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Mathematical Modeling of the Design Process of Additional Aerodynamic Surfaces of the Wing as a Component of the Configuration of a Commercial Airliner

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Abstract:

This paper introduces the methodology and the method of automated design of additional aerodynamic surfaces of the wing as a component of the configuration of a commercial airliner, based on the suggested mathematical model. The mathematical model makes it possible to solve a particular task of developing and designing an additional aerodynamic surface of the wing for a commercial airliner and to select the required type of additional aerodynamic surface for a certain aircraft. The decision on the required type of additional aerodynamic surface for a certain aircraft is made using a specialized software product, based on the implementation of a multi-criterial approach to the selection of the method in the process of defining the set tasks. The automation of the design process of additional aerodynamic surfaces of the wing as a component of the configuration of a commercial airliner is achieved by means of the developed software product, written in an object-oriented high-level programming language.

Keywords:

Additional aerodynamic surfaces, wing-tip mounted winglets, mathematical modelling, parametric synthesis, commercial airliner, structural optimization, automation of conceptual design, development and design procedures, model of characteristics

1. Introduction

Currently, the development and construction of new types of aircrafts and the improvement of the existing aircrafts (AC) and the components of their configuration according to chosen criteria should be performed in an automated way. The result of

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automated design is a set of design solutions [1-3]. When defining the rational parameters of additional aerodynamic surfaces (AAS) of the wing as a component of the configuration of a commercial airliner, in accordance with the criterion of technical and economic efficiency, a fast computing system for automated layout design is required. Utilizing such a system will allow for the optimization of the technical and flying characteristics of a commercial airliner in accordance with a selected criterion, and for defining the indicators of the functional efficiency of implementing new design and technological solutions.

Consequently, the goal of this research is to develop a method of automated design of a commercial AC, the configuration of which would feature the AAS of the wing, based on a formalized description of the design procedures and on utilizing the modern methods of modeling and engineering analysis.

The solution of the task requires using an effective design-computing system, equipped with the subsystems of specialized software, making it possible to carry out automated calculations of technical and flying characteristics of an AC, taking into account the AAS of the wing, which needs to be defined at the stage of preliminary design.

The proposed methodology and the developed method of automated design of the AAS of the wing as a component of the configuration of a commercial AC are based on the developed mathematical model. The functional layouts are obtained on the basis of the structural optimization of an AC, while the parameters of AC components and the technical and flying characteristics of AC operation modes in the process of air transportation and other tasks are defined, with regard to the pre-set limitations, by means of parametric synthesis.

2. The methodology

The purpose of this research is the automation of the design of the AAS of the wing as a component of the configuration of a commercial airliner. The design process is described by a mathematical model, which gives the opportunity to proceed from solving separate problems to developing and designing an entire complex system. The object of the research is the process of designing the AAS of the wing as a component of the configuration of a commercial AC. The subject of the research is the formalization of the AAS design process of the wing as a component of the configuration of a commercial airliner. The methodology of the research is based on the methods of systems analysis, mathematical modeling, statistical analysis, modern information technologies, structural and parametric synthesis, and mathematical programming.

The mathematical model allows to solve the problem of the optimization of the design process of the AAS of the wing for a commercial AC, and also, to make a selection of the required type of AAS for a certain AC. After uncovering the characteristics, affecting the design of the AAS of the wing as a component of the configuration of an airliner, it is necessary to focus on those with the most significant effect, while the function of the model has to be not merely descriptive, since the predictive function of the process is also important [4 - 5]. The mathematical modeling of the design process of the AAS of the wing as a component of the configuration of a commercial AC consists of several stages:

- conceptualizing the rationale behind the mathematical model;
- identifying the model by means of experiments;

- comparing the mathematical and theoretical studies of the model;
- checking whether the model is adequate.

