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Underwater Bearings-Only Passive Target Tracking Using Estimate Fusion Technique

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Abstract:

Estimate Fusion Technique (EFT) for Bearings–Only passive target tracking involves a process of estimating the state of a moving target by fusing the estimates given by different Nonlinear estimators which are driven by different Bearing measurements supplied by towed array. The estimates are fused with the help of a Weighted Least squares Estimator. This novel method has an advantage over the traditional nonlinear Estimators such as the Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) in terms of estimation errors which is proved in this paper by performing simulation in Matlab R2009a for a wartime scenario.

Keywords:

Estimate Fusion Technique, Bearings–Only Passive Target Tracking, Towed Array, Extended Kalman Filter, Unscented Kalman Filter.

1. Introduction

Tracking (a process of estimating the present and future state of a moving target) is an essential signal processing concept in war environment to remain safe or to blast the foe. It normally involves estimation of the target motion parameters i.e. range, bearing, course and velocity with the help of the noisy measurements i.e. range and bearing in case of active tracking and only bearing in case of passive tracking. Depending on the

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relative position of the sensors with respect to the propeller of the observer's vehicle, the intensity of the noise in the measurements varies and the measurements can be classified as hull Mounted and towed array. In detail, if the sensor lies on the body of the ship, it experiences more noise (because sensor is close to the propeller). These sensors are termed as hull Mounted sensors (hull meaning body). On the other hand the sensors which are far from the body of the ship experiences less noise (because sensor is far from the propellers noise). These sensors are termed as the towed sensors (tow means drag). In this paper the passive target tracking is done using towed array measurements.

Tracking a target with active sonar measurements,which involves the linear state and measurement equations is dealt with traditional Kalman filter equations 5.17, 5.18 and 15.19 of [2]. Here the assumption is that the measurement noise in rectangular coordinates has a mean of zero even after the transformation of measurements from polar system to rectangular system as shown in [8].The performance of Kalman filter is improved by precise computation of the mean and covariance of the sensor noise after the system transformation, followed by subtraction of the calculated mean from the measurements. This new Kalman filter for active tracking process with the removal of the bias from the measurements showed a great promise according to results shown in [8].

Tracking a target with passive measurements, where the measurement equation is nonlinear is dealt with the conventional nonlinear estimators, such as Extended Kalman Filter (EKF) in modified polar coordinates. This filter linearizes a nonlinear measurement equation using the Taylor series expansion [6]. The EKF when applied to Bearings-Only Tracking (BOT) struggles to perform well occasionally in terms of convergence of estimation error. This is not acceptable in war environment. The solution to this problem is given by Modified Gain Extended Kalman Filter (MGEKF) where a modified gain function in covariance matrix of the state vector is introduced [5].

Uhlman and Juliers Unscented Kalman Filter (UKF) [3], which works on the principle of unscented transformation of estimate and covariance over a nonlinear function, made BOT life easier as shown by Kotewara Rao et al. [7]. BOT with towed array measurements using UKF showed its upper hand over EKF in [4]. Occasionally some hybrid methods and additional input filters came and showed their importance for tracking. An example of hybrid method is given in [9] where Pseudo Linear Estimator (PLE) and MGEKF are combined together to result in a new filter with an improved performance. [10] shows an example of additional input filters where it is assumed that the Doppler measurement is also available in addition to range and bearing measurements.

Recently the particle filter (PF) and its derivatives [1, 2] entered almost all fields of engineering, like BOT where the estimation is a primary requirement. The popularity of PF-based algorithms is due to their capability of handling highly nonlinear state and measurement equations and their ability to deal with any type of noise at the cost of large computational time and sophisticated processor requirements.

A novel method based on fusion of estimates is proposed in this paper. In this technique, multiple measurements supplied by different sensors of towed array are applied to different filters; for example: UKF, which in turn produces different estimates along with the confidence levels in the estimates. These estimates are fused together using a weighted least squares estimator to get the final estimate of the state of a moving target. The block diagram of this approach is shown in Fig. 3. Based on the principle of operation, this filter can produce improved performance in estimation error over EKF and UKF as shown in Fig. 1 and less computational cost than PF as shown in Fig. 2.

Section 2 deals with the mathematical modelling of a moving target, towed array, Estimate Fusion technique (EFT), EFT based UKF expressions and at the end, the performance comparison parameters namely RMS error in position, RMS error in velocity and Estimator convergence time are defined. Simulation and explanation of results for a wartime scenario is shown in Section 3 and finally the paper is concluded in Section 4.
Notation *X(aïb) = X(a/b)* in the paper = the value of *X* at time 'a' considering the measurement at time 'b'.

