AiMT Advances in Military Technology
Vol. 10, No. 2, December 2015

Application of Fuzzy Graph Models for Finding Rational Route

N. Lytvynenko[1*], O. Lytvynenko[2]

1 Military institute of Taras Shevchenko National University of Kyiv
2 Military institute of Taras Shevchenko National University of Kyiv

The manuscript was received on 19 August 2015 and was accepted after revision for publication on 18 December 2015.

Abstract:

A commander solves a lot of problems which contain uncertainties during the planning and organization of the army unit march due to the impact of combat situation factors and meteorological conditions. The best model for the problem solving of finding routes for movement of military units between two points is the marked graph. The article is devoted to the question of finding the rational (in the sense of the most comfortable) route through fuzzy graph models.

Keywords:

Rational route, Army, movements, march, membership function, graph.

1. Introduction

Many military technical problems, including determining the route during the army march organization, are resolved by methods of graph theory. However, it is difficult to put into practice the solutions obtained by using graph theory because of the incompatibility of the graphic representation of result from the modern military command and control technology, based on computers. Therefore, the theoretical work is required for effective representation of graph theory methods in analytical form that is equally quite easy for a person and for a computer, for example, through the vertexes' neighbourhoods and limits.

Also it should be noted that the meteorological conditions that can change soil and water obstacle properties has a great influence on the route defining and it is difficult to formalize them. In turn, this causes fuzziness in the graph model of the route choice.

For a formalized description of fuzzy situations, so-called fuzzy relations should be used which describe the degree of knowledge about the investigated objects (in this case – graphs) via the membership function. Usage of fuzzy graph models enables a formal

[*] *Corresponding author: Ministry of Defense, Military institute of Taras Shevchenko National University of Kyiv, Lomonosov' Street 81, Kyiv, Ukraine, 03680, phone: +38044 5213289, fax: +38044 5213292, E-mail: n123n@ukr.net*

definition of terrain passability for military technique that depends on weather conditions and the choice of the most comfortable route.

2. Formulation of the Problem

A march is an organized movement of troops in columns on roads and column's ways for timely arrival in the determined area or line in full readiness for next actions. Tracked vehicles and vehicles with a small cruising reserve can be transported by trains (trailers) that are included into a structure of columns.

A march can take place in battle or piece conditions, from rear to front, along front or from front to rear. In all cases, a march takes place in secret, usually at night or in other low visibility conditions, and during a battle and in a deep rear of own troops – even in the afternoon [1].

To solve the problem of finding routes for movements of army units between two points, the best model is a marked graph. The presentation of graph in the form of the vertexes and edges, which connect some of these vertexes, is the most common. A directed graph is a pair of sets $L=(X,F)$, where $X=\{x_i\}$, $i \in I=\{1,2,\dots,n\}$ – set of graph's vertexes;, $<x_i,x_j>\in X^2$ – set of edges, moreover x_i – the beginning and x_j – the end of edge.

Supposing, for example, there is the graph $\varphi=(X_1,F_1)$, where $X_1=\{x_1,x_2,x_3,x_4\}$, $F_1=\{<x_1,x_2>,<x_1,x_4>,<x_3,x_4>,<x_3,x_2>,<x_4,x_2>\}$. This graph is represented in Fig. 1.

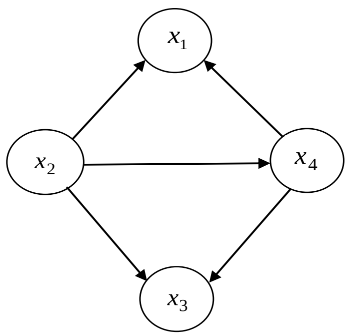

Fig. 1 Graphic presentation of graph

A graph can be marked by different indicators:

1) the distance (route's length on edge);

2) the time of edge passing;

3) the membership function.

Different definitions of route finding between two determined points are available for different types of graph marking:

a) the shortest route is a minimization of route edges sum marked by distance;

b) the fastest route is a minimization of edges sum marked by time, this problem coincides with the previous problem's algorithm;

c) the most comfortable route is a maximization of route's resulting function of belonging under graph marked by the function of belonging.

The problem solving of finding the most comfortable route with the help of fuzzy graph models (via graph marking by the membership function) is proposed in this article.

3. Research Results

The various procedures are proposed for problem solving (1) and (2), including linear programming scheme, such as the Dijkstra algorithm, the Bellman-Ford algorithm, the algorithm of the shortest route in acyclic directed graph, the simplex method, the ellipsoid algorithm, the Karmarkar algorithm, and others, among them [2].