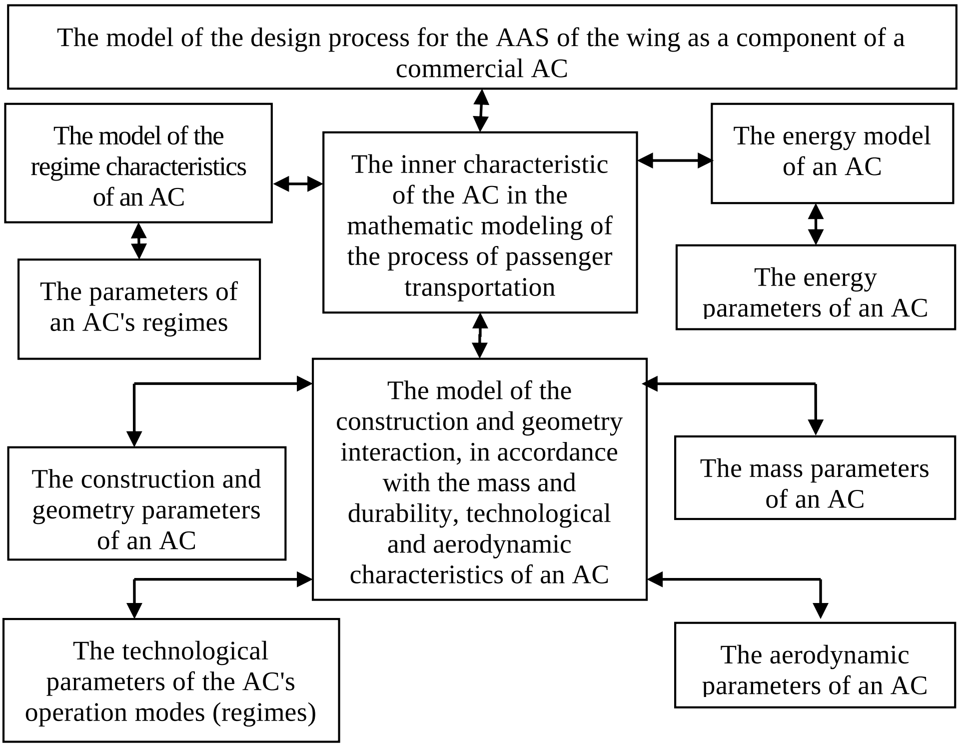

Fig. 1 The structure of the mathematical model of the design process of the AAS *of the wing as a component of the configuration of a commercial* AC

The structure of the mathematical model of the design process of the AAS of the wing as a component of the configuration of a commercial AC, within the framework of the air transportation process, consists of a separate set of characteristics:

- the model of the characteristics of operational modes (regimes) (RD);
- the model of construction and geometry characteristics (CGD);
- the model of mass and durability characteristics (MD);
- the model of energy characteristics (ED);
- the model of technological characteristics (TD);
- the model of aerodynamic characteristics (AD).

Using this structure-generating principle, it is possible to generate the structure of the mathematical model of the design process of the AAS of the wing as a component of the configuration of a commercial AC within the framework of air transportation process (Fig. 1).

Energy characteristics (ED) are the parameters of fuel consumption and the thrust of AC engines. The interaction of construction and geometry characteristics (CGD), of regime characteristics (RD) and aerodynamic characteristics (AD) of the process are viewed as a model of interaction of the structural elements of an air company with an AC. Based on the found connections, we obtain a complex of the efficiency parameters of an AC – the production costs (a_{pcij}), including the fuel consumption per hour *(C_{hour})*, the productivity of an AC (A_{ij}), the intensity of traffic on the route (*N*), which set the relationship with the inner characteristic of an AC – its economic efficiency (*E*).

The structure of the mathematical model of the design process of the AAS of the wing as a component of the configuration of a commercial AC on the basis of the economic efficiency of an AC consists of the target function, the limitations and the variables. Its core is assumed to be the total sum of costs of all the flights on all the routes, at which the profit parameter is still maintained [6 - 8].

3. The main part

Developing and designing the AAS of the wing for a commercial AC is a dynamic process. The changes of the process in question are mirrored by the economic efficiency of an AC, which is in fact a transfer function of “cost-efficiency” [9 - 11].

The task of designing and defining the best AAS for a commercial AC, set as demanding to minimize the production costs in the process of passenger transportation and to fulfil a certain plan, can be represented as the target function, which is calculated by the formula:

$$z = \sum_{i=1}^{n} \sum_{j=1}^{m} c_{pcij} \cdot I_E \rightarrow \min \tag{1}$$

where c_{ij} stands for the production costs on route *i* of type *j*;
I_E is the index of efficiency.

Construction and geometry characteristics CGD, the characteristics of regimes (operation modes) RD, mass and durability characteristics MD, energy characteristics ED and aerodynamic characteristics AD are used as limitations.

$$\text{The limitations} \begin{cases} V_{ij}; H_{ij}; L_{ij}. \\ \lambda_{wgj}; \bar{c}_j; l_{fj}; d_{fj}; S_{mfj}; \lambda_{fj}; \lambda_{ckpj}; \lambda_{t.pj}; A_{hej}; A_{vej}; \\ \lambda_{\text{AAS}}; \eta_{\text{AAS}}; \bar{c}_{\text{AAS}}; \overline{S_{\text{AAS}}}. \\ m_{0ij}. \\ c_{a.e.}; Q_{\max}; r_{xi}. \\ C_{\text{hour}j}; \gamma_{engj}; D_j. \end{cases} \tag{2}$$