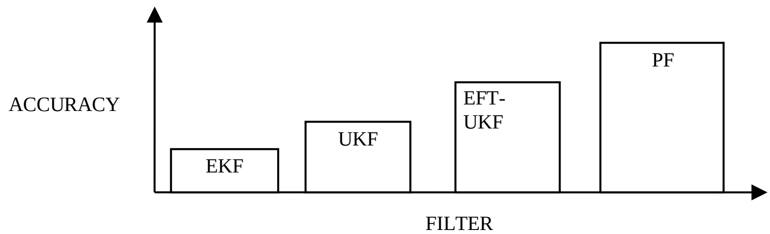

Fig.1 Comparison of the Nonlinear Estimators in Terms of Accuracy

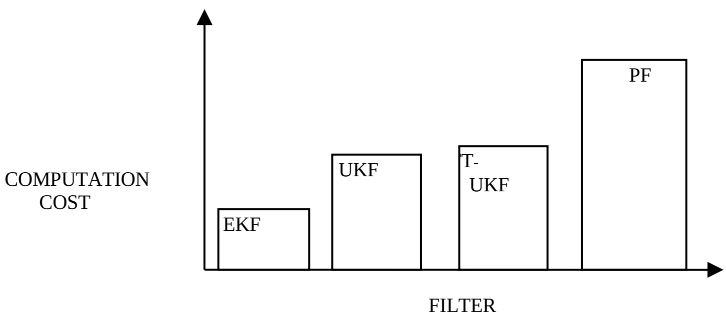

Fig. 2 Comparison of the Nonlinear Estimators in terms of Computational Cost.

2. Mathematical Modelling

2.1. Mathematical Modelling of a Moving Target

The components of the position of a target at time 'k' in x and y directions are denoted by x(k), y(k) and the components of the velocity of a target at time 'k' in x and y directions are denoted by $vx(k)$ and $vy(k)$. State vector as shown in (1) can be considered for tracking process.

$$X(k) = [x(k) \quad y(k) \quad vx(k) \quad vy(k)]^T \tag{1}$$

The state equation of a moving target as per linear, discretized wiener velocity model is shown in (2)

$$X(K+1) = FX(K) + Q(k) \tag{2}$$

F is the state transition matrix= $\begin{bmatrix} 1 & 0 & \Delta T & 0 \\ 0 & 1 & 0 & \Delta T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Where ΔT is the time interval between the states, Q(k) is the Gaussian process noise with a mean zero and a co-variance as given in eqn.13 of [4] is as follows

$$E[Q(k)Q(k)^T] = \begin{bmatrix} \Delta T^3/3 & 0 & \Delta T^2/2 & 0 \\ 0 & \Delta T^3/3 & 0 & \Delta T^2/2 \\ \Delta T^2/2 & 0 & \Delta T & 0 \\ 0 & \Delta T^2/2 & 0 & \Delta T \end{bmatrix} q \tag{3}$$

Where q is the spectral density of the acceleration errors.

