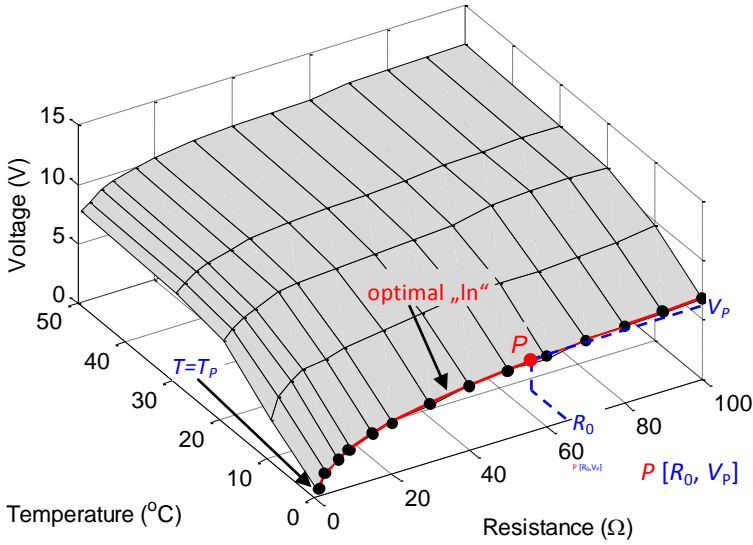


Fig. 6 Process of optimal logarithmic VR-approximation

The equation (5) determines the last coordinate of the operating point. The algorithms for the determination of the optimal degree  $n$  and for the calculation of coefficients of the  $K_1(T)$  and  $K_2(T)$  polynomials are not presented in this article. They have been presented in [4] for instance, where the Euclidean norm of residues is used as a criterion function.

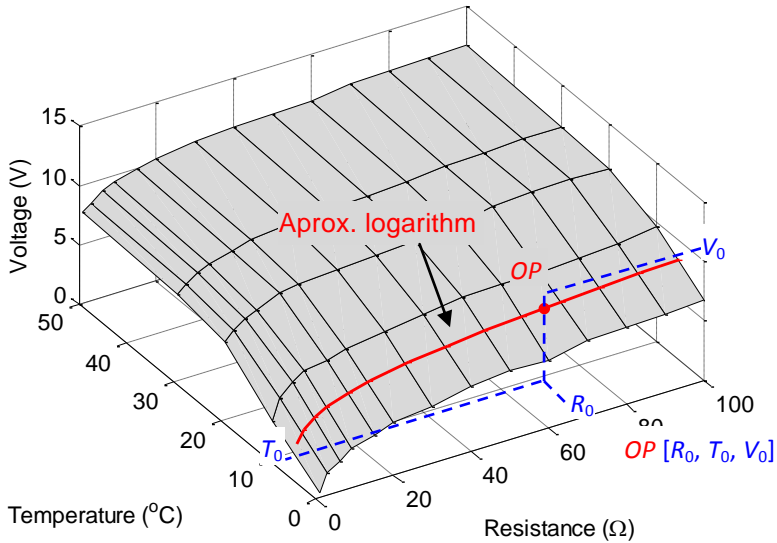


Fig. 7 Finding coordinates of operating point using VT-approximation

Fig. 7 shows the course of the approximation logarithmic function for a specific temperature  $T_0$  (beyond the values measured) along with the position of the resulting operating point, see (4).

Fig. 8 shows the result of the approximation in the plane  $V(R)$ , as well as particular numerical values of the operating point coordinates that will be used in the numerical example below, in the next section.

The whole process described in Fig. 5 was implemented to the environment of MATLAB system. The developed algorithm of model building in 3-D can be connected with the system of automatic measuring of a given characteristic for greater effectiveness. The resulting model has been tested in practice and is fully functional. For broader experiments it is more suitable to build a block model in the Simulink system on its basis. It is also possible to join the two environments and to offer a quite robust and powerful system of fuel cells modelling, i.e. the commands `get_param` and `set_param` in MATLAB.