The following technological characteristics are used as variables:

$$\text{The variables} \begin{cases} P_{ij}. \\ c_{pc\text{ij}}. \\ Y_j. \end{cases} \tag{3}$$

The maximum revenue of an air company within a period ϑ_{ij}^{pl} depends on the usage of a commercial AC of type *j* with the AAS on route *i* with the minimal production costs and it is calculated by the formula:

$$c^*_{pcij} = \arg\min_{\{x_{ij}\}} \sum_{j=1}^{mn} c_{pcij} I_E = c^*_{pcij}\left(\vartheta_{ij}\right) \quad (4)$$

The suggested mathematical model takes into account the influence of different groups of characteristics of the designed commercial AC with the AAS on its economic efficiency in the process of carrying out aviation tasks and transportation, which is exactly the criterion that most fully reflects the technical effectiveness of a commercial AC and takes into account the connection between the specific fuel consumption and the cost of transportation of one passenger or one unit of payload. The mathematical model serves as the basis for the formulated methodology of the automated design of AAS in particular, as well as of separate components of the configuration of a commercial AC.

On the basis of the suggested methodology, we have developed the method of automated design for the components of the configuration of a commercial AC, grounded on the human-computer interaction, when the heuristic actions of an aircraft designer are complemented by the computational capacities of the computer, which are realized by means of certain algorithms.

The proposed method for the automated design of the components of the configuration of a commercial AC includes the implementation of information processing, application software and algorithmic provision. The software realizes the following stages of the preliminary design:

- the calculation of the construction and geometry characteristics;
- the calculation of the mass characteristics;
- the calculation of the regimes characteristics;
- the calculation of the aerodynamic characteristics;
- the calculation of the durability characteristics;
- the study and calculation of the aerodynamic characteristics;
- the calculation of the ergonomic characteristics;
- the generation of the report on the performance of the automated design software, with the possibility to print and preview it (in the form of the values of the characteristics obtained and as graphical dependencies).

The structural scheme of the method of the automated design of the AAS of the wing as a component of the configuration of a commercial AC is shown in Fig. 2.

The calculation of the groups of characteristics of a commercial AC, the configuration of which includes the AAS, at the stage of preliminary design, is realized by means of software written in the object-oriented high-level programming language C#, which is featured in the product line of the Microsoft Visual Studio.

The preliminary design, involving the interconnection of all the major aspects of AC layout, related to the study of the characteristics mentioned above, is rationally based on the following principles that are included in the software execution: the iterative character of design, the principle of uniformity, and the principle of controllability of each stage. A fragment of the operation algorithm of a software module is shown in Fig. 3.