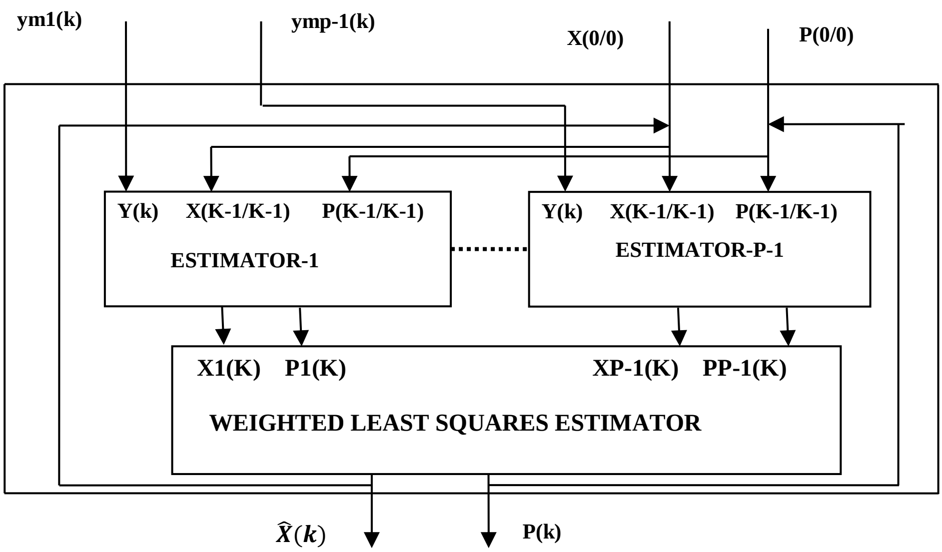

Fig. 3 Block diagram of EFT-UKF

2.2 Mathematical Modelling of Towed Array

Let (x(k), y(k)) be the targets position coordinates at time ‘k’. The towed array is comprised of two sensors S1 and S2 located at (S1(1), S1(2)), (S2(1), S2(2)) respectively. The azimuth or the bearing at time ‘k’ as viewed at S1and S2 denoted by B1(k), B2(k) is computed by using the geometry in the Fig. 4.

$$B1(k) = \arctan\left(\frac{y(k)\text{-}S1(2)}{x(k)\text{-}S1(1)}\right) \tag{4.1}$$

$$B2(k) = \arctan\left(\frac{y(k)\text{-}S2(2)}{x(k)\text{-}S2(1)}\right) \quad (4.2)$$

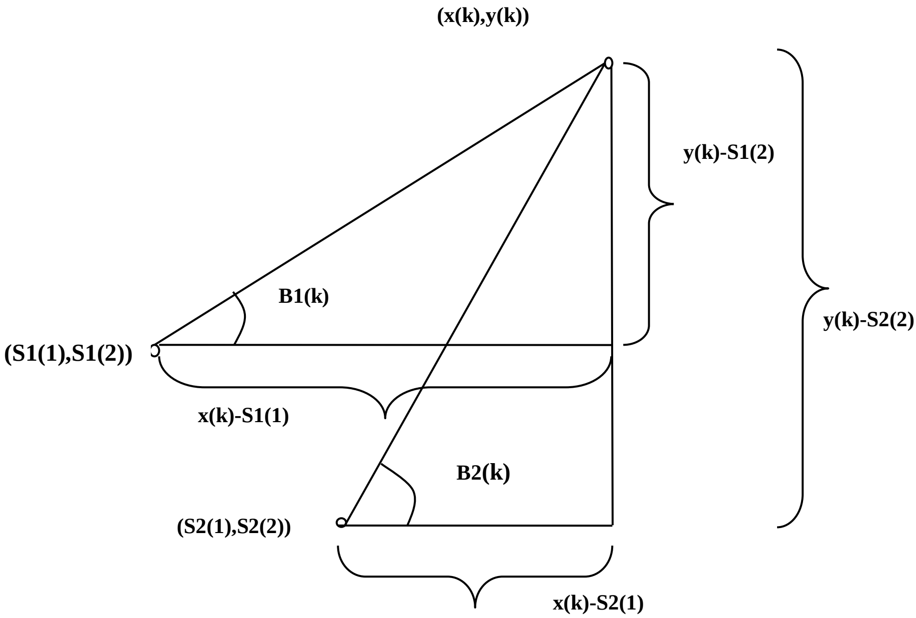

Fig 4 Geometry to calculate azimuth or bearing.

The noise corrupted bearing measurements at S1 and S2 denoted by Bm1(k) and Bm2(k) are expressed as

$$Bm1(k)=B1(k)+Bn1(k) \quad (5.1)$$

$$Bm2(k)=B2(k)+Bn2(k) \quad (5.2)$$

If the measurement vector is considered as $y(k)=\begin{bmatrix} Bm1(k) \\ Bm2(k) \end{bmatrix}$ (6)

The measurement equation can be written as $y(k)=h(x(k),k)+v(k)$ (7)

The transfer function is denoted by h(x(k), k)) and is computed using (8)

$$h(x(k),k))=\begin{bmatrix} arctan\left(\frac{y(k)\text{-}S1(2)}{x(k)\text{-}S1(1)}\right) \\ arctan\left(\frac{y(k)\text{-}S2(2)}{x(k)\text{-}S2(1)}\right) \end{bmatrix} \quad (8)$$

$v(k)$ in (7) is a sensor noise of type 'Gaussian' with a null mean and co-variance R of the form as shown in (9)

$$R=E[\ v(k)\,v(k)^T\ ]=diag\left(\sigma_1^2,\sigma_2^2\right) \tag{9}$$

Where σ_1^2,σ_2^2 are the variances of noise in the measurements given by sensors S1 and S2.