The procedure of finding the shortest route which is described by Sang M. Lee, L.J. Moore, B.W. Taylor [3], is the simplest to use and the fastest to implement.

As an example, let's have a look at the network that is shown in Fig. 2. The figure shows the marked graph, where the tops are matched to the points, and the distances are marked on the edges (matching to roads).

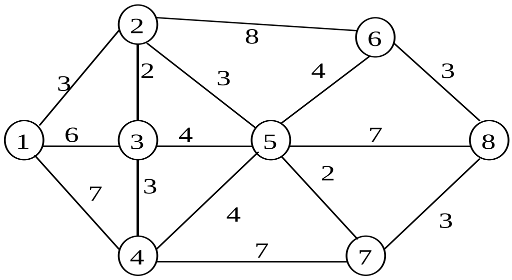

Fig. 2 Network for determining the shortest route

The shortest route from the vertex 1 to the vertex 8 is highlighted on Fig. 3 [2, 3]. The length of this route is D=11.

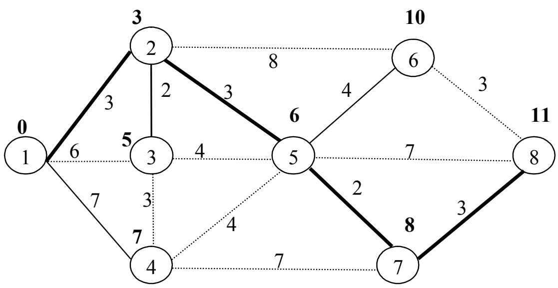

Fig. 3 Network where the shortest route is highlighted

However, special attention should be paid to the problem (3) which relates to the formulation of choice of the most comfortable route. This concept allows to take into consideration the impact of not only one factor (the length of route or time of movement), but of more of them (the value of slope steepness, the passability of routes, the passability of water barriers, the passability of vegetal barriers etc.), while finding the rational route.

As noted above, the most comfortable route is determined via fuzzy graph model. This method is as follows.

It is necessary to specify the graph (Fig. 2) in the form of vertex neighbourhoods [4]. The first neighbourhood S_i^1 of the vertex x_i is the set of end-vertexes for edges, which are incident with x_i, and the vertex x_i.

For this set, the following expression is true (1):

$$\forall x_j \in X \left\{ x_j, x_i \in S_i^1 \leftrightarrow \exists < x_i, x_j > \left[\left(< x_i, x_j > \in F < x_i, x_j > \right) \vee \left(x_i = x_j \right) \right] \right\}, \quad (1)$$
$$i \in I = \{1, 2, \ldots, n\},\ j \in J = \{1, 2, \ldots, m\},$$

where n – the number of the beginning vertex and m – the number of the end vertex on edge.

Then the n-th neighbourhood of vertex x_i by induction is determined as (2):

$$S_i^n = \bigcup_{x_j \in S_i^{n-1}} S_j^1. \quad (2)$$

It means that the n-th neighbourhood of a vertex could be received by adding the set of neighbourhoods to the $n-1$-th, namely the end-vertexes of edges, that are incident with S_i^{n-1}. Where

$$S_i^1 \subseteq S_i^2 \subseteq \ldots \subseteq S_i^n. \quad (3)$$

Let's do the presentation of graph via counting the first neighbourhoods of its vertexes (4):

$$L = \left\{ S_i^1 \middle| \, x_i \in X \right\},$$
$$S_i^1 = \left(\{x_j\}, x_i \right) \quad i \in I = \{1, 2, \ldots, n\},\ j \in J = \{1, 2, \ldots, m\}. \quad (4)$$

For the graph (Fig. 2) we have (5):

$$\begin{aligned} S_1^1 &= \{2, 3, 4, 1\}; \\ S_2^1 &= \{1, 3, 5, 6, 2\}; \\ S_3^1 &= \{1, 2, 4, 5, 3\}; \\ S_4^1 &= \{1, 3, 5, 7, 4\}; \\ S_5^1 &= \{2, 3, 4, 6, 7, 8, 5\}; \\ S_6^1 &= \{2, 5, 8, 6\}; \\ S_7^1 &= \{4, 5, 8, 7\}; \\ S_8^1 &= \{5, 6, 7, 8\}. \end{aligned} \quad (5)$$

To find the rational route, it is necessary to consider a lot of factors that will influence movements of the army units [5]. The main factors which the march depends on are following:

1. the value of slope steepness on a route. A possible permissible speed of military vehicles depends on the slope steepness [6]:

1) less than 5º – all types of vehicles may easy overcome such slope while the soil is dry;

2) 5-10º – the traffic is complicated, the speed reduces;

3) 10-20º – it is very difficult to overcome such slope by wheeled vehicles, the movement of tracked vehicles is complicated;

4) 20-30º – it is critical for all types of wheeled vehicles and such slope is overcome by tracked machines with great difficulties;

5) more than 30º – practically inaccessible for all types of wheeled and tracked vehicles [7].