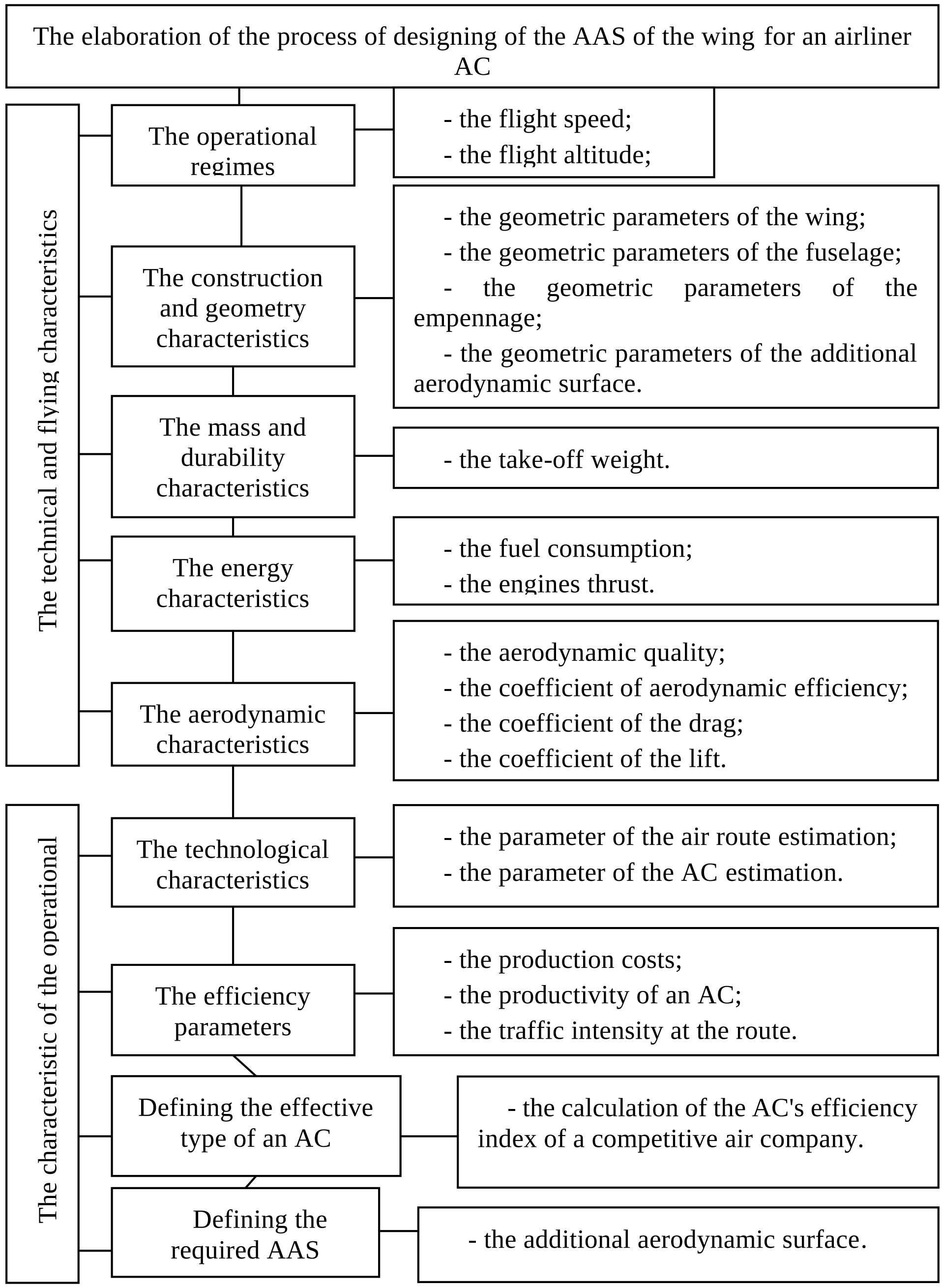

Fig. 2 The structural scheme of the design process of the AAS *of the wing as a component of the configuration of a commercial* AC