2.3 Estimate Fusion Technique

If sensors which are numbered 1 and j+1 of towed array supply a measurement $ymj(k)$ using (6). Then $yml(k)$ with l=1, 2... p-1 are the measurements obtained from p elements of the towed array with the corresponding transfer functions $hl(x(k),k)$ obtained using (8), covariance matrices Rl obtained using (9). Different UKFs will process these measurements to produce $\overline{\text{Xl}}(\text{k\"{ı}k})$ with l=1, 2.... p-1 as estimates with the corresponding covariance matrices as Pl(kïk). Matrix $\overline{\text{Xl}}(\text{k\"{ı}k})$ is of the form as shown in (10)

$$\overline{Xl}(k\ddot{ı}k)=\left[\overline{xl}_{(1)}(k\ddot{ı}k)\quad \overline{xl}_{(2)}(k\ddot{ı}k)\quad .\quad .\quad \overline{xl}_{(n)}(k\ddot{ı}k)\right]^T \tag{10}$$

The i^{th} element of the estimate at k^{th} instant of time considering the k^{th} measurement given by the l^{th} UKF is of the form $\overline{\text{xl}}_{(1)}(\text{k\"{ı}k})$. Covariance matrix of the estimate is denoted by $pl(k\ddot{ı}k)$ and can be expressed in the form of (11)

$$pl(k\ddot{ı}k)=\text{diag}\left[pl1\quad pl2\quad .\quad .\quad pl\,n\right] \tag{11}$$

p_{li} notation in (11) denotes l^{th} UKFs variance in the estimate of i^{th} element of its state. The estimates given by all the p-1 UKFs i.e. ($\overline{\text{x}}1_{(\text{i})}(\text{k\"{ı}k})$ $\overline{\text{x}}2_{(\text{i})}(\text{k\"{ı}k})$ .... $\overline{\text{x}}(p-1)_{(\text{i})}(\text{k\"{ı}k})$) are fused to give the consolidated estimate $X_i(k\ddot{ı}k)$. The suffix i indicates the i^{th} element of the estimate.

$$\begin{bmatrix} \overline{x1}_{(i)}(k\ddot{ı}k) \\ \overline{x2}_{(i)}(k\ddot{ı}k) \\ . \\ . \\ \overline{xp-1}_{(i)}(k\ddot{ı}k) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ . \\ . \\ 1 \end{bmatrix}_{(p-1)x1} X_i(k\ddot{ı}k)+l_i(k) \tag{12}$$

$l_i(k)$ is Gaussian noise of dimensions (p-1)x1, mean of zero and covariance $R_{(i)}$. It is the error in i^{th} element of the estimate.

$$R_{(i)}=\text{diag}(\ p_{1i}\quad p_{2i}\quad .\quad .\quad p_{(p-1)i}\ ) \tag{13}$$

Eq. (13) is inherited from Eq. (11) as follows

If we are trying to fuse two estimator outputs with four elements in the state vector. Then (11) will be in the form of

$$p1(k\ddot{\imath}k) = \begin{bmatrix} p_{11} & 0 & 0 & 0 \\ 0 & p_{12} & 0 & 0 \\ 0 & 0 & p_{13} & 0 \\ 0 & 0 & 0 & p_{14} \end{bmatrix}$$

$$p2(k\ddot{\imath}k) = \begin{bmatrix} p_{21} & 0 & 0 & 0 \\ 0 & p_{22} & 0 & 0 \\ 0 & 0 & p_{23} & 0 \\ 0 & 0 & 0 & p_{24} \end{bmatrix}$$

with $p1(k\ddot{\imath}k)$ and $p2(k\ddot{\imath}k)$ as the covariance matrices associated with the estimates given by the first and the second UKF respectively.

Then the covariance matrix associated with the first element is denoted by $R_{(1)}$

$$R_{(1)} = \begin{bmatrix} p_{11} & 0 \\ 0 & p_{21} \end{bmatrix} \quad R_{(2)} = \begin{bmatrix} p_{12} & 0 \\ 0 & p_{22} \end{bmatrix} \quad R_{(3)} = \begin{bmatrix} p_{13} & 0 \\ 0 & p_{23} \end{bmatrix} \quad R_{(4)} = \begin{bmatrix} p_{14} & 0 \\ 0 & p_{24} \end{bmatrix}$$

From the above we can write that the covariance corresponding to the ith element is $R_{(i)} = diag(p_{1i}, p_{2i})$ for 2 estimator fusion. Similarly for p-1 estimator fusion $R_{(i)} = \mathrm{diag}(\ p_{1i} \quad p_{2i} \quad . \quad . \quad p_{(p-1)i}\ )$ which is Eq. (13). In this way Eq. (13) is inherited from Eq. (11).