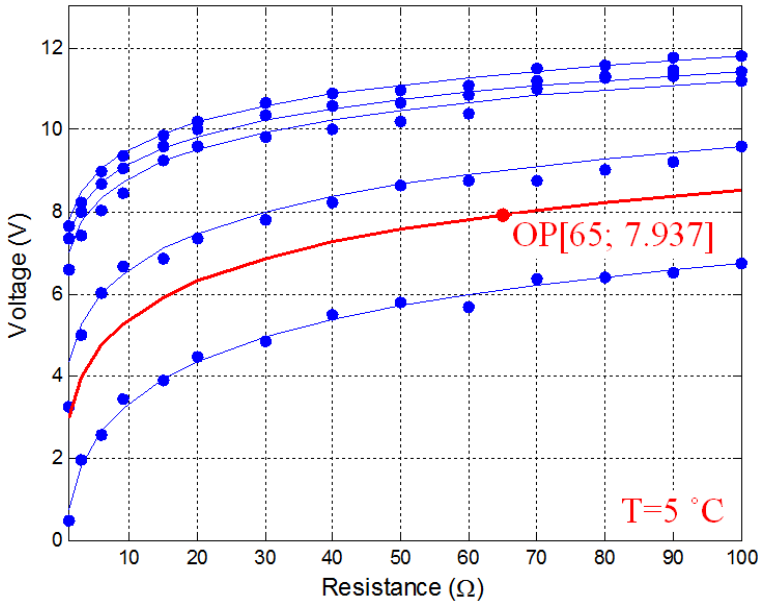
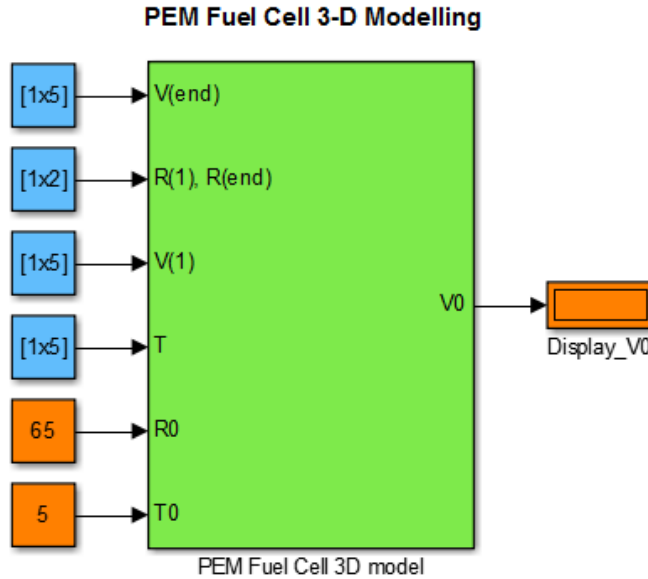


Fig. 8 Operating point coordinates in the VR-plane

### 3. Block Model in Simulink

Fig. 9 shows a block model of the whole algorithm described above in the Simulink system, as a form of subsystem.

The main input scalar quantities are the resistance  $R_0$  and the temperature  $T_0$ . The output is a particular value of the voltage  $V_0$ . Other inputs are the vectors of beginning and final values of the measured data  $V(1)$  and  $V(end)$ , the vector of measured temperatures  $T$  and beginning and final values of the vector  $R$ . The subsystem's content is presented in Fig. 10.



*Fig. 9 Block model in Simulink as a subsystem*

For this example we used five measured temperatures:

$$\begin{aligned}
 T &= [0, 10, 20, 30, 50] \text{ } ^\circ\text{C. From the measured data we found:} \\
 V(1) &= [0.48, 3.25, 6.58, 7.35, 7.65] \text{ V,} \\
 V(\text{end}) &= [6.76, 9.58, 11.21, 11.43, 11.79] \text{ V,} \\
 R(1), R(\text{end}) &= [1, 100] \text{ } \Omega.
 \end{aligned}$$

As Fig. 8 shows, we used  $R_0 = 65 \text{ } \Omega$ ,  $T_0 = 5 \text{ } \Omega$ . These values were inserted to the fuel cell model according to Fig. 9. After the activation of simulation, the output shows the calculated value of the operating point voltage coordinate  $V_0 = 7.937 \text{ V}$ , see Fig. 8 (the numerical format was left in the short implicit setting). The ODE Solver: Ode 45 (Dormand-Prince) was used. Fig. 10 shows, for comparison, a part of the algorithm listing in the MATLAB environment with the value of the constants  $K_1$  and  $K_2$  and the operating point coordinates.

Fig. 11 shows the detailed structure of the block model from Fig. 9. The violet blocks (subsystems) process the calculation of the  $K_1$  and  $K_2$  constant vector according to (2). In the numerical example, the chosen degree of the polynomial 3 was determined for the given data as optimal.

The described type of the fuel cell is used, along with others, at the department of the authors of this article. The model is, however, universal and suitable for computer experiments with similar types of fuel cells. The created model is valid within the framework of the voltage, current and temperature ranges prescribed by the manufacturer. While measuring, the power loss of the resistor needs to be regarded.



| Command Window       |                     |                    |
|----------------------|---------------------|--------------------|
| R <sub>o</sub> (Ohm) | T <sub>o</sub> (oC) | V <sub>o</sub> (V) |
| 65.0000              | 5.0000              | 7.9373             |
| K1                   |                     | K2                 |
| -0.2131              | 1.5357              |                    |
| 2.5569               | 1.3358              |                    |
| 5.8869               | 1.0674              |                    |
| 6.6569               | 1.0140              |                    |
| 6.9569               | 1.0188              |                    |