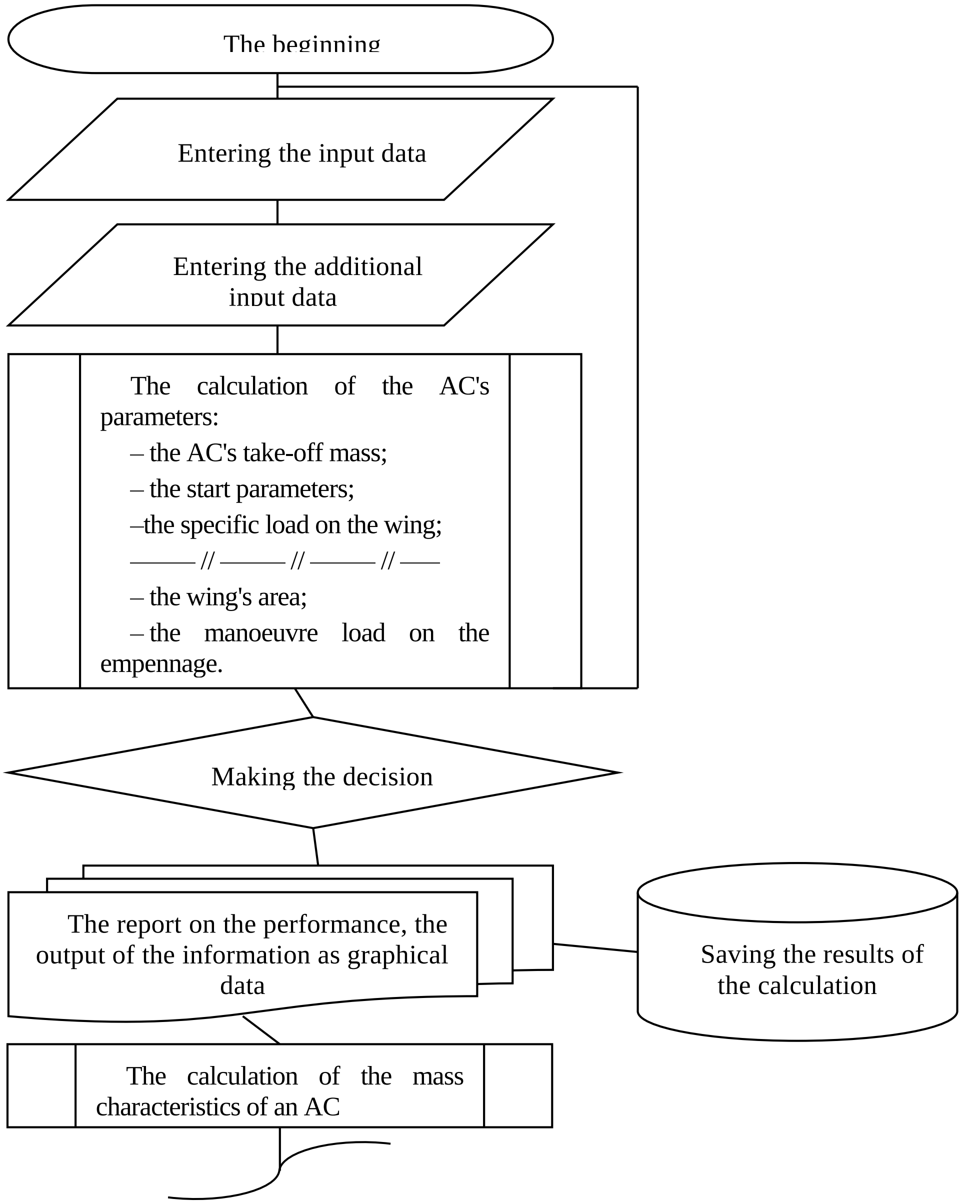

Fig. 3 A fragment of software operation algorithm

The input data is defined by the aircraft designer or by the design specification; the screen for data input is shown in Fig. 4. Each stage of the calculation is realized by means of a separate software module (Fig. 5).

After each stage of calculation of a certain group of characteristics, the user makes a decision on how well the obtained characteristics correspond to the required ones, in accordance with the chosen criterion. If the decision is positive, the system goes on to the next stage of calculation, if the decision is negative, it returns the user to the previous stage for re-calculation [12 - 15]. After the completion of the calculation, the system plots the graphical dependencies and prints the report on the software operation.

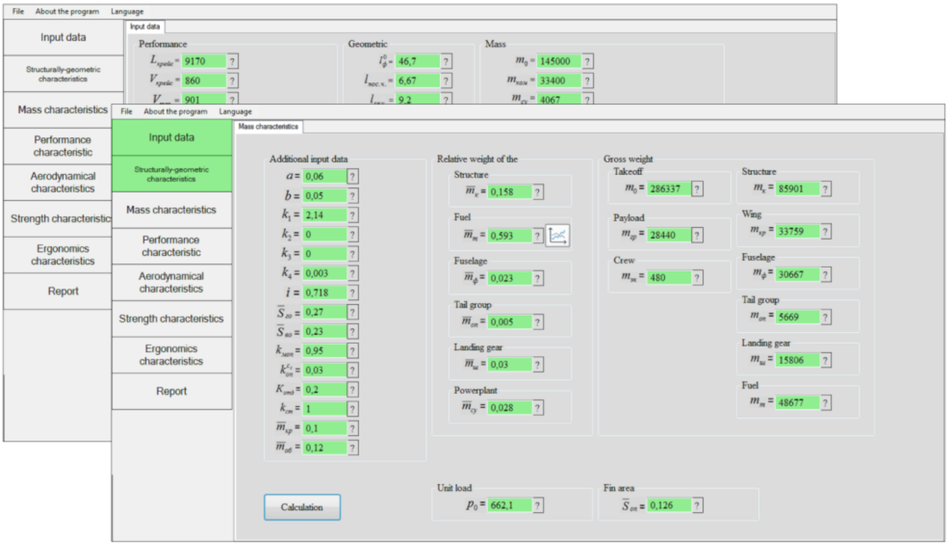

Fig. 4 The dialog window for entering the input data

In accordance with the calculated set of characteristics, the user builds a 3D-model, using a CAD-system of three dimensional modeling, in order to conduct further calculation operations in the programs of computational aero- and hydrodynamics; in case the results of the high-accuracy mathematical modeling do not meet the selected criteria of efficiency, the calculation is corrected by inputting additional initial data.