Now Eq. (12) is rewritten as $\quad Y_{(i)}(k\ddot{\imath}k) = HX_{(i)}(k\ddot{\imath}k) + l_i(k) \qquad (14)$

$$\text{Where } Y_{(i)}(k\ddot{\imath}k) = \begin{bmatrix} \overline{x1}_{(i)}(k\ddot{\imath}k) \\ \overline{x2}_{(i)}(k\ddot{\imath}k) \\ . \\ . \\ \overline{xp-1}_{(i)}(k\ddot{\imath}k) \end{bmatrix} \qquad (15)$$

$$\text{And } H = \begin{bmatrix} 1 & 1 & . & . & 1 \end{bmatrix}^T_{(p-1)x1} \qquad (16)$$

The least squares estimator, Eq. 3.15 of [2] is used to obtain the estimate of $X_i(k\ddot{\imath}k)$ of (14). It is denoted by $\overline{X}_{(i)}(k/k)$.

$$\overline{X}_{(i)}(k\ddot{\imath}k) = \left(H^T R_{(i)}{}^{-1} H\right)^{-1} H^T R_{(i)}{}^{-1} Y_{(i)}(k\ddot{\imath}k) \qquad (17)$$

Eq. (17) is rewritten as Eq. (18) using Eq. 3.19 of [2]

$$\overline{X}_{(i)}(k|k) = \left(\sum_{j=1}^{p-1} \frac{1}{p_{ji}^{2}} \right)^{-1} \left(\sum_{j=1}^{p-1} \frac{Xj_{(i)}(k|k)}{p_{ji}^{2}} \right) \tag{18}$$

The fusion-based estimate taking k^{th} measurement into consideration denoted by $\overline{X}$(k/k) is composed using (18)

$$\overline{X}(k|k) = \left[\overline{x}_{(1)}(k|k) \quad \overline{x}_{(2)}(k|k) \quad . \quad . \quad \overline{x}_{(n)}(k|k) \right]^{T} \tag{19}$$

The covariance in the fused- estimate is as follows

$$P(k|k) = \frac{1}{p-1} \sum_{i=1}^{p-1} Pi(k|k) \tag{20}$$

2.4 Estimate Fusion Technique based UKF(EFT-UKF) Equations

Identify the state vector *X(k)* of n elements. Then the state equation expressed in the form of (21) and measurement equation in the form of (22) are derived.

$$\textit{State equation}: \quad X(k+1) = FX(k) + w(k) \tag{21}$$

$$\textit{Measurement equation}: \; ym(k) = h(x(k),k) + v(k) \quad w(k) \sim (0,Q) \;\; v(k) \sim (0,R) \tag{22}$$

There after the initialization of EFT-UKF is performed, which involves the initialization of state as shown in (23) and initialization of the covariance as shown in (24).

$$\overline{X}(0|0) = E[X(0)] \tag{23}$$

$$P(0|0) = E[(X - \overline{X}(0|0))(X - \overline{X}(0|0))^{T}] \tag{24}$$

As the state equation is linear in nature,a priori estimate of the state and the covariance are computed using the prediction step of normal kalman filter.

$$\overline{X}(k|k-1) = F\overline{X}(k-1|k-1) \tag{25}$$

$$P(k|k-1) = FP(k|k-1)F^{T} + Q \tag{26}$$

Correction step starts with the computation of *2n+1 sigma points* using the following

$$X^{0}(K) = \overline{X}(k|k-1)$$

$$X^{i}(K) = \overline{X}(k|k-1) - \left[\sqrt{(n+\lambda)P(k|k-1)} \right]_{i^{th}row}^{T} i = 1,2,3\ldots n$$

$$X^{n+i}(K) = \overline{X}(k|k-1) - \left[\sqrt{(n+\lambda)P(k|k-1)} \right]_{i^{th}row}^{T} i = 1,2,3\ldots n \tag{27}$$

Where λ *is a scaling parameter given by*

$$\lambda = \alpha^{2}(n+ka) - n$$

The weights corresponding to the sigma points are computed using the following relations

$$w_m^0 = \frac{\lambda}{n+\lambda}$$

$$w_c^0 = \frac{\lambda}{n+\lambda} + 1 - \alpha^2 + \beta$$

$$w_m^i = \frac{1}{2(n+\lambda)} i = 1,2 \ldots 2n$$

$$w_c^i = \frac{1}{2(n+\lambda)} i = 1,2 \ldots 2n \tag{28}$$

Where α, β and ka are the parameters used in the algorithm.