Fig. 10 List of selected parameters of algorithm in MATLAB environment

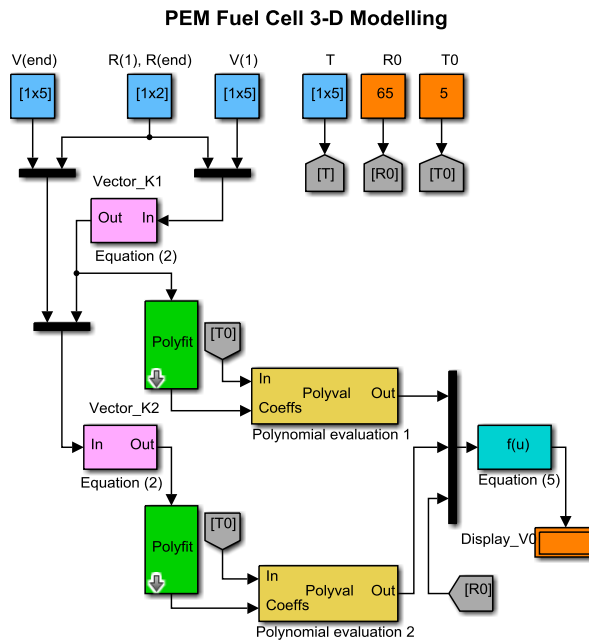


Fig. 11 Detailed structure of the block model subsystem

#### 4. Conclusion

The article describes the original algorithm of the fuel cell Horizon H-30 model building in 3-D. It has been based on the real measured data of the load voltage-resistance-temperature characteristic, ergo  $V = f(R, T)$ . The essence of modelling is the use of two different approximation functions. The basic characteristics  $V = f(R)$ ,  $T = const.$  are approximated by a modified natural logarithmic function  $\ln$ . For the movement of the operating point in the plane  $R = const.$ , the polynomial approximations are used. The model is implemented in MATLAB (text form) and

---

Simulink (block approach) environments. The model enables us to calculate the output voltage of the fuel cell  $V_0$  for any input values of resistance and temperature  $R_0$  and  $T_0$  in a given measured scale. The mathematical bases of the algorithm are described along with a numerical example. Approximation errors are not discussed.

### Acknowledgements

The work presented in this paper has been supported by research program of Brno University of Defence, DZRO K217+K215, Czech Republic. MATLAB<sup>®</sup> and Simulink<sup>®</sup> are registered trademarks of The MathWorks, Inc.

### References

- [1] TÖPLER, J. and LEHMANN, J. *Hydrogen and Fuel Cell*. London : Springer, 2015. p. 180.
- [2] REVANKAR, S. T. and MAJMUDAR, P. *Fuel Cells: Principles, Design and Analysis*. CRC Press, 2014. p. 250.
- [3] STOLTEN, D. and EMONTS, B. *Fuel Cell Science and Engineering: Materials, Processes, Systems and Technology*. London : John Wiley&Sons, 2012.p. 230.
- [4] ZAPLATILEK, K. and LEUCHTER, J. Optimal polynomial approximation of photovoltaic panel characteristics using a stochastic approach. *Advances of Military Technology*, 2013, vol. 8, no. 2, p. 43-51.
- [5] JEMEI, S. A. A new modeling approach of embedded fuel cell power generators based on artificial neural network. *IEEE Transactions on Industrial Electronics*, 2008, vol. 55, no. 1, p. 437-447.
- [6] PASRICHA, S. A. Dynamic PEM fuel cell model. *IEEE Transactions on Energy Conversion*, 2006, vol. 21, no. 2, p. 484-490.
- [7] ISHAKU, J., LOTFI, N., ZOMORODI, H., LANDERS, R. G. Control-oriented modeling for open-cathode fuel cell systems. In *Proceedings of the American Control Conference (ACC)*. Portland, OR, 2014, p. 268-273.
- [8] KARTHIK, M. Design, modeling and simulation study of a cascade optimal neural network based fuel cell powered vehicle. In *Proceedings of the IEEE 2<sup>nd</sup> International Conference on Electrical Energy Systems (ICEES)*. Chennai, 2014, p. 71-76.
- [9] HERB, F., AKULA, P. R., TRIVEDI, K., JANDHYALA, L. Theoretical analysis of energy management strategies for fuel cell electric vehicle with respect to fuel cell and battery aging. In *Proceedings of the World Electric Vehicle Symposium and Exhibition (EVS27)*. Barcelona, 2013, p. 1-9.
- [10] YU-JEN, C., JING-LUN, S. An analytical overpotential model of a direct liquid-feed fuel cell. In *Proceedings of the 10<sup>th</sup> IEEE International Conference on Power Electronic and Drive Systems (PEDS)*. Kitakyushu (Japan), 2013, p. 924-929.
- [11] *Horizon Fuel Cell Technologies* [on line]. Singapore [cited 2016-01-01]. Available from: <<http://www.horizonfuelcell.com>>.
- [12] *Vötsch industrial technique* [on line]. Wien, Austria [cited 2016-01-01]. Available from: <<http://www.v-it.com>>.