The choice of the best type of AAS for a commercial AC depends on the economic efficiency of a commercial AC of type *j* on route *i*, which is expressed by the inner characteristic – the efficiency index. By the "economic efficiency" of an AC we mean the effectiveness of AC operation, expressed as the proportion of the useful end results of its functioning to the resources spent. In order to obtain an unambiguous dependence, let us suppose that the revenue of an air company within a period can vary, and it is described by the relationships that are calculated by the formula:

$$\vartheta_{ij}^{pl} \Big/ \vartheta_{ij+1}^{pl} = I_{Eij},\ i=\overline{1,n},\ j=\overline{1,m}. \tag{5}$$

The inner characteristic of an AC is a value that reveals a multitude of specific characteristic parameters of an AC, which act as interrelated. The index of the efficiency of an AC is used as an indicator of AC efficiency. The index of the efficiency of an AC seems necessary from the technological point of view, when used in the process of designing additional aerodynamic surface for a commercial AC. The index of the efficiency of an AC is a linear function of five characteristics, calculated by the formula:

$$I_E = \mathrm{RD} + \mathrm{CGD} + \mathrm{MD} + \mathrm{ED} + \mathrm{TD} + \mathrm{AD} \tag{6}$$

where *RD* stands for the characteristics of regimes, calculated by the formula:

$$\mathrm{RD} = k_{\mathrm{weight}i} \cdot \overline{\mathrm{M}} + k_{\mathrm{weight}i} \cdot \overline{\mathrm{H}} \tag{7}$$

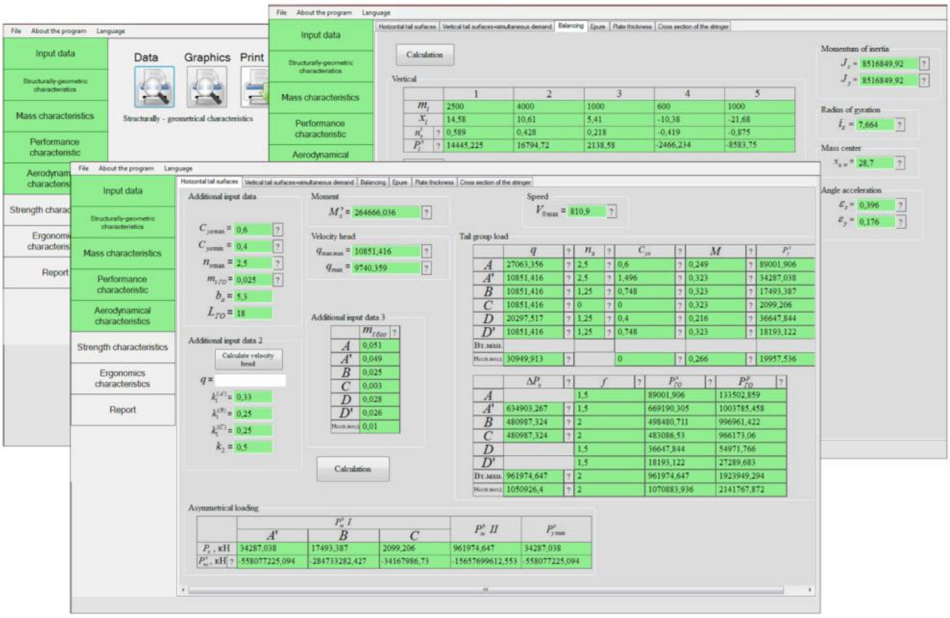

Fig. 5 Screens of calculation of separate groups of characteristics

where $k_{\text{weight}i}$ is the weight coefficient, allocated to the i-th parameter;

$\overline{\mathrm{M}}$ is the flight speed in relative units;

$\overline{\mathrm{H}}$ is the flight altitude in relative units.

CGD – the construction and geometry characteristics, calculated by the formula:

$$\mathrm{CGD} = k_{\text{weight}i} \cdot \overline{l_{wg}} + k_{\text{weight}i} \cdot \overline{l_f} + k_{\text{weight}i} \cdot \overline{d_f} + k_{\text{weight}i} \cdot \overline{\lambda_f} + k_{\text{weght}i} \cdot \overline{\lambda_{mw}} + \\ + k_{\text{weight}i} \cdot \overline{l_{\text{AAS}}} + k_{\text{weight}i} \cdot \overline{\lambda}_{\text{AAS}} + k_{\text{weight}i} \cdot \overline{\eta_{\text{AAS}}} \qquad (8)$$

where $\overline{l_{wg}}$ is the wing span in relative units;

$\overline{l_f}$ – the length of the fuselage in relative units;

$\overline{d_f}$ – the fuselage diameter in relative units;

$\overline{\lambda_f}$ – the fuselage elongation in relative units;

$\overline{\lambda_{mw}}$ – the mid-wing elongation in relative units;

$\overline{l_{\text{AAS}}}$ – the AAS span in relative units;

$\overline{\lambda}_{\text{AAS}}$ – the AAS elongation in relative units;

$\overline{\eta_{\text{AAS}}}$ – the AAS taper in relative units.

MD – the mass and durability characteristics are calculated by the formula:

$$\mathrm{MD} = k_{\text{weight}i} \cdot \overline{m_0} + k_{\text{weight}i} \cdot \overline{m_{pl}} \qquad (9)$$

where $\overline{m_0}$ – is the normal take-off mass of an AC in relative units;

$\overline{m_{pl}}$ – the payload mass of an AC in relative units.