The 2n+1 sigma points are transformed over the nonlinear measurement equation i.e. Eq. (22)

$$ym^i(k) = h(x^i(k), k) \quad i = 1,2,3 \ldots 2n. \tag{29}$$

A posterior estimate computation starts by taking the weighted average of the transformed sigma points as given below

$$\overline{ym}(k) = \sum_{i=0}^{2n} w_m^{(i)} ym^{(i)}(k) \tag{30}$$

variances corresponding to the a posterior estimate denoted by P_y is computed according to Eq. 14.65 of [2] as follows

$$P_y = \sum_{i=0}^{2n} w_c^{(i)} \left[ym^i(k) - \overline{ym}(k) \right] \left[ym^i(k) - \overline{ym}(k) \right]^T + R \tag{31}$$

Crosscovariances of the a posterior estimate denoted by P_{xy} is computed according to Eq. 14.66 of [2] as follows.

$$P_{xy} = \sum_{i=0}^{2n} w_c^{(i)} \left[X^{(i)}(k) - \overline{X}(k\ddot{\imath}k-1) \right] \left[ym^i(k) - \overline{ym}(k) \right]^T \tag{32}$$

The Kalman gain kk is calculated using Eq. 14.67 of [2] as follows

$$kk = P_{xy} P_y^{-1} \tag{33}$$

Finally the a posterior estimate denoted by $\overline{X}(k\ddot{\imath}k)$ is computed according to Eq. 14.67 of [2] as follows.

$$\overline{X}(k\ddot{\imath}k) = \overline{X}(k\ddot{\imath}k-1) + kk(ym(k) - \overline{y}m(k)) \quad (34)$$

Covariance matrix corresponding to the a posterior estimate denoted by $P(k\ddot{\imath}k)$ is computed according to Eq. 14.67 of [2] as follows.

$$P(k\ddot{\imath}k) = P(k\ddot{\imath}k-1) + (kk)P_y(kk)^T \quad (35)$$

Equations (23) to (35) are the UKF equations applied for all the p-1 measurements (*ymj(k) j=1,2,...p-1*) obtained from the sensors of the towed array to produce $\overline{X}j(k/k)$, $Pj(k/k)$ *with j=1,2,......p-1* as the aposterior estimates and covariances respectively using the corresponding transfer functions *hj(x(k),k)*. The least squares estimator is used to fuse all the p-1 estimates to give $\overline{X}(k\ddot{\imath}k)$, as fused estimate with $P(k/k)$ as the corresponding covariance matrix.
The fused estimate is composed of n elements as follows

$$\overline{X}(k\ddot{\imath}k) = \left[\overline{x}_{(1)}(k\ddot{\imath}k) \quad \overline{x}_{(2)}(k\ddot{\imath}k) \quad . \quad . \quad \overline{x}_{(n)}(k\ddot{\imath}k)\right]^T \quad (36)$$

Where the i^{th} element of the fused estimate is derived in estimate fusion technique i.e. section 2(c) and is shown below in (37)

$$\overline{X}_{(i)}(k\ddot{\imath}k) = \left(\sum_{j=1}^{p-1} \frac{1}{p_{ji}^2}\right)^{-1} \left(\sum_{j=1}^{p-1} \frac{Xj_{(i)}(k\ddot{\imath}k)}{p_{ji}^2}\right) \quad (37)$$

If we assume all the weights are equal, then the fused estimate can be written as

$$\overline{X}(k\ddot{\imath}k) = \frac{1}{p-1} \sum_{i=0}^{p-1} \overline{X}i(k\ddot{\imath}k) \quad (38)$$

The related covariance matrix is given below

$$P(k\ddot{\imath}k) = \frac{1}{p-1} \sum_{i=1}^{p-1} Pi(k\ddot{\imath}k) \quad (39)$$

2.5 Performance Comparison Parameters
(i) Root mean square error in position: as given in Eq. 6.104 of [1]

$$\text{RMS position error } (k) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left(\left(x_i(k) - \overline{x}_i(k)\right)^2 + \left(y_i(k) - \overline{y}_i(k)\right)^2 \right)} \quad (40)$$

'N' in the above equation denotes the total number of Montecarlo (MC) runs. The actual components of position in x and y directions at the instant of time 'k' in the MC run 'i' are denoted by $x_i(k)$, $y_i(k)$, while the estimated components are represented by $\overline{x}_i(k)$, $\overline{y}_i(k)$.