2. the passability of routes by coverage type (roads and off-road). All routes can be divided into two groups:

- roads (motorways, developed highways, highways);
- dirt roads (developed dirt roads, dirt roads, field and forest roads) [8].

The main factors that affect the dirt road passability are the nature and type of soil. Studying characteristics of soils, it is important above all to describe the terrain conditions for different types of military vehicles. For this purpose areas with the main soil types, their chemical and mechanical texture, hardness, dustiness over the seasons etc. are determined [9].

3. The quantity and type of water barriers depending on season and weather conditions. The main data that must be considered while studying tactical properties of rivers are: width, depth and flow speed of river, soil nature of the bottom, coast and neighboring areas, the presence of fords and hydraulic structures [10].

4. The area and type of vegetal barriers. Forests cause the greatest influence on combat operations among the vegetal barriers. They affect significantly the passability of routes, limit the maneuverability and visibility and complicate the orientation and organization of interaction and the control of military troops. The passability of forests depends on its size, shape and density, species and thickness of trees [9].

The degree of mentioned factors which influence finding a rational route, depends on tactical and technical characteristics of vehicles, seasonal natural events and meteorological conditions which are the source of uncertainty, because this information is unpredictable, incomplete and inaccurate [11]. Therefore, it is advisable to apply fuzzy set theory and to determine the membership function for marking of graph.

Let $X=\{x\}$ – the universal set which is complete and covers all problem area. Fuzzy set $A \subseteq X$ is a set of pairs $\{(x,\mu_A(x))\}$, where $x \in X$ and $\mu_A(\tilde{o}) \to [0,1]$ – the membership functions which describe some subjective degree of element's x adequacy to fuzzy set A. $\mu_A(x)$ can have a value from zero (absolute non-belonging) to one (absolute belonging of element x to fuzzy set A).

If fuzzy set is determined in the universal set $X=\{x_1,x_2,\ldots,x_n\}$, it can be represented as follows:

$$A=\mu_A(x_1)/x_1+\mu_A(x_2)/x_2+\ldots+\mu_A(x_n)/x_n,$$

where $\mu_A(x_i)/x_i$ – a pair of „membership function / element", and symbol „+" means a set of pairs.

For example, let's have a look at Fig. 2 and find the membership functions for neighborhoods sets of graph vertexes. The accepted condition: a column of vehicles consists of URAL-4320 military trucks [12], see Table 1, however, meteorological conditions are stable dry weather.

Table 1 Tactical and technical characteristics of URAL-4320 military truck

Vehicle type	Weight, t	Size, m			Power, hp	Maximum speed, km/h	Cruising reserve, km	Overcoming obstacle	Maximum steepness, grad
		Length	Width	Height				Ford, m	
URAL-4320	8.6	7.37	2.72	2.87	210	85	607	1.5	30

Using the data received after GIS-analysis for routes (roads and off-roads), the slope steepness, water and vegetal barriers (using GIS) in some sections of routes, a group of experts has defined the membership functions for each segment and the indicator of passability, see Tables 2, 3, 4, 5.

Table 2 Membership functions for indicator of routes coverage passability for URAL-4320 military truck (dry soil conditions)

Vehicle type	Hard cover	Broken stone	Gravel	Large sand	Middle size sand	Shallow sand	Dusting sand	Sanding	Loamy	Clayey	Peating
URAL - 4320	1.0	0.8	0.8	0.4	0.4	0.3	0.3	0.4	0.5	0.6	0.3

Table 3 Membership functions for indicator of slope steepness passability for URAL-4320 military truck

Vehicle type	Value of slope steepness				
	less than 5°	5-9°	10-19°	20-30°	more than 30°
URAL - 4320	1.0	0.8	0.5	0.2	0

Table 4 Membership functions for indicator of water barriers passability for URAL-4320 military truck