ED – the energy characteristics are calculated by the formula:

$$\text{ED} = k_{\text{weight}i} \cdot \overline{\text{C}_{\text{hour}}} + k_{\text{weight}i} \cdot \overline{m} + k_{\text{weight}i} \cdot \overline{\gamma_{\text{eng}}} + k_{\text{weight}i} \cdot \overline{D} \quad (10)$$

where $\overline{\text{C}_{\text{hour}}}$ – is the fuel consumption per hour in relative units;

$\overline{m}$ – the bypass ratio of the engine in relative units;

$\overline{\gamma_{\text{eng}}}$ – the specific weight of the engine in relative units;

$\overline{\text{D}}$ – the maximal diameter of the engine in relative units.

TD – the technological characteristics, calculated by the formula:

$$\text{TD} = k_{\text{weight}i} \cdot \overline{a_{pc}} + k_{\text{weight}i} \cdot \overline{P} + k_{\text{weight}i} \cdot \overline{Y} \quad (11)$$

where $\overline{a_{pc}}$ – the production costs in relative units;

$\overline{P}$ – the parameter of the air route assessment in relative units;

$\overline{Y}$ – the parameter of the AC assessment in relative units;

AD – the aerodynamic characteristics, calculated by the formula:

$$\text{AD} = k_{\text{weight}i} \cdot \overline{c_{a.e.}} + k_{\text{weight}i} \cdot \overline{Q_{\max}} + k_{\text{weight}i} \cdot \overline{r_{xi}} \quad (12)$$

where $\overline{c_{a.e.}}$ – the coefficient of aerodynamic efficiency in relative units;

$\overline{Q_{\max}}$ – the maximal aerodynamic quality in relative units;

$\overline{r_{xi}}$ – the inductive resistance of an AC with the AAS in relative units.

All the components of the index have equal specific weights, as otherwise it would be necessary to use another assessment method, for instance, the method of expert estimations. All the parameters are directly related to the indicator of the efficiency of an AC; at the same time, some of the indicators of the index have an adverse effect on the efficiency of an AC.

In order to obtain the efficiency index of an AC, we need to put it in some comparable form. For this, we shall employ the method of linear scaling. Its essence is in reflecting the value of each parameter within the range from 0 to 1, while preserving all the proportions between separate values. Thus, all the structural characteristics of the initial parameter are preserved.

The scaled value is calculated by the formula:

$$x_i^s = \frac{(x_i - x_{\min})}{(x_{\max} - x_{\min})} \quad (13)$$

where x_i is the value observed;

$x_{\min}$ – the minimal value of the examined parameter;

$x_{\max}$ – the maximal value of the examined parameter.

In case the directly measured parameter is negatively associated with the efficiency of an AC, reversed linear scaling is applied.

The mathematical model of the parametric synthesis establishes the connections between the sets of RD, CGD, MD, ED, TD, AD and the set of the efficiency parameters, selected for the process through the inner characteristic of an AC, E_1 – the economic efficiency:

1) the production costs – a_{pcij};
2) the productivity of an AC – A_{ij}.

The production costs for one flight on route *i* of an AC of type *j*, are calculated by the formula:

$$a_{pcij} = a_{ij} + a_{cap.inv.j} \tag{14}$$

where a_{ij} is the cost price of transportation, in rubles.

The specific fuel consumption, one kilogram of fuel per one newton of thrust per hour, is calculated by the formula:

$$C_{C.cruise} = 0.95\left[\left(0.82/(1+0.525\sqrt[3]{m})\right) + M\left(0.494 - 0.0145H\right)\right], \tag{15}$$

where m – the bypass ratio of the engine;
M – the M number of the flight;
H – the altitude of the flight, km.

The productiveness on route *i* of an AC *j* with the AAS *m* is calculated by the formula:

$$A_{ij} = m_{\mathrm{pay}j} / t_{ij} \tag{16}$$

where $m_{\mathrm{pay}j}$ is the flight payload, corresponding to a given flight distance;
t_{ij} is the time of flight, h.

The same calculation is performed for every type of an AC for a given route, with different types of the AAS of the wing, utilizing the developed software (No. 2013613814) [16]. As a result, we receive the efficiency index for each type of an AC on a given route with specified AAS. The obtained results allow to make a selection of a required type of the AAS for a certain AC; in order to do it, a comprehensive analysis of the problematic situation is carried out, utilizing computational devices. Due to the fact that the efficiency index includes numerous groups of characteristics and parameters, the software allows to pick the characteristics with the necessary parameters, which are currently important.