(ii) Estimator convergence time: It is the duration spent by the estimator to drag root mean square error in position to less than three hundred metres.

(iii) Root mean square error in velocity:

$$\text{RMS velocity error(k)}=\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\left(vx_i(k)-\overline{vx_i}(k)\right)^2+\left(vy_i(k)-\overline{vy_i}(k)\right)^2\right)} \quad (41)$$

$vx_i(k)$ and $vy_i(k)$ are the true velocity components while $\overline{vx_i}(k)$ and $\overline{vy_i}(k)$ are the corresponding estimated components.

3. Simulation, Results and Analysis

War time scenario: The target is assumed to be initially at a distance of 18 km and a bearing of 100 degrees with respect to the true north. Thereafter it tries to move at a constant velocity of 10 meters per second in the direction of true north but it is disturbed to some extent due to the acceleration errors (in both x and y directions) of Gaussian type with a mean of zero and a standard deviation of 0.01. The towed array used is provided with three sensors which are located at (0, 0), (0, 500) and (0, 1000). These sensors have the capability of providing the bearing measurements to the signal processing unit of the submarine after every 1 second. The measurements are not pure as expected. They are mixed with some additive noise of Gaussian type. The mean and the variance of this Gaussian noise is assumed to be 0 and 0.28 degree r.m.s respectively. The UKFs whose estimates are fused are initialized with a deviation (+3000 m in x,-3000 m in y in terms of position and +5 m/s in x, -5 m/s in y in terms of velocity) from the true values. The covariance matrix is initialized with a diagonal matrix [900 k, 900 k, 25, 25]. The scaling parameter α is set to a value very close to zero while β is set to 2, which is an optimal value while dealing with Gaussians. The total number of Montecarlo runs performed is 50 with the simulation duration of 800 seconds. In this the authors have fused the estimates of UKFs and the resultant is named as Estimate Fusion Technique based UKF (EFT-UKF).

The processor requirements of KF, EKF, UKF and the proposed EFT-UKF are nearly the same (complexity of all the above mentioned algorithms is same) while PF requires nearly 1000 times more sophisticated processor as per [1] and [2].

In Fig. 7 the estimated path by EFT-UKF is almost superimposed over the actual path of the target indicating the success of the EFT-UKF.

Fig. 6 shows that the estimation error in velocity using EFT-UKF gradually tends to the smallest value compared to those of UKF and EKF after the transition period suggesting the improvement attained by using Estimate fusion technique. The superiority of this proposed method is well illustrated by Fig. 5 and Tab. 1 which shows that the RMS position error and convergence time of the estimator of the novel method is much smaller than those of the conventional nonlinear estimators.