Vehicle type	Depth of water barriers, m				
	less than 0.3	0.3-0.7	0.8-1.2	1.3-1.5	more than 1.5
URAL - 4320	1.0	0.7	0.5	0.3	0

Table 5 Membership functions for indicator of vegetal barriers passability for URAL-4320 military truck

Vehicle type	Distance between trees, m				
	less than 3.0	3.0-3.9	4.0-4.9	5.0-6.0	more than 6
URAL - 4320	0.2	0.4	0.6	0.8	1

For example, GIS-analysis showed the area (1,2), see Fig. 2, has the following parameters: the route coverage – hard coating, the slope steepness – 10-19°, there is one water barrier with the depth of 0.6 meters, there are no vegetal barriers. The applied formula (6) is:

$$\mu_{\grave{A}}(\chi) = \min_i [\mu_{Ai}(\chi)], \qquad (6)$$

where $\mu_{\grave{A}}(\chi)$ – the membership function of the route segment x, μ_{Ai} – the membership function of the influence factor A_i, $i = \overline{1,n}$ – the number of influence factor, we have (7):

$$\mu_{\grave{A}}(1,2) = \min[1, 0.5, 0.7, 1] = 0.5. \qquad (7)$$

By analogy, the resulting membership functions $\mu_{\grave{A}}(\chi)$ for each route segment are (8):

$$\begin{aligned}
S_1^1 &= \{<0.5/2>, <1.0/3>, <0.9/4>\};\\
S_2^1 &= \{<0.5/1>, <0.9/3>, <0.9/5>, <0.8/6>\};\\
S_3^1 &= \{<1.0/1>, <0.9/2>, <0.7/4>, <0.8/5>\};\\
S_4^1 &= \{<0.9/1>, <0.7/3>, <0.7/5>, <0.7/7>\};\\
S_5^1 &= \{<0.9/2>, <0.8/3>, <0.7/4>, <1.0/6>, <0.5/7>, <1.0/8>\}; \qquad (8)\\
S_6^1 &= \{<0.8/2>, <1.0/5>, <0.5/8>\};\\
S_7^1 &= \{<0.7/4>, <0.5/5>, <0.5/8>\};\\
S_8^1 &= \{<1.0/5>, <0.5/6>, <0.5/7>\}.
\end{aligned}$$

The graph marked by the membership functions is shown in Fig. 4.

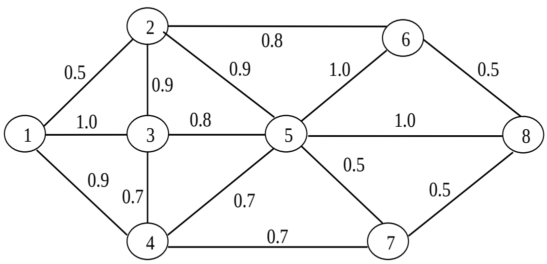

Fig. 4 Graph marked by membership functions

The next step is to find the membership functions for the route's segments of the graph (Fig. 4) using the elements of fuzzy sets theory [13]: for successive sections (a,b), and (b,c) with known $\mu(a,b)$ and $\mu(b,c)$ we have $\mu(\grave{a},c)=\min[\mu(\grave{a},b);\mu(b,c)]$.

For example: $\mu(1{,}6)=[\min(1,2,6); \min(1,2,5,6); \min(1,3,5,6); \min(1,2,3,5,6);$ $\min(1{,}3{,}2{,}5{,}6)]$. Then $\mu(1{,}6)=\max[0.5{,}0.5, 0.8, 0.5{,}0.9]=0.9$.

The final result of finding route via fuzzy graph model (Fig. 5) that has a physical meaning of the route passability lightness has the following form: $\max\mu(1,8)=\mu(1,3,2,5,8)=0.9$.

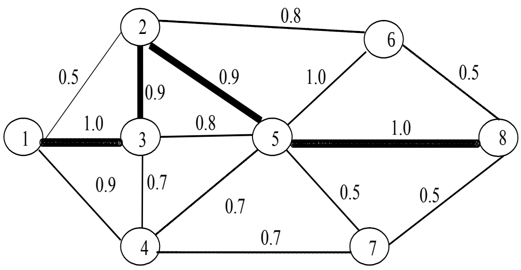

Fig. 5 Final result of finding route via graph model with marking by the membership functions

4. Conclusion

1. Fuzzy graph models are proposed as the apparatus for formalizing the solution of the problem of rational route finding.