In this case, we apply a multi-criteria approach to the selection of the method in the process of defining the set tasks, employing a number of methods, such as:

- the method of averaging individual estimations;
- the method of additive reconciliation of individual values;
- the method of group analytical hierarchy;
- the technique for order preference by similarity to ideal solution (TOPSIS);
- the method of aggregating paired comparisons.

In the process of working out the solution on choosing the required type of AAS for a particular AC, the software was utilized that allows to unify all the methods above. The screen of the developed software is shown in Fig. 6.

Using the specialized software when choosing the AAS type for a commercial AC, we can define the required type of the AAS for carrying out certain kinds of air transportation and other tasks.

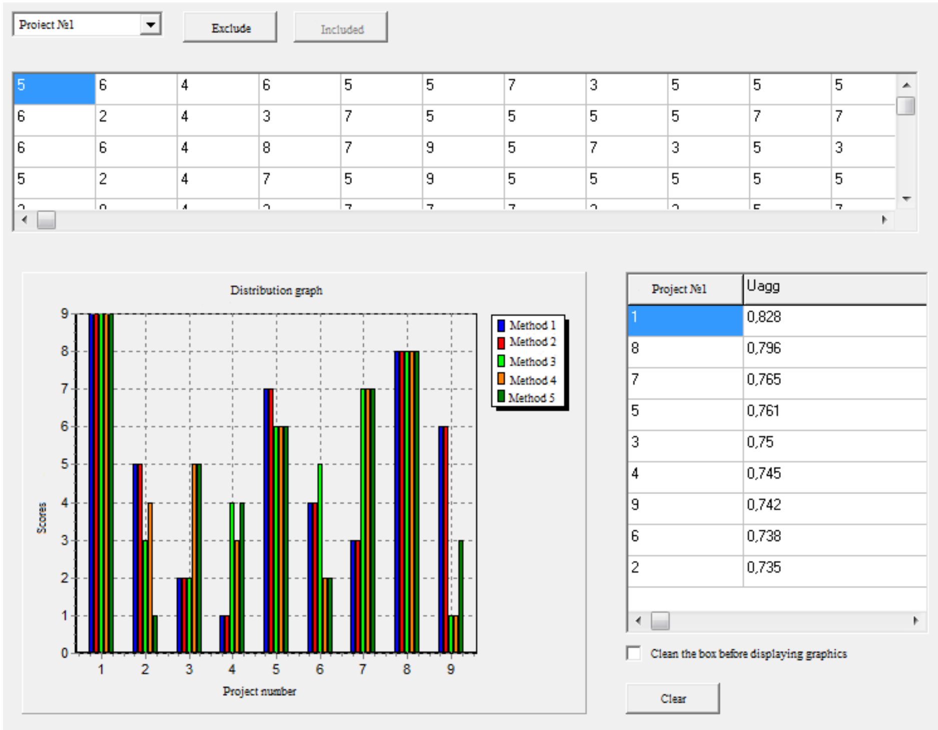

Fig. 6 Dialog window of software for decision making on type of AAS for a particular AC (Method TOPSIS, competence factor – 0.2, weight ration criterion – 0.091)

4. Conclusions

The proposed software allows to carry out complex, multi-variant iteration calculations for the new constructions of the AAS at the stage of preliminary design, regarding them as a component of the configuration of a commercial AC, to guarantee the high quality of design solutions and to shorten the terms of developing, designing and introducing into service specialized variants of flying machines. The selection of the required type of AAS is made on the basis of rational, calculations-based aerodynamic characteristics of the AAS for a specific type of an AC.

5. Findings

1. The method of automated design of the AAS of the wing as a component of the configuration of a commercial AC, based on a mathematical model, has been implemented in the form of specialized information and application software and algorithms.
2. The software is a functionally complete tool, capable of storing and translating information; the result of its work is a design solution, using which the user can build a 3D-model of the AAS and use it for carrying out the further computation in the programs of computational aero- and hydrodynamics.
3. The efficiency index reflects the inner characteristic of the process of choosing the required type of the AAS of the wing of a commercial AC on the basis of economic

efficiency and with regard to the aerodynamic efficiency, which interacts with all the parameters of an AC. Utilizing a specialized software tool makes it possible to determine the required type of the AAS with a particular set of characteristics and given parameters that are currently relevant for carrying out certain kinds of air transportation and other tasks.

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The results of the research have been accepted for further implementation in the CJSC "The inter-branch center for innovation KAI-Engineering" (the city of Kazan) and in the OJSC "Aviatechpriemka" (Moscow).

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