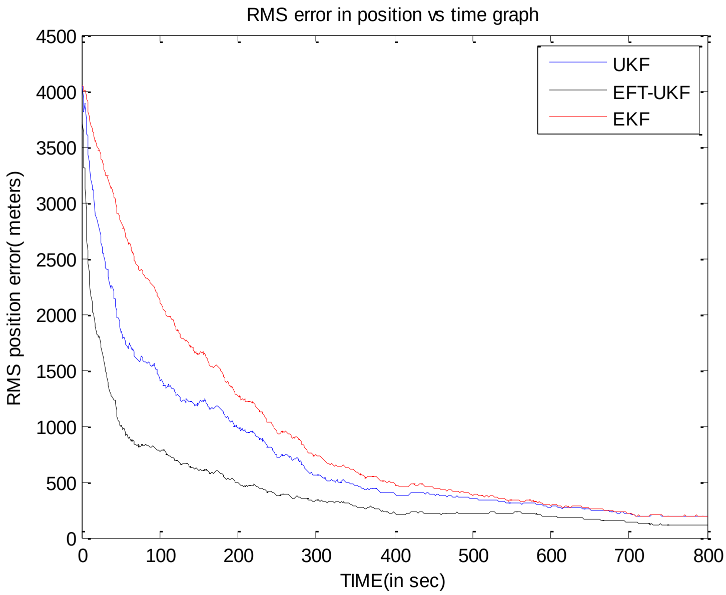

Fig. 5 Comparison of RMS position errors

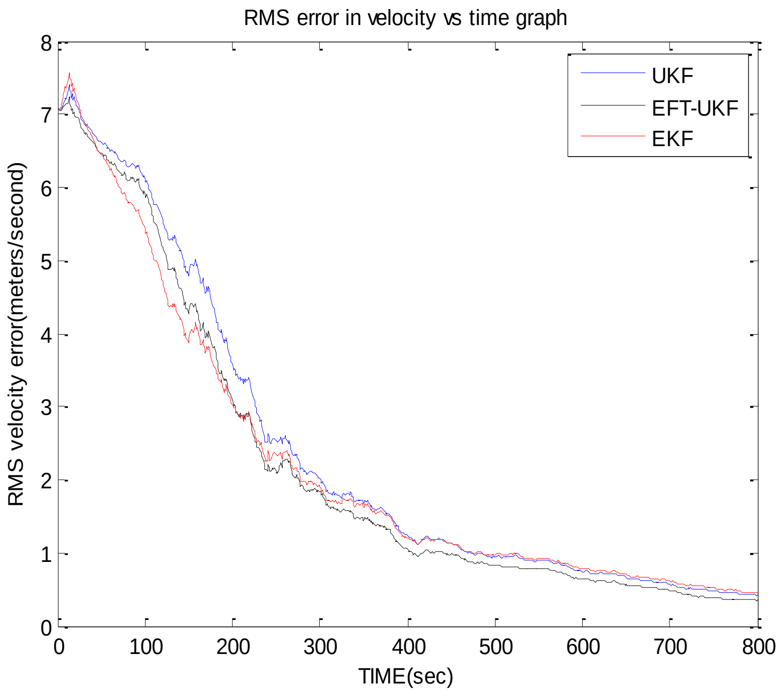

Fig. 6 Comparison of RMS velocity errors

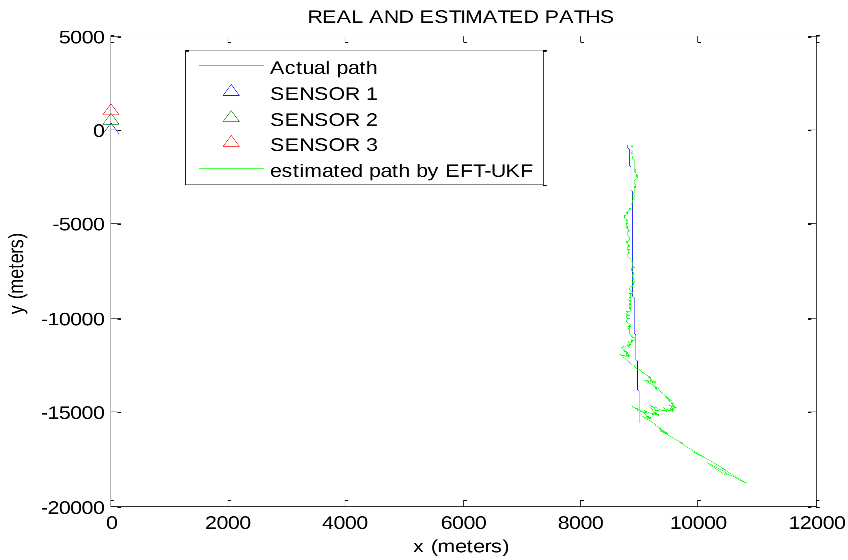

Fig. 7 Actual path and estimated path provided by EFT-UKF

Tab. 1 Comparison table of nonlinear Estimators

Filter	**RMS position error (m)**	**RMSE velocity error (m/s)**	**Estimator Convergence Time(s)**
EKF	194	0.457	606
UKF	194	0.424	583
EFT-UKF	**108**	**0.353**	**343**

4. CONCLUSIONS

EFT based UKF has a superior performance over the conventional nonlinear estimators such as EKF and UKF in terms of the estimation errors in position and velocity and at the same time it needs less computational requirements than PF which suggests that, the estimate fusion technique can provide an optimal solution for tracking a moving target in passive mode. The performance of all the existing nonlinear estimators can be sent to a new level by employing the EFT.

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