The implementation of the proposed models requires software tools that allow automating a design process. In turn, it requires the development of appropriate analytic methods for graph's presentation. The approach to presentation of the graphs through vertexes neighborhoods and limits enables to solve this problem, avoiding redundancy that is common for traditional methods of representation.

2. The various procedures are proposed to solve the problem of finding the shortest routes; the procedure described by Sang M. Lee, L.J. Moore, B.W. Taylor, is the easiest to use and the fastest to do.

3. Fuzzy set theory is the modern mathematical apparatus that enables considering the initial information uncertainty.

4. The marking of graph edges by the membership functions allows finding the most comfortable route. This concept makes possible to consider the impact on choice of the rational route not only of one factor (the length or the time), but of more of them (quantity of slope steepness, passability of route coverage, passability of water barriers or vegetal barriers etc.).

5. The most comfortable route is defined under certain weather conditions (stable dry weather) on the graph marked by the membership functions for a conditional vehicles column (URAL-4320 military truck).

6. The research showed that it is necessary to choose the most comfortable route for the army unit movements because the shortest route, in terms of passability, is not always rational, and its choice as the route of movement can lead to untimely execution

or complete failure of the combat mission because of low ability of vehicles to pass over certain route segments.

Acknowledgement

This paper is a partial output from the defense research project „ISPPR-GIS” managed by the Military institute of Taras Shevchenko National University of Kyiv.

References

[1] *Boyovyy statut Sukhoputnykh viys'k. Chastyna 2. Batal'yon, rota* (in Ukrainian). Kyiv: Komanduvannya Sukhoputnykh viys'k ZS Ukrayiny, 2005, p. 172-184.

[2] *Osnovy modelyuvannya boyovykh diy viys'k: Pidruchnyk* / Kolektyv avtoriv (in Ukrainian). Kyiv: Vyd. NAOU, 2005, p. 481-496.

[3] SANG, M. LEE, MOORE, L.J., TAYLOR, B.W. *Management Science.* Allyn and Bacon, Inc. Boston, 1985, p. 910.

[4] TOLUBKO, V.B., SBITNYEV, A.I., PERMYAKOV, O.Y. *Metodolohichni osnovy proektuvannya prykladnoho prohramnoho zabezpechennya dlya ASU voyennoho pryznachennya: monohrafiya* (in Ukrainian). Kyiv: NAOU, 2004, 249 p.

[5] RYBANSKÝ, M. *Effect of the Geographic Factors on the Cross Country Movement during Military Operations and the Natural Disasters.* In: International Conference on Military Technologies, University of Defence, Brno (Czech Republic), 2007, p. 590 – 596, ISBN 978-80-7231-238-2.

[6] RYBANSKÝ, M., VALA, M. *Relief Impact on Transport.* In.: ICMT’09 - International Conference On Military Technologies, Brno (Czech Republic), 2009, 9 p., ISBN 978-80-7231-649-6 (978-80-7231-648-9 CD).

[7] LAHIN, A.F., BYIZOV, B.E., PRISCHEPA, I.M. *Voennaya topografiya* (in Russian). Moscow: Voenizdat, 1973, p. 7-33.

[8] *Klasyfikator* (in Ukrainian). Kyiv: HUHKK, 2002, p. 11.

[9] POVSHEDNYY, V.A. *Voyenna heohrafiya: navch. posib* (in Ukrainian). Kyiv: NAOU, НАОУ, 2000, p. 15-37.

[10] POVSHEDNYY V.A. *Shchodo vyvchennya mistsevosti pry orhanizatsiyi zahal'noviys'kovoho boyu u vodnoyi pereshkody: navch. posib* (in Ukrainian). Kyiv: NAOU, Kyiv, 2000. 20 p.

[11] RYBANSKÝ, M., VALA, M. *Geographic Conditions of Military Transport Using Roads and Terrain.* In: ICMT’09 - International conference on military technologies 2009, Brno (Czech Republic), 2009, 9 p, ISBN 978-80-7231-649-6 (978-80-7231-648-9 CD).

[12] KAZANTSEV, O.Y., SHVAB, V.K. *Ozbroyennya pidrozdiliv Sukhoputnykh viys'k Zbroynykh Syl Ukrayiny (u skhemakh i tablytsyakh)* (in Ukrainian). Kyiv: Vydavnycho-polihrafichnyy tsentr „Kyyivs'kyy universytet”, 2004, 71 p.

[13] KOFMAN, A.V. *Vvedenie v teoriyu nechetkih mnozhestv* (in Russian). Moscow: Sov. Radio, 1982, 432 p.